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Applications of Intuitionistic Multi L–Fuzzy Sets in Decision-Making Problems by Using the Normalized Euclidean Distance Approach

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Abstract:- Intuitionistic Multi L-Fuzzy Sets (IMLFS) offer a powerful framework for handling uncertainty and hesitation in decision-making. They incorporate degrees of membership, non-membership, and hesitation over a multi-level lattice structure. This paper explores the application of IMLFS in multi-criteria decision-making (MCDM) problems, offering a flexible and comprehensive framework for evaluating alternatives under uncertainty. To measure the similarity or dissimilarity between different alternatives in the IMLFS context, distance measures play a pivotal role. We examine four commonly used distance metrics: Hamming distance, Euclidean distance, Normalized Hamming distance, and Normalized Euclidean distance. Among these, the Normalized Euclidean distance emerges as the most effective in decision-making scenarios due to its ability to account for relative deviations while mitigating the impact of scale differences. It maintains sensitivity to changes in fuzzy values and supports a balanced evaluation in ranking alternatives. Thus, the integration of IMLFS with Normalized Euclidean distance provides a robust decision-making tool, enabling improved handling of uncertainty, ambiguity, and linguistic variability.

Keywords: *Intuitionistic Multi L-Fuzzy Sets (IMLFS) – Multi-Criteria Decision-Making (MCDM) – Hamming Distance – Euclidean Distance – Normalized Hamming Distance - Normalized Euclidean Distance.*

1. Introduction

Decision-making problems often involve uncertainty, vagueness, and imprecision, which traditional crisp models fail to handle effectively. Intuitionistic Multi L-Fuzzy Sets (IMLFS) provide a flexible mathematical framework to represent and process such imprecise information by incorporating degrees of membership, non-membership, and hesitation in a multi-level structure. This makes them particularly suitable for multi-criteria decision-making (MCDM) problems [1-8], where decisions depend on evaluating various conflicting attributes under uncertain conditions. They allow decision-makers to express not just how much an alternative satisfies a criterion, but also how much it does not and how uncertain they are about the evaluation. To evaluate and rank alternatives in decision-making [9,10], distance measures play a crucial role in comparing fuzzy information.

This study applies four prominent techniques: Hamming Distance, Euclidean Distance, Normalized Hamming Distance, and Normalized Euclidean Distance [11,12,13] for measuring the similarity between alternatives and ideal solutions. While all four methods offer insights, the normalized Euclidean distance measure proves to be the most effective, as it maintains proportionality across different scales and yields more accurate and consistent rankings in diverse decision-making scenarios. These tools collectively enhance the decision-making process by offering a richer and more flexible representation of human reasoning under uncertainty.

This approach has been successfully applied across various domains such as engineering, economics, medical diagnosis, supplier selection and expert systems, where nuanced judgment is crucial [14,15].

2. Objectives

In this section, we site the fundamental definitions that will be used in the sequel.

2.1 Definition (L.A. Zadeh [16], 1965)

Let X be a non-empty set. A fuzzy set A drawn from X is defined as $A = \{\langle x, \mu_A(x)\rangle : x\epsilon X\}$, where $\mu_A(x): X \to [0,1]$ is the membership function of the fuzzy set A, and also which is a collection of objects with graded membership, which means degrees of membership.

2.2 Definition (K.T. Atanassov [17,18], 1983)

Let X be a non-empty set. An Intuitionistic fuzzy set (IFS) A in X is an object having the form $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x\epsilon X\}$, where the functions $\mu_A(x), \gamma_A(x): X \to [0,1]$ define respectively, the degree of membership and non-membership of the element $x \in X$ to the set A, which is a subset of X, and for every element $x \in X$ such that $0 \le \mu_A(x) + \gamma_A(x) \le 1$.

Furthermore, we have $\pi_A(x) = 1 - \mu_A(x) - \gamma_A(x)$ called the intuitionistic fuzzy set index (or) hesitation margin (or) degree of indeterminacy of x in A. For, if $\pi_A(x): X \to [0,1]$ and $0 \le \pi_A(x) \le 1$ for every $x \in X$, which expresses the lack of knowledge of whether x belongs to intuitionistic fuzzy set A or not.

2.3 Definition

Let $A = \{\langle x, \mu_{A_i}(x), \gamma_{A_i}(x)\rangle : x\epsilon X\}$ where $\mu_{A_i}(x) = \left(\mu_{A_1}(x), \mu_{A_2}(x), \ldots\ldots, \mu_{A_k}(x)\right)$ and $\gamma_{A_i}(x) = \left(\gamma_{A_1}(x), \gamma_{A_2}(x), \ldots\ldots, \gamma_{A_k}(x)\right)$ such that $0 \le \mu_{A_i}(x) + \gamma_{A_i}(x) \le 1$, for all i, $\forall\, x\epsilon X$. Also for each $i = 1,2,\ldots.,k$, $\mu_{A_i}(x): X \to [0,1]$, $\gamma_{A_i}(x): X \to [0,1]$.

Here, $\mu_{A_1}(x) \ge \mu_{A_2}(x) \ge \ldots. \ge \mu_{A_k}(x)$, for all $x\epsilon X$. That is, μ_{A_i}'s are decreasingly ordered sequence, the corresponding non-membership sequence γ_{A_i}'s are may not be in decreasing or increasing order.

i.e., $0 \le \mu_{A_i}(x) + \gamma_{A_i}(x) \le 1$, $\forall\, x\epsilon X$, for all $i = 1,2,\ldots.,k$. Then the set A is said to be an intuitionistic multi L-fuzzy set (IMLFS) with dimension k of X.

Furthermore, we have $\pi_{A_i}(x) = \left(1 - \mu_{A_i}(x) - \gamma_{A_i}(x)\right)$ is called the index of intuitionistic multi L-fuzzy set (or) hesitation margin (or) degree of indeterminacy of x in A.

For if $\pi_{A_i}(x): X \to [0,1]$ and $0 \le \pi_{A_i}(x) \le 1$, $\forall\, i = 1,2,\ldots.,k$ for every $x\epsilon X$, which expresses the lack of knowledge of whether x belongs to intuitionistic fuzzy set A or not.

We use the notation $\left(\mu_{A_i}(x), \gamma_{A_i}(x), \pi_{A_i}(x)\right)$ to represent an intuitionistic multi L-fuzzy set.

2.4 Examples to define membership values and linguistic terms

In the context of fuzzy set theory, linguistic terms such as “Excellent”, “Good”, “Average”, and “Poor” are commonly used to represent qualitative assessments of quantitative data, such as student marks. These linguistic terms are mapped to numerical values using membership functions, which assign a degree of membership ranging from 0 to 1. This approach enables the modeling of uncertainty and subjectivity inherent in human judgment.

The following examples illustrate how membership values can be defined for various linguistic terms based on student’s marks, providing a foundation for more nuanced and flexible evaluation in decision-making processes.

Marks	Linguistic Terms	Membership Values
0	Absent Performance	0.0
1-10	Very Very Poor	0.1
11-20	Very Poor	0.2
21-30	Poor	0.3
31-40	Below Average	0.4
41-50	Average	0.5
51-60	Above Average	0.6
61-70	Good	0.7
71-80	Very Good	0.8
81-90	Excellent	0.9
91-100	Outstanding / Perfect	1.0

2.5 Example

Let A be an IMLFS of dimension two with $\mu_A(x) = (0.3,0.5)$ and $\gamma_A(x) = (0.6,0.2)$.

Then $\pi_A(x) = \left(1 - \mu_{A_1}(x) - \gamma_{A_1}(x), 1 - \mu_{A_2}(x) - \gamma_{A_2}(x)\right)$

$$= (1 - 0.3 - 0.6, 1 - 0.5 - 0.2) \quad = (0.1,0.3)$$

It can be interpreted as “the degree that the object x belongs to IMLFS A is $(0.3,0.5)$, the degree that the object x does not belongs to IMLFS A is $(0.6,0.2)$ and the degree of hesitancy is $(0.1,0.3)$".

3. Methods

3.1 Definition

Let X be non-empty set. An intuitionistic multi L-fuzzy subsets $A, B, C \in X$ then the distance measured is a mapping $d: X \times X \to [0,1]$, if $d(A, B)$ satisfies the following axioms:

(i) $0 \leq d(A, B) \leq 1$

(ii) $d(A,B) = 0 \Leftrightarrow A = B$

(iii) $d(A,B) = d(B,A)$

(iv) $d(A,C) + d(C,B) \geq d(A,B)$

(v) If $A \subseteq B \subseteq C$, then $d(A,C) \geq d(A,B)$ and $d(A,C) \geq d(C,B)$.

Then $d(A,B)$ is a distance measure between IMLFS's of A and B.

3.2 Definition

The Cardinality of the membership function $\mu_{A_i}(x)$ and the non-membership function $\gamma_{A_i}(x)$ is the length of an element x is an IMLFS of A denoted as $n(A)$ and it is defined as

$$n(A) = |\mu_{A_i}(x)| = |\gamma_{A_i}(x)|.$$

If A,B,C are the IMLFS's defined on X, then their Cardinality $n = max\{n(A), n(B), n(C)\}$.

3.3 Definition

Let, $A = \{\langle x_j, \mu_{A_i}(x_j), \gamma_{A_i}(x_j), \pi_{A_i}(x_j)\rangle : x_j \epsilon X\}$ *and* $B = \{\langle x_j, \mu_{B_i}(x_j), \gamma_{B_i}(x_j), \pi_{B_i}(x_j)\rangle : x_j \epsilon X\}$ be two IMFS's in $X = \{x_1, x_2, \dots \dots, x_k\}$, $j = 1,2, \dots \dots, k$ based on the geometric interpretation of IMFS Szmidt and Kacprzyk proposed [19].

1. The Hamming Distance

$$d_H(A,B) = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{k}\left(|\mu_{A_i}(x_j) - \mu_{B_i}(x_j)| + |\gamma_{A_i}(x_j) - \gamma_{B_i}(x_j)| + |\pi_{A_i}(x_j) - \pi_{B_i}(x_j)|\right)$$

2. The Euclidean Distance

$$d_E(A,B) = \sqrt{\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{k}\left[\left(\mu_{A_i}(x_j) - \mu_{B_i}(x_j)\right)^2 + \left(\gamma_{A_i}(x_j) - \gamma_{B_i}(x_j)\right)^2 + \left(\pi_{A_i}(x_j) - \pi_{B_i}(x_j)\right)^2\right]}$$

3. The Normalized Hamming Distance

$$d_{N-H}(A,B) = \frac{1}{2n}\sum_{i=1}^{n}\sum_{j=1}^{k}\left(|\mu_{A_i}(x_j) - \mu_{B_i}(x_j)| + |\gamma_{A_i}(x_j) - \gamma_{B_i}(x_j)| + |\pi_{A_i}(x_j) - \pi_{B_i}(x_j)|\right)$$

4. The Normalized Euclidean Distance

$$d_{N-E}(A,B) = \sqrt{\frac{1}{2nk}\sum_{i=1}^{n}\sum_{j=1}^{k}\left[\left(\mu_{A_i}(x_j) - \mu_{B_i}(x_j)\right)^2 + \left(\gamma_{A_i}(x_j) - \gamma_{B_i}(x_j)\right)^2 + \left(\pi_{A_i}(x_j) - \pi_{B_i}(x_j)\right)^2\right]}$$

Where, $n = max\{n(A_i), n(B_i)\}$ and $X = \{x_1, x_2, \ldots \ldots, x_k\}$.

Remark

Here X denotes the set of all multi-criterias and n represents the cardinality of the IMLFS.

Example

Let $A = \{\langle 0.5,0.3,0.2\rangle\langle 0.6,0.2,0.2\rangle\}$ and $B = \{\langle 0.4,0.5,0.1\rangle\langle 1.0,0.0,0.0\rangle\}$ be intuitionistic fuzzy sets in X. If $X = \{x_1, x_2\}$ we use the above distance measures to calculate the distance between A and B.

(i) Hamming Distance of A and B

$$d_H(A,B) = \frac{1}{2}(|0.5-0.4| + |0.3-0.5| + |0.2-0.1| + |0.6-1.0| + |0.2-0.0| + |0.2-0.0|)$$

$$= \frac{1}{2}(0.1 + 0.2 + 0.1 + 0.4 + 0.2 + 0.2) = \frac{1}{2}(1.2) = \mathbf{0.6000}$$

(ii) Euclidean Distance of A and B

$$d_E(A,B) = \sqrt{\frac{1}{2}[(0.1)^2 + (0.2)^2 + (0.1)^2 + (0.4)^2 + (0.2)^2 + (0.2)^2]}$$

$$= \sqrt{\frac{1}{2}[0.01 + 0.04 + 0.01 + 0.16 + 0.04 + 0.04]} = \sqrt{\frac{1}{2}[0.30]} = \sqrt{0.15} = \mathbf{0.3873}$$

(iii) Normalized Hamming Distance of A and B

$$d_{N-H}(A,B) = \frac{1}{2(2)}(0.1 + 0.2 + 0.1 + 0.4 + 0.2 + 0.2) = \frac{1}{4}(1.2) = \mathbf{0.3000}$$

(iv) Normalized Euclidean Distance of A and B

$$d_{N-E}(A,B) = \sqrt{\frac{1}{2(2)(1)}[(0.1)^2 + (0.2)^2 + (0.1)^2 + (0.4)^2 + (0.2)^2 + (0.2)^2]}$$

$$= \sqrt{\frac{1}{4}[0.01 + 0.04 + 0.01 + 0.16 + 0.04 + 0.04]} = \sqrt{\frac{1}{4}[0.30]} = \sqrt{0.075} = \mathbf{0.2739}$$

These findings demonstrated that the Normalized Euclidean distance provides the most accurate measure of the distance between points A and B. We will therefore utilize the Normalized Euclidean distance in the applications due to its high accuracy confidence rate.

3.4 Application of Intuitionistic Multi L-Fuzzy Sets in College Student Determination

Let $C = \{C_1, C_2, C_3\}$ be a set of colleges, $A = \{\,Subject\ Marks, Personal\ Interview, Sports,$ $NCC/NSS\ others, Co-Curricular\ Activities\}$ be the set of students' abilities, and let $S = \{S_1, S_2, S_3, S_4\}$ be a set of students.

Structure for Student Selection Committee Evaluation

For a college student selection committee, a structured and fair evaluation system is essential. This system ensures that all applicants are assessed consistently and impartially, ultimately leading to the selection of the most suitable candidates. It should outline clear criteria, evaluation methods, and a transparent process. The following tables illustrate how these elements can be effectively addressed in this section.

Table 1: Arts College Committee Vs Criteria's

		Criteria's				
Arts College Committee		Subject Marks (Academics)	Personal Interview	Sports	NCC/NSS Others	$Co-Curricular\ Activities$
	C_1	((0.7,0.5,0.8) (0.2,0.3,0.1) (0.1,0.2,0.1))	((0.6,0.7,0.5) (0.3,0.1,0.2) (0.1,0.2,0.3))	((0.8,0.7,0.5) (0.0,0.1,0.2) (0.2,0.2,0.3))	((0.5,0.6,0.8) (0.1,0.3,0.0) (0.4,0.1,0.2))	((0.4,0.5,0.7) (0.3,0.2,0.2) (0.3,0.3,0.1))
	C_2	((0.8,0.7,0.5) (0.1,0.1,0.3) (0.1,0.2,0.2))	((0.8,0.6,0.7) (0.0,0.2,0.1) (0.2,0.2,0.2))	((0.5,0.7,0.9) (0.0,0.1,0.1) (0.5,0.2,0.0))	((0.6,0.5,0.4) (0.1,0.1,0.3) (0.3,0.4,0.3))	((0.8,0.7,0.7) (0.1,0.0,0.1) (0.1,0.3,0.2))
	C_3	((0.9,0.7,0.8) (0.1,0.1,0.2) (0.0,0.2,0.0))	((0.5,0.5,0.6) (0.2,0.1,0.0) (0.3,0.4,0.4))	((0.7,0.4,0.5) (0.1,0.3,0.2) (0.2,0.3,0.3))	((0.6,0.7,0.7) (0.2,0.0,0.1) (0.2,0.3,0.2))	((0.6,0.5,0.4) (0.1,0.1,0.2) (0.3,0.4,0.4))

Table 2: Students Abilities Vs Criteria's

		Criteria's				
Student Abilities		Subject Marks (Academics)	Personal Interview	Sports	NCC/NSS Others	$Co-Curricular\ Activities$
	S_1	((0.4,0.6,0.7) (0.4,0.3,0.2) (0.2,0.1,0.1))	((0.5,0.7,0.4) (0.2,0.1,0.3) (0.3,0.2,0.3))	((0.6,0.8,0.4) (0.2,0.0,0.3) (0.2,0.2,0.3))	((0.5,0.5,0.6) (0.2,0.3,0.1) (0.3,0.2,0.3))	((0.6,0.5,0.6) (0.3,0.2,0.1) (0.1,0.3,0.3))
	S_2	((0.2,0.2,0.4) (0.0,0.2,0.1) (0.8,0.6,0.5))	((0.4,0.5,0.7) (0.3,0.4,0.2) (0.3,0.1,0.1))	((0.1,0.2,0.1) (0.9,0.8,0.6) (0.0,0.0,0.3))	((0.7,0.5,0.7) (0.1,0.2,0.1) (0.2,0.3,0.2))	((0.2,0.0,0.4) (0.0,0.2,0.2) (0.8,0.8,0.4))
	S_3	((0.6,0.3,0.5) (0.2,0.5,0.2) (0.2,0.2,0.3))	((0.4,0.3,0.1) (0.5,0.5,0.9) (0.1,0.2,0.0))	((0.5,0.2,0.2) (0.1,0.3,0.1) (0.4,0.5,0.7))	((0.0,0.1,0.1) (0.2,0.1,0.0) (0.8,0.8,0.9))	((0.7,0.6,0.3) (0.1,0.2,0.1) (0.2,0.2,0.6))
	S_4	((0.3,0.4,0.3) (0.5,0.1,0.4) (0.2,0.5,0.3))	((0.5,0.4,0.6) (0.2,0.4,0.1) (0.3,0.2,0.3))	((0.2,0.0,0.3) (0.6,0.4,0.2) (0.2,0.6,0.5))	((0.1,0.4,0.3) (0.7,0.5,0.6) (0.2,0.1,0.1))	((0.6,0.5,0.7) (0.0,0.2,0.1) (0.4,0.3,0.2))

Table 3: Arts College Committee Vs Students Abilities

$n = \max\{n(A), n(B)\} = \max\{3,4\} = 4$ and $k = 5$ (i.e., Criteria's)

The Normalized Euclidean shortest distance gives:

		Arts College Committee		
Student Abilities		C_1	C_2	C_3
	S_1	**0.1245**	0.1987	0.1817
	S_2	0.3647	0.4093	**0.3536**
	S_3	0.3507	**0.3286**	0.3536
	S_4	0.3162	0.3178	**0.3000**

Example Calculation:

College C_1

Student S_1 $= \left[\frac{1}{2(4)(5)}\left((0.3)^2 + (-0.1)^2 + (0.1)^2 + (-0.2)^2 + 0^2 + (-0.1)^2 + (-0.1)^2 + (0.1)^2 + 0^2 + (0.1)^2 + 0^2 + (0.1)^2 + (0.1)^2 + 0^2 + (-0.1)^2 + (-0.2)^2 + 0^2 + 0^2 + (0.2)^2 + (-0.1)^2 + (0.1)^2 + (-0.2)^2 + (0.1)^2 + (-0.1)^2 + 0^2 + 0^2 + 0^2 + 0^2 + (0.1)^2 + (0.2)^2 + (-0.1)^2 + 0^2 + (-0.1)^2 + (0.1)^2 + (-0.1)^2 + (-0.1)^2 + (-0.2)^2 + 0^2 + (0.1)^2 + 0^2 + 0^2 + (0.1)^2 + (-0.2)^2 + 0^2 + (-0.2)^2\right)\right]^{\frac{1}{2}}$

$$= \left[\frac{1}{40}\left((0.09 \times 1) + (0.01 \times 21) + (0.04 \times 8)\right)\right]^{\frac{1}{2}} = 0.1245$$

Results: Table 3 depicts that the Normalized Euclidean shortest distance between each student and each college is given that the student will enroll in the college. According to Table 3, the student S_1 is to enroll in C_1 college, the student S_2 is to enroll in C_3 college, the student S_3 is to enroll in C_2 college, the student S_4 is to enroll in C_3 college.

3.5 Application of Intuitionistic Multi L-Fuzzy Sets to Determine the Performance of the Students in TNPSC Group 4 Exam Preparation

This study explores the use of Intuitionistic Multi L-Fuzzy Sets (IMLFS) to assess the performance of students preparing for the TNPSC Group 4 examination. Traditional evaluation methods often lack the capacity to handle uncertainty and hesitation inherent in human assessments. IMLFS integrates membership, non-membership, and hesitation degrees along with linguistic terms (e.g., Good, Average, Poor) to better reflect real-world judgments. Performance indicators such as test scores, mock test consistency, time management, and subject-wise strength are mapped into multi-level linguistic variables. By applying normalized distance measures, such as normalized Euclidean distance, the closeness of each student's performance to an ideal benchmark is evaluated. This method provides a more nuanced and flexible approach to ranking and guiding students, especially in large-scale competitive exam settings like TNPSC.

Table 1: Benchmark for each subject and each model test

Criteria's					
General ($Tamil/$ English)	General Studies	Aptitude & Mental Ability	$Indian -$ ($Polity$ & $Economy$)	$General$ $Science$	History & Geography
((0.8,1.0,0.9) (0.1,0.0,0.1) (0.1,0.0,0.0))	((0.6,0.7,0.5) (0.2,0.2,0.1) (0.2,0.1,0.4))	((0.9,0.7,0.8) (0.1,0.2,0.2) (0.0,0.1,0.0))	((0.8,0.6,0.6) (0.1,0.1,0.3) (0.1,0.3,0.1))	((0.5,0.6,0.5) (0.2,0.1,0.3) (0.3,0.3,0.2))	((0.7,0.6,0.7) (0.2,0.2,0.1) (0.1,0.2,0.2))

Table 2: Each student's marks in each subject

	Criteria's					
	General ($Tamil/$ English)	General Studies	Aptitude & Mental Ability	$Indian -$ ($Polity$ & $Economy$)	$General$ $Science$	History & Geography
S_1	((0.5,0.5,0.6) (0.3,0.2,0.3) (0.2,0.3,0.1))	((0.7,0.5,0.6) (0.1,0.3,0.1) (0.2,0.2,0.3))	((0.8,0.6,0.8) (0.2,0.1,0.1) (0.0,0.3,0.1))	((0.7,0.9,0.5) (0.2,0.0,0.2) (0.1,0.1,0.3))	((0.6,0.6,0.7) (0.0,0.2,0.1) (0.4,0.2,0.2))	((0.4,0.6,0.5) (0.4,0.2,0.2) (0.2,0.2,0.3))
S_2	((0.8,0.9,0.8) (0.1,0.0,0.2) (0.1,0.1,0.0))	((0.7,0.6,0.7) (0.1,0.4,0.2) (0.2,0.0,0.1))	((0.5,0.5,0.6) (0.2,0.3,0.1) (0.3,0.2,0.3))	((0.4,0.7,0.8) (0.3,0.2,0.1) (0.3,0.1,0.1))	((0.6,0.8,0.4) (0.1,0.2,0.5) (0.3,0.0,0.1))	((0.7,0.5,0.5) (0.1,0.4,0.3) (0.2,0.1,0.2))
S_3	((0.7,0.8,1.0) (0.2,0.1,0.0) (0.1,0.1,0.0))	((0.6,0.7,0.7) (0.3,0.1,0.2) (0.1,0.2,0.1))	((0.9,0.9,0.8) (0.1,0.0,0.1) (0.0,0.1,0.1))	((0.6,0.8,0.7) (0.2,0.1,0.2) (0.2,0.1,0.1))	((0.5,0.4,0.4) (0.1,0.5,0.3) (0.4,0.1,0.3))	((0.8,0.5,0.8) (0.1,0.2,0.2) (0.1,0.3,0.0))
S_4	((0.9,0.5,0.6) (0.0,0.4,0.2) (0.1,0.1,0.2))	((0.9,0.6,0.8) (0.1,0.4,0.1) (0.0,0.0,0.1))	((0.5,0.7,0.6) (0.1,0.2,0.1) (0.4,0.1,0.3))	((0.8,0.7,0.8) (0.2,0.2,0.1) (0.0,0.1,0.1))	((0.6,1.0,0.7) (0.1,0.0,0.2) (0.3,0.0,0.1))	((0.8,0.8,0.6) (0.0,0.0,0.3) (0.2,0.2,0.1))
S_5	((0.6,0.4,0.5) (0.2,0.3,0.2) (0.2,0.3,0.3))	((0.7,0.5,0.7) (0.2,0.1,0.1) (0.1,0.4,0.2))	((0.4,0.4,0.8) (0.1,0.2,0.1) (0.5,0.4,0.1))	((0.9,0.6,0.7) (0.0,0.3,0.1) (0.1,0.1,0.2))	((0.6,0.2,0.3) (0.1,0.3,0.1) (0.3,0.5,0.6))	((0.5,0.7,0.4) (0.3,0.3,0.2) (0.2,0.0,0.4))

Table 3: The Normalized Euclidean distance between the benchmark and each student's mark

S_1	S_2	S_3	S_4	S_5
0.1549	0.1572	**0.1237**	0.1879	**0.2128**

Results: From the above table, the minimum distance gives the optimal solution to the given intuitionistic multi-criteria decision-making problem. The Normalized Euclidean distance between the benchmark given by the coaching centre and the marks obtained by the student S_3 is minimum. Hence, it is identified that the student S_3 performs well in the model examinations. Also, the student S_5 is maximum. Hence, it is identified that for the student S_5, more attention is needed.

And the students S_1, S_2 and S_4 is study average performance of the others.

4. Discussions

This study explores the use of intuitionistic multi L-fuzzy sets in solving complex decision-making problems. By incorporating uncertainty and hesitation values, these sets provide a more flexible framework than traditional fuzzy models. The research applies the Normalized Euclidean distance approach to evaluate and compare alternatives effectively. This method enhances accuracy in decision-making where multiple-criteria and vague information are involved.

Atanassov KT introduced the theory of intuitionistic fuzzy sets in 1986, extending classical fuzzy sets by incorporating both membership and non-membership degrees, along with a hesitation margin. In 2016, Ananthakanagajothi and others wrote an article titled “On Some Distance Measures in Intuitionistic Fuzzy Sets.” The authors explore distance measures in intuitionistic fuzzy set (IFS) theory—tools essential for quantifying dissimilarities in systems characterized by uncertainty. The paper surveys selected models and applications, such as career determination and pattern recognition, demonstrating how these measures function in real-life decision-making scenarios. A focus is placed on the Normalized Euclidean distance, highlighting its efficacy in capturing nuanced differences between IFSs in applied contexts. By presenting these models, the study provides a lucid and comprehensive understanding of distance-based approaches within the IFS framework.

Ejegwa PA, Akubo AJ, and Joshua OM present the theory of intuitionistic fuzzy sets (IFS), which extend traditional fuzzy sets by incorporating degrees of membership, non-membership, and hesitation. They apply IFS to career determination, providing a more nuanced decision-making framework uncertainty in high. The method uses Normalized Euclidean distance measures to compare a candidate’s attributes with career profiles. This approach enhances accuracy in matching individuals to suitable careers under vague or incomplete information.

Next, “MCDM by Normalized Euclidean Distance in Intuitionistic Multi-Fuzzy Sets” by Muthuraj R and others introduces intuitionistic multi-fuzzy sets (IMFS)—an extension of intuitionistic fuzzy sets, where each element has multiple membership and non-membership values across dimensions—enabling richer modeling of uncertainty. They apply this IMFS framework to a multi-criteria decision-making (MCDM) problem—specifically choosing the best heavy motor vehicle mechanism aligned with rider aptitude across various criteria. The core method uses the normalized Euclidean distance, computing the distance between each HMV's mechanism profile and each rider’s aptitude profile to quantify closeness. The optimal choice emerges as the mechanism with the smallest normalized Euclidean distance—i.e., the shortest "gap" between the IMFS representations of the rider’s aptitude and the alternative.

Finally in this study, the applications of Intuitionistic Multi L-Fuzzy Sets (IMLFS) have been explored in real-life decision-making scenarios using the Normalized Euclidean Distance measure. Specifically, Section 3.4 demonstrates how IMLFS can be effectively applied in a college student selection committee evaluation, providing a systematic and objective framework for ranking candidates based on multiple qualitative and quantitative criteria. Section 3.5 further extends the approach to assess student performance in the context of TNPSC Group 4 examination preparation, offering clear examples and step-by-step solutions to illustrate the method's practicality. These applications highlight the versatility and robustness of IMLFS in handling uncertainty, hesitation, and vagueness inherent in human judgment, ultimately supporting fair and transparent decision-making processes.

5. Conclusions

In this study, four distance measures – Hamming distance, Euclidean distance, Normalized Hamming distance, and Normalized Euclidean distance – were analyzed and compared in the context of decision-making and similarity evaluation. However, among these techniques, the Normalized Euclidean distance measure emerged as the most effective and reliable.

Also, we explored the application of intuitionistic multi L-fuzzy sets in decision-making problems using the Normalized Euclidean distance approach. By incorporating multiple degrees of membership, non-membership, and hesitation values across lattice structures (L), IMLFS allows for a richer and more comprehensive modeling of real-world decision parameters.

The use of the Normalized Euclidean distance enables a consistent and quantifiable comparison of alternatives by measuring their closeness to ideal solutions. This method effectively ranks alternatives even in the presence of conflicting or incomplete information. It also enhances the objectivity of the decision-making process by standardizing the influence of diverse criteria through normalization.

Therefore, the Normalized Euclidean distance approach under the intuitionistic multi L-fuzzy environment provides a powerful and reliable tool for solving a wide range of decision-making problems in fields such as engineering, management, healthcare, and finance.

Future research can explore hybrid models combining IMLFS with other distance measures or optimization techniques to further strengthen its practical applications.

6. Use of Generative-AI tools declaration

This article was not created using Artificial Intelligence (AI) tools.

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8. Conflict of Interests

The authors declare that they have no conflicts of interest.

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