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# Theoretical Development of Hydraulic Jump in Trapezoidal Channel Forced by a Broad-Crested Sill

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***Abstract:-*** Applying the momentum equation reveals that there are three key factors that control the hydraulic jump in a trapezoidal channel caused by a sill: the inflow Froude number $Fr_1$, the sequence depth ratio Y, which is the ratio of the downstream depth $h_2$ to the upstream depth $h_1$, the relative upstream flow depth M, and the relative height of the broad-crested sill "S" which is the ratio of the step's geometric height "s" to the height $h_1$ of the jump at its upstream section. The aim of this study is to establish the empirical connections governing the broad-crested sill-induced hydraulic leaps in a trapezoidal channel. The goal is to come up with a concrete representation of the theoretical relationship between $Fr_1$, Y, M, and "S". Applying the momentum equation between the upstream and downstream parts, which limits the leap, enables this. The resulting connection will be contrasted with the traditional hydraulic leap. After that, a comparison with the outcomes of the experiments serves to confirm the theoretical method.

***Keywords:*** hydraulic jump, stilling basin, trapezoidal channel, broad-crested sill, Froude number.

## 1. Introduction

Several works have been presented on hydraulic jumps in rectangular and triangular channels. These studies have analyzed the various parameters characterizing the hydraulic jump such as the sequence depths, the jump characteristic lengths, and the free surface profile.

Concerning hydraulic jumps in rectangular channels, a recent experimental contribution was presented by Achour et al. [1] on the hydraulic jump controlled by a thin sill. The study was based on the determination of the value of the sill position X so that the hydraulic jump is completely formed in the dissipation basin, where Lj is approximately equal to X. The jump compactness in the rectangular channels has been studied by Achour et al. [2]. The authors also analyzed the effect of the jump compactness by considering the length of the classical jump, Lj*. On the same vein, Benmalek et al. [3] analyzed the jump compactness in a compound rectangular channel. This technique offers a technical and economic advantage, in particular during periods of low water levels when the flow has largely been exploited.

Hydraulic jumps in a triangular channel have been experimentally examined by Debabeche and Achour [4]. In this study, investigations were conducted on the main characteristics of controlled and B-minimum jumps under various inflow conditions. A thin-crested or a broad-crested sill was used to ensure jump development. Based on an extensive experimental program, relationships were derived by modeling the effect of the inflow Froude number on the relative sill height, the sequence depth ratio, and the relative sill position. Other works have been presented by Achour and Debabeche [5]. The authors investigated the controlled hydraulic jump by a continuous thin sill in a triangular channel with a 90° opening angle. A relation translating the controlled jump under any formation conditions is presented in a dimensional form to give it general validity.

The hydraulic jump in trapezoidal channels has been widely studied in the past few years. Among the earliest known studies in this area are those of Posey and Hsing [6]. The authors applied the momentum equation between the upstream and downstream sections of the jump. The results obtained were used to calculate the sequence depth ratio as a function of the inflow Froude number. Sandover and Holmes [7], Mohed and Sharp [8], and Ohtsu [9] developed the theoretical and experimental analysis for this type of jump as a function of flow conditions. Later, Wanoschek and Hager [10] studied the internal flow structure of hydraulic jumps in trapezoidal channels. The authors analyzed the main flow parameters such as the sequence depths, the length of the bottom roller, and the characteristic lengths of the phenomenon.

More recently, Afzal and Bushra [11] used Reynolds' equations for the mean turbulent motion in an incompressible two-dimensional flow for any section; the authors obtained the free surface profile and the axial length of the hydraulic jump in a trapezoidal channel. Kateb et al. [12] experimentally examined the positive step effect on the hydraulic jump characteristics in a trapezoidal channel. A comparison study of controlled jump characteristics and type A-forced jump by positive step was proposed. The sill effect in trapezoidal channels was examined by Benmalek et al. [13]. The authors developed empirical formulas relating the sequence depth ratio and the relative sill height as functions of the inflow Froude number. The determination of these parameters allows for sizing the stilling basin.

For this type of channel, the phenomenon of compactness was analyzed by Benmalek et al. [14], and the result obtained shows that the jump compactness has a reducing effect on the jump sequence depths and the sill relative height.

The effect of abrupt enlargement on trapezoidal jumps has been studied by Benmalek and Debabeche [15]; the authors theoretically and experimentally analyzed the enlargement effect on the jump sequence depths, energy loss, and free surface profile. All these studies aim to give an economic dimensioning of trapezoidal stilling basin. The hydraulic jump in a sloped trapezoidal channel is theoretically and experimentally examined by Kateb et al. [16]. The study aims to propose for different configuration of jump, a theoretical relation $Fr_1 = f(Y, \lambda, \alpha)$ expressing the inflow Froude number $Fr_1$ as a function of the angle of inclination $\alpha$of the channel with regard to the horizontal, the sequent depth ratio $Y = h_2/h_1$) and the relative length ($\lambda = Lj/h_1$) of the jump.

A much more compelling substitution relation is thus put forth, demonstrating the detrimental impact of the simplifying hypotheses of the Forster and Skrinde approach and the ensuing flaws in the equation, as a result of a thorough theoretical development free of any simplifying hypotheses.

This study concerns the analysis of the broad-crested sill effect on the forced hydraulic jumps, in a trapezoidal channel. Functional relations have been developed, and the results obtained have been validated by the experimental results.

## 2. Theoretical considerations

The bibliographical review shows that the hydraulic jump is governed by the momentum equation applied between its initial and final sections. In this case, it is necessary to use the momentum equation for a type A hydraulic jump forced by a broad-crested sill developing in a trapezoidal channel, as shown in Fig. 1.The cross-section of the sill is therefore trapezoidal, characterized by its height. The side face of the sill, then, forms an obstacle, which is the seat of a reaction opposed to the flow direction. This results from the pressure acting on the side surface of the sill. This pressure is studied as pressure acting on a vertical flat surface of a trapezoidal geometric shape. The reaction created by the presence of the sill exerts a compressive force Fs on the center of gravity. Using the momentum equation, this force is added to the external forces, ensuring a balance between the upstream and downstream parts of the jump.

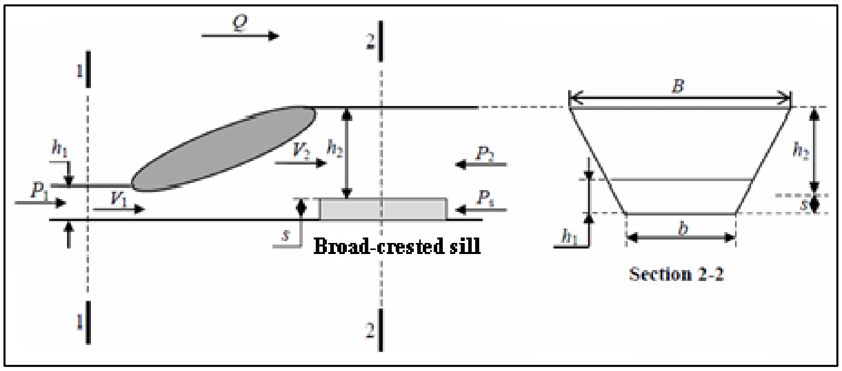



**Figure 1: Graphical representation of the different forces acting on the hydraulic jump forced by a broad-crested sill in a trapezoidal channel**

The momentum equation was applied taking into account the following simplifying assumptions:

• Pressure distribution in sections 1-1 and 2-2 is hydrostatic.

• Friction on the walls and the bottom of the channel is negligible compared to the load loss due to turbulence created by the jump.

• The velocities of the various liquid streams in each of sections 1-1 and 2-2 are parallel to the mean velocity V and are considered uniform.

• Resistance of the area is negligible.

Applying Newton's second law gives:

$$\xi_1 \rho Q V_2 - \xi_2 \rho Q V_1 = P_1 - P_2 - P_S \tag{1}$$

Where: $P_1$, $P_2$ and $P_s$: represents the external forces. These strengths are:

- The hydrostatic pressure force $P_1$ applied to section 1-1.[$Kg.m.s^{-2}$]
- The hydrostatic pressure force $P_2$ applied above the sill in Section 2-2.[$Kg.m.s^{-2}$]
- The reaction $P_s$ generated by the presence of the sill in Section 2-2.[$Kg.m.s^{-2}$]
- $\zeta_1$ and $\zeta_2$ are the momentum correction factors that are considered equal to the unit since the velocity distribution is assumed to be uniform.
- $\rho$ is the density of the moving liquid [$Kg.m^{-3}$]
- Q is the volume flow. [$m^3.s^{-1}$]
- V is the average velocity of the flow. [$m.s^{-1}$]

Considering all its forces, Eq. (1) is written;

$$\rho Q V_1 + P_1 = \rho Q V_2 + P_2 + P_S \tag{2}$$

The point of application of each of these forces coincides with the center of gravity of the section under consideration. According to Fig. 1, all forces are applied to trapezoidal profiles. These forces can be expressed by applying the laws of hydrostatics:

$P_1 = \overline{\omega h_1} A_1$, $P_2 = \overline{\omega h_2} A_2$ and $P_S = \overline{\omega s} A_S$, Where: $\omega$ represents the unit weight of the flowing liquid; $h_1$, $h_2$, and s respectively represent the distance between the center of gravity of the cross sections 1, 2, and s and the free surface of the flow; $A_1$ and $A_2$ are respectively the areas of the wetted sections 1 and 2; and As represents the area of the side face of the sill. By replacing the expressions of $P_1$, $P_2$, and Ps in Eq. (2), it gives:

$$\rho Q V_1 + \overline{\omega h_1} A_1 = \rho Q V_2 + \overline{\omega h_2} A_2 + \overline{\omega s} A_S \tag{3}$$

Thus, the average flow velocities $V_1$ and $V_2$ at the upstream and downstream sections of the jump, respectively, are expressed as: $V_1 = Q/A_1$ and $V_2 = Q/A_2$. Taking all these considerations into account and knowing that $\overline{\omega} = \rho.g$, Eq. (3) can be written by dividing all its members by ρ.g:

$$\overline{h}_1 A_1 + \frac{Q^2}{gA_1} = \overline{h}_2 A_2 + \overline{s} A_S + \frac{Q^2}{gA_2} \tag{4}$$

Based on Fig. 2, the determination of the expressions $A_1$, $A_2$, $A_s$, $\overline{h}_1$, $\overline{h}_2$ and $\overline{s}$ is as follows:

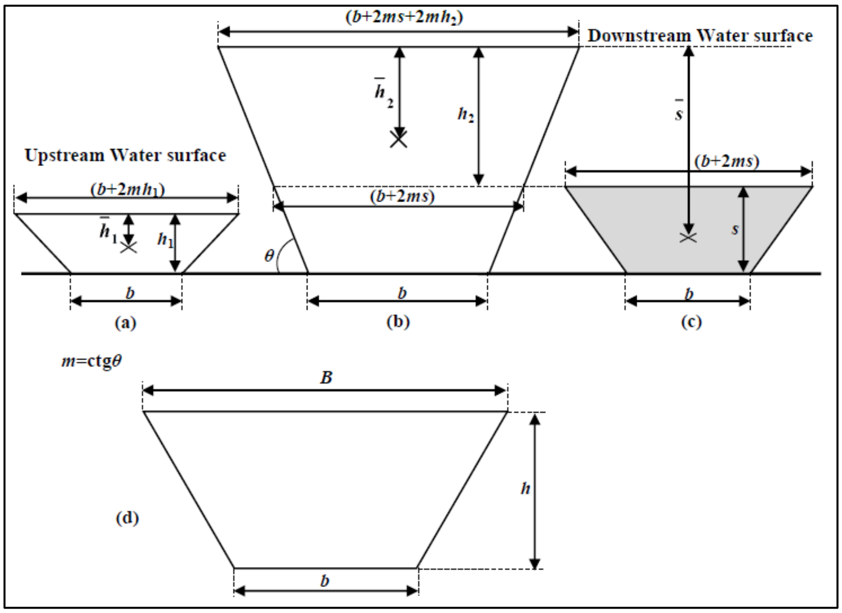



**Figure 2: Geometric characteristics of the trapezoidal section**

Figure 2a shows the geometric characteristics of the trapezoidal channel at the toe of the jump, where $h_1$ is the upstream depth. The parameter m is defined as the cotangent of the inclination angle θ of the trapezoidal channel walls. Geometrically, the section $A_1$ and the distance $h_1$ of the gravity center were given by Eqs. (5) and (6):

$$A_1 = bh_1 + mh_1^2 \tag{5}$$

$$\overline{h}_1 = \frac{h_1^2}{6}\left(\frac{3b + 2mh_1}{A_1}\right) \tag{6}$$

Figure 2b shows the trapezoidal section downstream of the forced jump, where the end of the roller is at the upstream end of the sill. Downstream of the roller, the liquid crosses the step above. The flowing liquid profile above the sill always takes a trapezoidal shape. The area of the section $A_2$ and the distance $h_2$ from the center of gravity are given by Eqs. 7 and 8:

$$A_2 = bh_2 + 2msh_2 + mh_2^2 \tag{7}$$

$$\overline{h}_2 = \frac{h_2^2}{6}\left(\frac{3b + 6ms + 2mh_2}{A_2}\right) \tag{8}$$

The same steps are taken for Fig. 2c, but in this case the distance of the center of gravity from the horizontal axis of the free surface s is the center of gravity specific to the thick sill by adding the downstream water depth $h_2$. So, the area of the trapezoidal section of the sill and the distance s are written as follows:

$$A_S = bs + ms^2 \tag{9}$$

$$s = h_2 + \frac{s^2}{6}\left(\frac{3b + 2ms}{A_S}\right) \tag{10}$$

The hydraulic jump in a suddenly varied flow expresses the flow discharge Q and area of the wetted section A by the dimensional parameter $Fr_1$ or the inflow Froude number upstream of the jump. This number is given generally by the universal relation:

$$Fr_1^2 = \frac{Q^2}{gA_1^3}\frac{\partial A_1}{\partial h_1} \tag{11}$$

Equation (11) shows that the Froude number $Fr_1$ depends on four parameters: Q is the flow discharge through the trapezoidal channel.
G is the acceleration of gravity;
$A_1$ is the area of the section wet at level $h_1$;
$(\partial A_1)/(\partial h_1)$ is the partial derivative of the area of the initial wet section relative to the conjugate depth $h_1$. In reality, the ratio $(\partial A_1/(\partial h_1) = B$ is simply the width of the surface flow.
The partial derivative of $A_1$compared to $h_1$ for a trapezoidal section results in:

$$\frac{\partial A_1}{\partial h_1} = \frac{\partial}{\partial h_1}(bh_1 + mh_1^2) = b + 2mh_1$$

Equation (11) takes the form:

$$Fr_1^2 = \frac{Q^2\left(b + 2mh_1\right)}{gA_1^3} \tag{12}$$

When equations (6), (8), (10), and (12) were inserted into Eq. (4), the following result occurs:

$$\frac{h_1^2}{6}\left[\frac{3b + 2mh_1}{A_1}\right]A_1 + \frac{A_1^3}{\left(b + 2mh_1\right)}\frac{Fr_1^2}{A_1} = \frac{h_2^2}{6}\left[\frac{3b + 6ms + 2mh_2}{A_2}\right]A_2 + \\ + \left[h_2 + \frac{s^2}{6}\left(\frac{3b + 2ms}{A_S}\right)\right]A_S + \frac{A_1^3}{\left(b + 2mh_1\right)}\frac{Fr_1^2}{A_2} \tag{13}$$

After simplifying Eq. (13), it is written:

$$h_1^2\left(\frac{1}{2}+\frac{1}{3}\frac{mh_1}{b}\right)+\frac{h_1^2\left(1+\frac{mh_1}{b}\right)^2}{\left(1+2\frac{mh_1}{b}\right)}Fr_1^2=h_2^2\left(\frac{1}{2}+\frac{ms}{b}+\frac{1}{3}\frac{mh_2}{b}\right)+h_2s\left(1+\frac{ms}{b}\right)+$$
$$+s^2\left(\frac{1}{2}+\frac{1}{3}\frac{ms}{b}\right)+\frac{h_1^3\left(1+\frac{mh_1}{b}\right)^3}{h_2\left(1+2\frac{mh_1}{b}\right)\left(1+2\frac{ms}{b}+\frac{mh_2}{b}\right)}Fr_1^2 \tag{14}$$

## 3. Dimensional analysis

Equation (13) is obtained by applying the momentum equation between sections (1-1) and (2-2) (Fig. 1) of a type A hydraulic jump, forced by broad-crested sill motion in a channel of a trapezoidal section. In practice, the use of equation 13 presents a difficulty for the dimensioning of the dissipation basins because of its implicit and dimensional form, which makes the projection of the physical model to the real model difficult. So, it is necessary to move to the dimensionless form.

The literature shows that the dimensionless rapports can be expressed by the following relationships:

- The sequence depth ratio is $Y = h_2/h_1$.
- The relative height of the sill is $S = s/h_1$.
- The relative upstream flow depth is: $M = (mh_1)/b$.

Equation (14) is written as follows:

$$Fr_1^2=\frac{\frac{(1+2M)}{2(1+M)^2}\left[Y^2\left(1+2MS+\frac{2}{3}MY\right)+2SY(1+MS)+ +S^2\left(1+\frac{2}{3}MS\right)-\left(1+\frac{2}{3}M\right)\right]}{\left[1-\frac{(1+M)}{Y(1+2MS+MY)}\right]} \tag{15}$$

The implicit relation Eq. (15) expresses the variation of the sequence depth ratio as a function of the inflow Froude number, the relative upstream flow depth M and the relative height of the sill S.

In the case of the classical jump (S=0), Eq. (15) is written as follows:

$$Fr_1^2=\frac{\frac{(1+2M)}{2(1+M)^2}\left[Y^2(1+\frac{2}{3}MY)-(1+\frac{2}{3}M)\right]}{\left[1-\frac{(1+M)}{Y(1+MY)}\right]} \tag{16}$$

In the relation Eq. (15), the determination of Y will go through an iterative process; for this, it is necessary to propose an explicit relation which has the form $y = f(Fr_1, S, M)$. Simplifying circumstances of flow were considered in the theoretical development. For this purpose, the mathematical analysis of the experimental results requires a comparison between the theoretical Froude number Eq. (15) and that resulting from the universal relation Eq. (12).

## 4. Comparison of the Froude number resulting from the universal relation with that resulting from the theoretical relation

The relative deviation obtained between the ratio of conjugate depths from Eq. (12) of the theoretical development and that of the approximate relation Eq. (15) is due to the relative differences between the Froude numbers ($Fr_{1th}$ and $Fr_{1exp}$). This requires calculating this relative deviation and to propose an adjustment of the theoretical relationship by the method of least squares (Fig. 3), based on the experimental results.

Figure 3 shows the variation of the experimental Froude number $Fr_{1exp}$ as a function of the theoretical Froude number $Fr_{1the}$. This figure denotes a shift of the point cloud with respect to the first bisector, which increases with increasing Froude number. This shift is attributed largely to the neglect of the singular head loss due to the widening of the channel at the upstream end of the broad-crested sill.

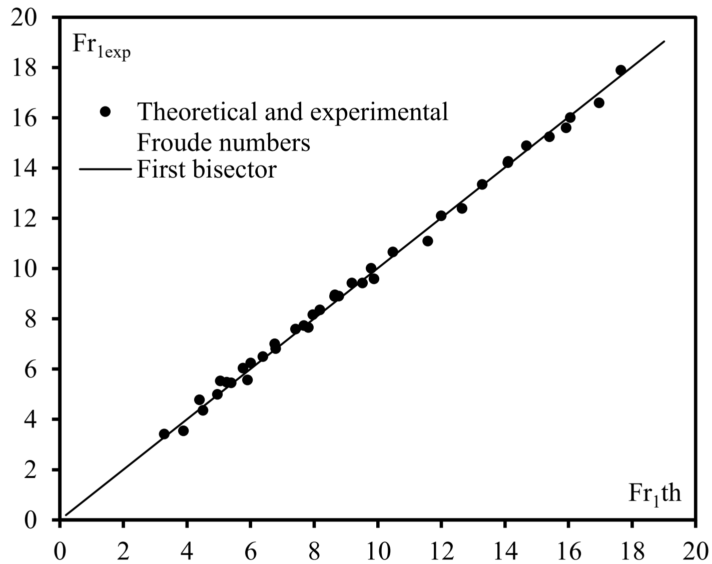



**Figure 3: Graphic representation of experimental ($Fr_{1exp}$) and theoretical ($Fr_{1th}$) Froude numbers**

To overcome this difference, which is based on the experimental results, an adjustment of the theoretical relationship by the least squares method is essential. The adjustment between two parameters resulted in a straight line (Fig. 3), which passes through the origin with a slope equal to 1.03.

$$Fr_{1exp} = 1.0015 Fr_{1th} \tag{17}$$

The purpose of this adjustment is to correct the theoretical relation Eq. (15), by the slope coefficient 1.0015.

$$Fr_1^2 = 0.5 \frac{\frac{(1+2M)}{(1+M)^2}\left[\begin{array}{l} Y^2\left(1+2MS+\frac{2}{3}MY\right)+ \\ +2SY(1+MS)+S^2\left(1+\frac{2}{3}MS\right)-\left(1+\frac{2}{3}M\right)\end{array}\right]}{\left[1-\frac{(1+M)}{Y(1+2MS+MY)}\right]} \tag{18}$$

## 5. The broad-crested sill effect on hydraulic jump

To facilitate hydraulic calculations and to replace the theoretical development's implicit relationship Eq. (18) with a more practical and explicit relationship, In this case, it is preferable to carry out an adjustment of the theoretical curves by a mathematical model.

Figure 4 represents the graphic representation of the theoretical variation of the sequence depth ratio Y as a function of the Froude number $Fr_1$ and the relative upstream flow depth M. The curves were drawn according to the theoretical relation Eq. (18) of the classical jump (S = 0) and M = (0.04, 0.07, 0.12 and 0.20).

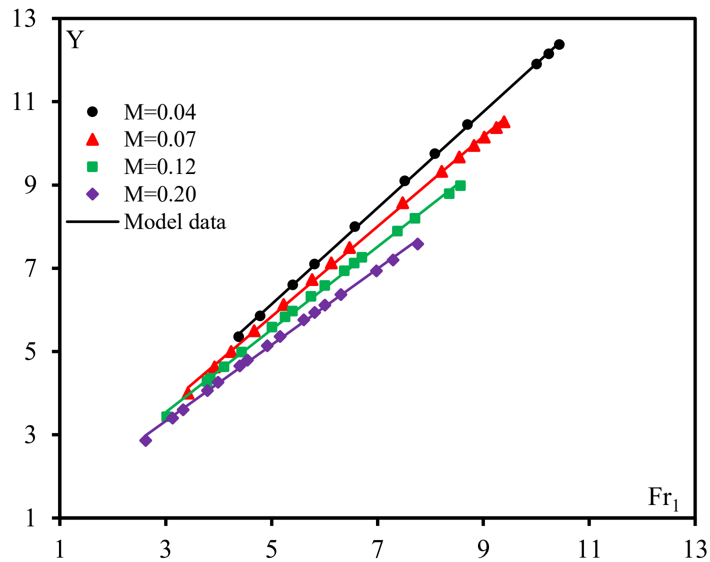



**Figure 4: Variation of Y as a function of $Fr_1$ and M according to Eq. (18) for a hydraulic jump, forced by broad-crested sill for a classical jump S=0.**

Figure 4 shows that the more the relative upstream flow depth M increases, the more the sequence depth ratio Y decreases for a fixed Froude number value. Figure 4 shows that the representation of Y = f($Fr_1$) is of the linear type:

$$Y = aFr_1 - b \tag{19}$$

For each value of S, the variation of Y as a function of $Fr_1$ and M is presented in the first case through an analysis of the experimental points. S is equal to (1, 2, 3, and 4). The graphical variation of Y = f($Fr_1$, M) for different S values is shown in (Fig. 5).

The values of M were determined by entering the experimental data for the initial height at the toe of the jump, more precisely $h_1$ = (1.5, 2.5, 4, and 7).

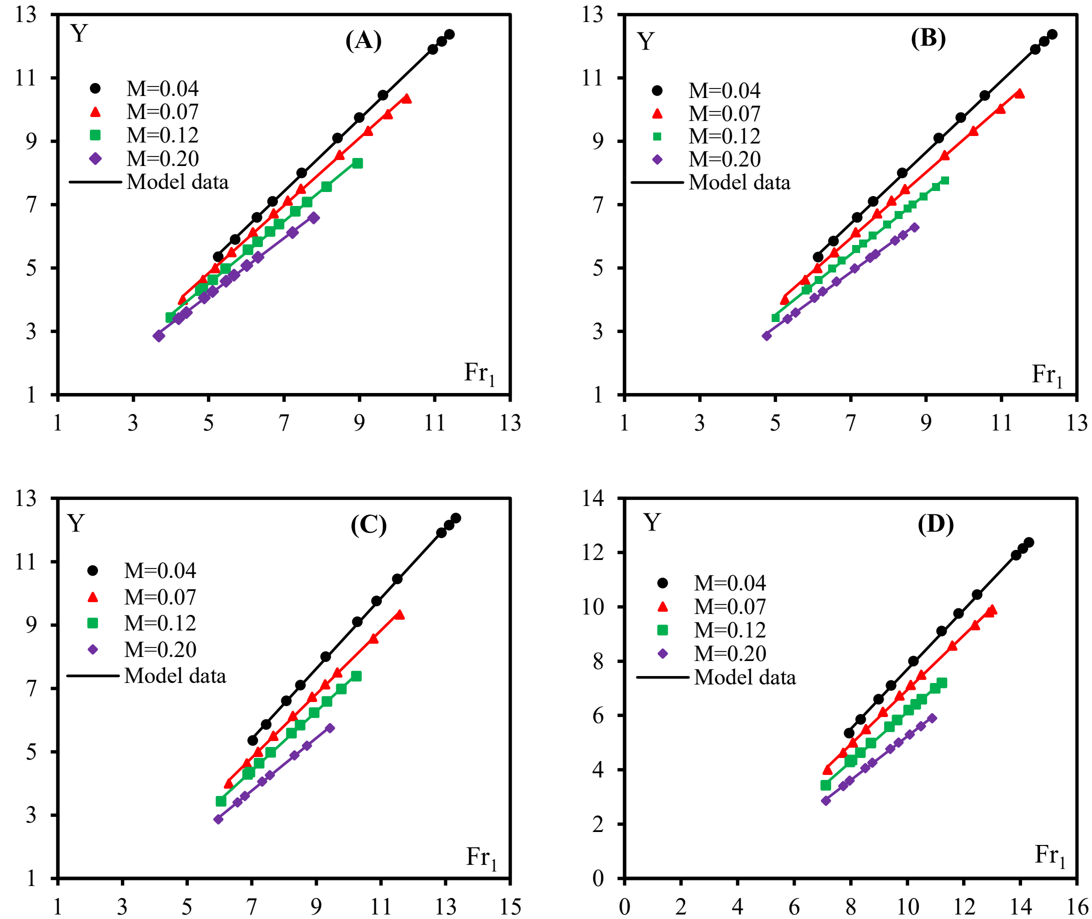



**Figure 5: For different S values, the variation of Y as a function of $Fr_1$ and M. (A) S = 1, (B) S = 2, (C) S = 3, (D) S = 4**

Figure 6 shows that the variation of the coefficients "a" and "b" as a function of M takes the following form:

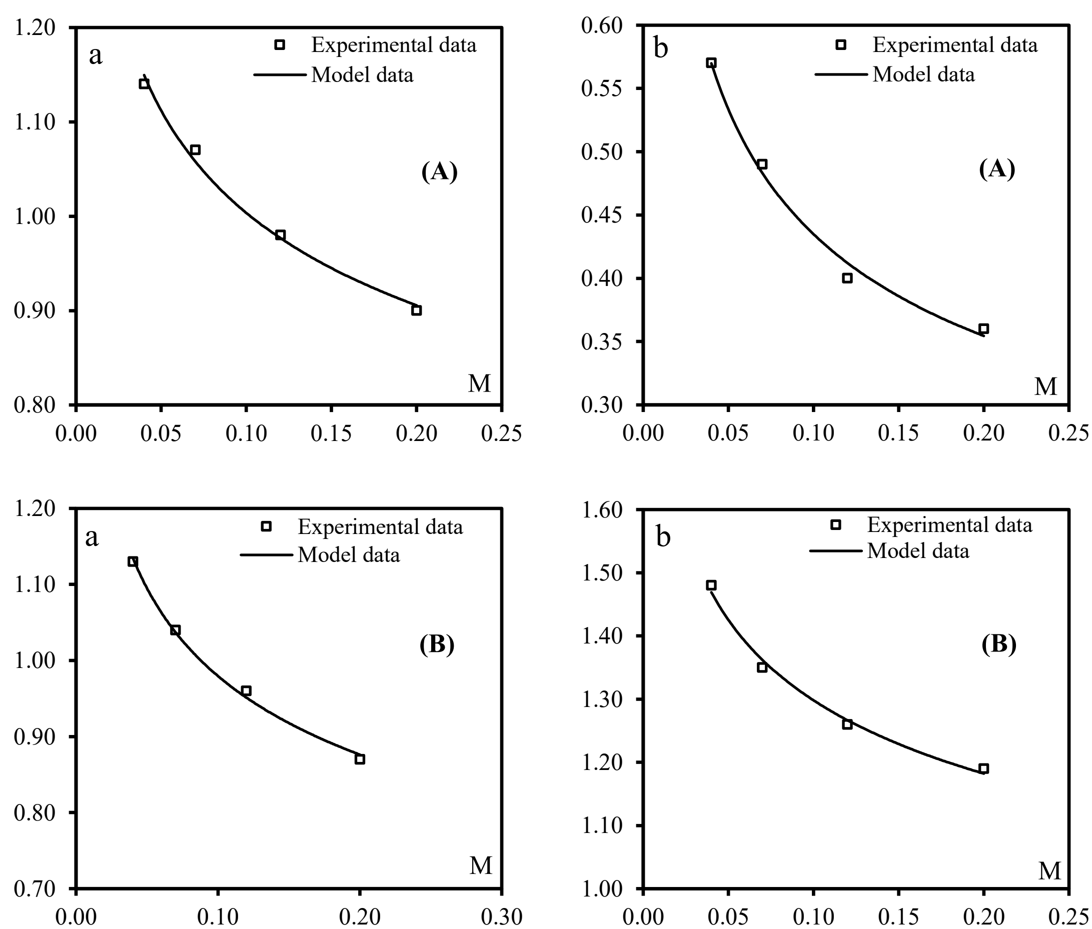
$$a = a_1 M^{a_2} \tag{20}$$

$$b = b_1 M^{b_2} \tag{21}$$

The adjustment of the experimental points of a = f(M) and b = f(M) for a given value of S showed that as M increases, the values of "a" and "b" decrease Fig. (6). Table 1 presents the equations that resulted from this adjustment.

**Table 1: Adjustment equations of a and b as a function of M**

| Values of S | Equations | |
|---|---|---|
| | a | b |
| 1 | $a=0.71M^{-0.15}$ | $b=0.22M^{-0.30}$ |
| 2 | $a=0.68M^{-0.16}$ | $b=0.95M^{-0.13}$ |
| 3 | $a=0.63M^{-0.17}$ | $b=1.77M^{-0.09}$ |
| 4 | $a=0.59M^{-0.20}$ | $b=2.44M^{-0.09}$ |



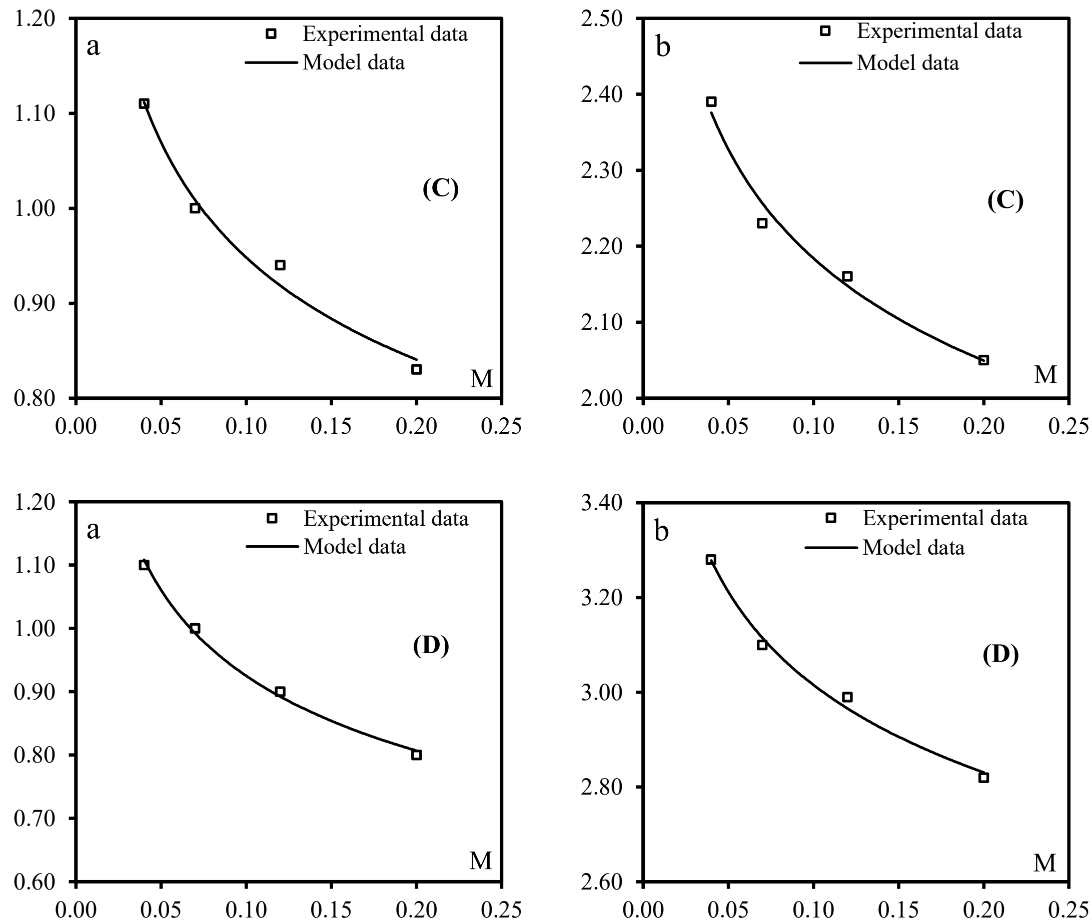



**Figure 6: Variation of the coefficients a = f(M) and b = f(M) for different values of S. (A) S = 1, (B) S = 2, (C) S = 3, (D) S = 4**

In a second mathematical adjustment of the experimental data (Fig. 7), the variation of the coefficients "$a_1$," "$a_2$," "$b_1$," and "$b_2$" as a function of S was established for a given value of M. The variations of $a_1$ = f(S) for M = 0.04, $a_2$ = f(S) for M = 0.07, $b_1$ = f(S) for M = 0.12 and $b_2$ = f(S) for M = 0.20 were given by Table 2 as follows:

**Table 2: Adjustment equations of $a_1$, $a_2$, $b_1$ and $b_2$ as a function of M**

| Values of M | Equations |
|---|---|
| 0.04 | $a_1$=-0.04S+0.75 |
| 0.07 | $a_2$=0.01S+0.14 |
| 0.12 | $b_1$=0.79S-0.59 |
| 0.20 | $b_2$=0.29S$^{(-1.09)}$ |

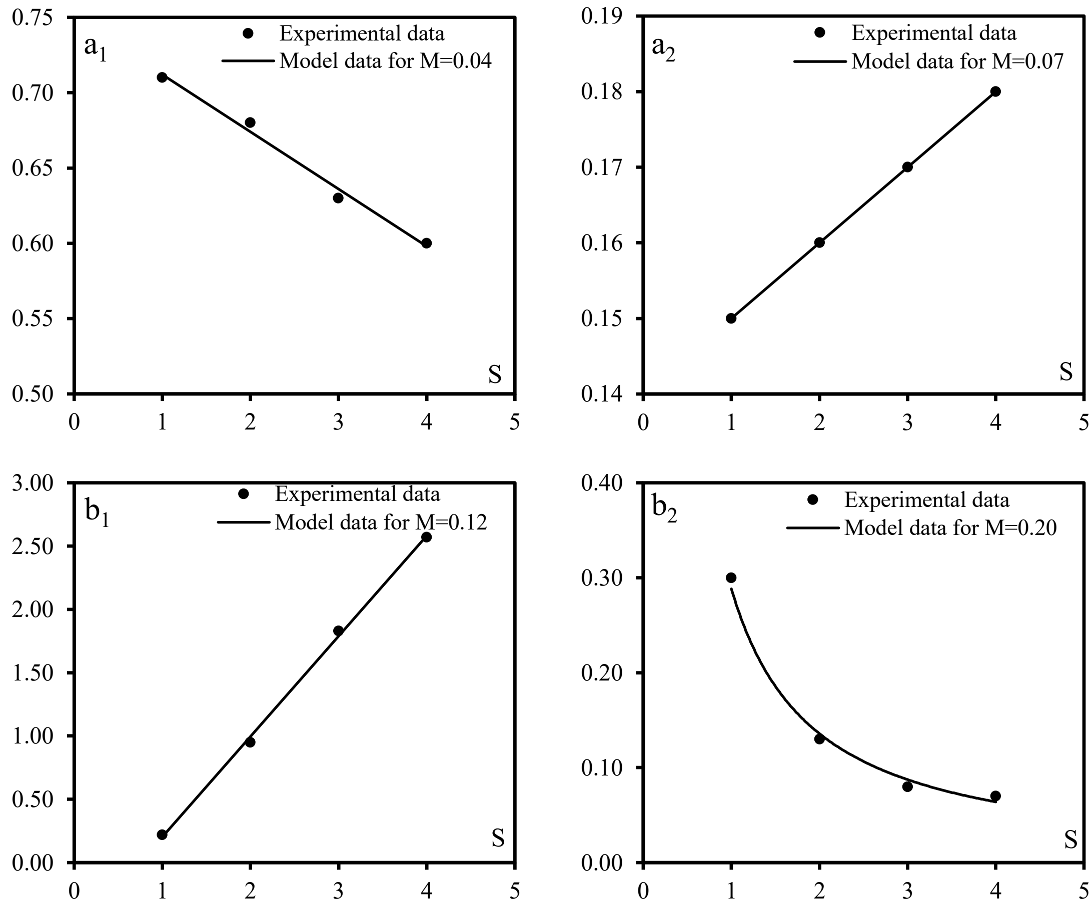



**Figure 7: Variation of coefficients "$a_1$", "$a_2$", "$b_1$" and "$b_2$" as a function of S.**

Substituting the equations in Table 1 and Table 2 into Eqs (19), (20) and (21) gives the expression for the sequence depth ratio as a function of the inflow Froude number $Fr_1$ for a range of relative upstream flow depth: $0.04 \leq M \leq 0.20$ and for $2.60 \leq Fr_1 \leq 14.20$.

$$Y = (-0.04S + 0.75)M^{(0.01S+0.14)}Fr_1 - (0.79S - 0.59)M^{(0.29S^{-1.09})} \quad (22)$$

The explicit relation Eq. (22) is deduced, which gives the possibility of determining the sequence depth ratio Y by knowing the value of the inflow Froude number $Fr_1$, the relative height of sill S, and the relative upstream flow depth M.

## 6. Discussion

Through this study, a theoretical development was presented in order to establish the relationship between the characteristics $Fr_1$, Y, M and S of a hydraulic jump, forced by broad-crested sill evolution in a trapezoidal channel. This development led to the proposal of a theoretical relationship Eq. (15) established by applying the momentum equation between the upstream and downstream sections of the jump.

Knowing the Froude number $Fr_1$ and the relative height S of the broad-crested sill, the sequence depth ratio $Y=h_2/h_1$ can be obtained from Eq. (15). Since the depth $h_1$ of the jump at its upstream section and the value of the aspect ratio M are known, and from the value of Y, it is possible to determine the depth $h_2$ of the jump at its downstream section.

Also, Eq. (15) leads to Eq. (16) of the classical hydraulic jump evolving in a trapezoidal channel for S = 0. This last consideration confirms the validity of the theoretical development undertaken.
However, the comparison of the values of the Froude numbers $Fr_1$, determined from the universal relation Eq. (13), using the experimental values with those determined by the application of Eq. (15), shows a more or less significant offset. This shift is largely attributed to the singularity at the upstream end of the broad-crested sill. This last observation helps to correct Eq. (15) by the experimental measurements and the relation Eq. (18) is then obtained.
Moreover, the application of the general relation Eq. (18) requires the use of an iterative process since it arises analytically in an implicit form with respect to the ratio of the conjugate depths Y. To solve the problem, it has been proposed to replace the latter relation by the explicit relation Eq. (22). The latter then allows easy determination of the sequence depth ratio Y as a function of the Froude number $Fr_1$ and the relative height S of the sill.

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