Tuijin Jishu/Journal of Propulsion Technology
ISSN: 1001-4055
Vol. 46 No. 3 (2025)

Soft Expert Set Approach For The Association Action Rules

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Abstract: Action rules are built from atomic expressions called atomic action terms, and they describe possible transitions of objects from one state to another. In this paper, we discuss the relationships between soft expert sets and information systems. It is shown that soft expert sets can be used as a class of special information systems. The usefulness of the soft expert set in mining association action rules also has been illustrated with the help of an example. Finally, the notion of the association action rule for the soft expert set in terms of support and confidence measures also has been introduced.

Keywords: Soft expert set, association rule, support, confidence and action rule.

1. Introduction

In association rule mining, the rules extracted from an information system using support- confidence framework are used by the domain experts who need to filter what information is interesting or trivial. Action rules were introduced in [16] as a new class of rule discovery that provides hints on possible actions to be taken in a business to achieve a desired target.

Mining action rules is defined as the process of identifying patterns in a decision system capturing the possible changes to certain object attributes that may lead to a change in the decision value [16]. Generally, action rule mining operates on a decision system [3] with objects having three classes of attributes: *stable*, *flexible* and *decision*. The stable attributes are fixed type attributes that cannot be changed or, in some approaches, require a prohibitive high cost to change them [4]. Some examples of stable attributes are: date of birth, weather conditions and conversely, flexible attributes are there on which the analysts have a certain degree of freedom for manipulation such as color, sale's percentage etc. The existing action rule discovery methods use a decision table as their primary search domain. The employed strategies are limited to candidate generation-and-test. In our approach, the discovery of action rules is based on a domain of experts that we create from the decision system, called the *experts opinion table*.

An action rule is a rule extracted from a decision system that describes a possible transition of objects from one state to another with respect to a distinguished attribute called a decision attribute [16]. We assume that attributes used to describe objects in a decision system are partitioned into stable and flexible. The values of flexible attributes can be changed. This change can be influenced and controlled by users. The action rule mining was initially based on comparing two groups of profiles of targeted objects i.e., desirable and undesirable.

A further investigation on action rules can be found in [18][20][17][19][21][22]. He et. al. [4] is probably the first attempt towards formally introducing the problem of mining action rules without pre-existing classification rules. The authors explicitly formulate it as a research problem within the support-confidence-cost framework. The authors introduce a new approach for the soft expert set generating association-type action rules. Im and Ra [6], present an effective algorithm called ARED, which is based on Pawlak's model of an information system [11]. Its goal was to identify certain relationships between granules defined by the indiscernibility relation on its objects. Some of these relationships uniquely define action rules for the information system.

Based on the detailed analysis of the inherent difficulties of some other theories such as interval analysis and fuzzy set theory for dealing with uncertainty and incompleteness of information. Molodtsov [10] propose soft set theory with sufficient parameters so that it is free from the corresponding difficulties. Also he introduces a series of interesting applications of the theory in stability and regularization, game theory, operations research, probability and statistics. Also, Maji et al. [8] propose several operations on soft sets, along with some basic properties of these operations. Many researchers have been studying soft set theory and they create some models to solve problems in decision making and medical diagnosis, but most of these models deal only with one expert. If we want to take the opinions of more than one expert, this causes a problem with the user, especially with those who use questionnaires in their work and studies. In our model, the user can know the opinion of all experts in one model without any operation. Even after any operation on our model, the user can know the opinion of all the experts.

On the other hand, information systems have been intensively studied by many authors from several domains containing knowledge engineering [2, 7, 12], rough set theory [13-15, 24], granular computing [23], data mining and knowledge discovery [25], and so on. Through a careful study, one can observe that there exists some compact connections between soft expert sets and information systems. We clarify the relationships of these two branches, and intend to unify them in this paper.

Moreover, Maji et al.[9] discuss the application of soft set theory in a decision making problem. Also, T. Herawan et. al.[5] develop a soft set approach for mining association patterns in transactional datasets. Meanwhile, S. Alkhazaleh et al. [1] introduce the concept of a soft expert set which can be found more effective and useful. Also the authors define its basic operations, namely, complement, union, intersection, AND and OR. Finally, we show an application of this concept in decision-making problem.

The rest of the paper is organized as follows. In section 2, we provide the background of our work. In section 3, we discuss the relationship between soft expert set and information system. We show that, soft expert set can be represented as an information system. In section 4, we have introduced and defined support confidence framework of association action rules using soft expert set approach. Also to establish the approach we have used approximate examples using the expert opinions in terms of stable and flexible values for a given attribute sets. We have also introduced the concept of attributes co-occurrence in expert's opinions for the given universal set. In section 5, we establish the applicability of soft expert set approach on the dependency of expert's opinions for the attributes and the concept of an information system for soft expert set approach for the association action rule mining.

2. Background and Objectives

In this section, we introduce the concept, background and definitions of action rules and soft expert set. Also we describe there with necessary illustration.

2.1 Knowledge Representation

We present the notion of a finite table representing knowledge, called an information system. An information system as basically a 4-tuple (quadruple) $S=(U,\ A,\ V,\ f)$, where $U=\{u_1,u_2,\ u_3,\ldots\ldots\ldots u_{|U|},\}$ is a non-empty finite set of objects, $A=\{a_1,a_2,\ a_3,\ldots\ldots\ldots a_{|A|},\}$ is a non-empty set of attribute, $V=\bigcup_{a\in A} V_a$, where V_a is the domain (value set) of attribute 'a' and $f:U\times A\rightarrow V$ is an information function such that $f(u,a)\in V_a$, for every $(u,\ a)\in U\times A$, called information (knowledge) function. An information system is also called a knowledge representation systems or an attribute-valued system and can be intuitively expressed in terms of a 2-D array as shown in table 1. The complexity for computing an information system $S=(U,\ A,\ V,\ f)$ is $|U|\times|A|$. Since there are $|U|\times|A|$ values of $f(u_i\ ,\ a_j)$ to be computed, where $i=1,2,3,\ldots.|U|$, $j=1,2,3,\ldots.|A|$.

Note that t induces a set of maps, $t=f(u\ ,\ a):U\times A\rightarrow V$. Each map is tuple
$t_i=[f(u_i\ ,\ a_1),\ f(u_i\ ,\ a_2),\ f(u_i\ ,\ a_3),\ldots\ldots\ldots f(u_i\ ,\ a_{|A|})]$, where $i=1,2,3,\ldots.|U|$.

Table 1: An Information System

U	a_1	a_2		a_k		a_A
u_1	$f(u_1,a_1)$	$f(u_1,a_2)$	………	$f(u_1,a_k)$	……….	$f(u_1,a_{\|A\|})$
u_1	$f(u_2,a_1)$	$f(u_2,a_2)$	………	$f(u_2,a_k)$	………	$f(u_2,a_{\|A\|})$
u_1	$f(u_3,a_1)$	$f(u_3,a_2)$	………	$f(u_{31},a_k)$	……….	$f(u_3,a_{\|A\|})$
⋮						
⋮						
u_1	$f(u_{\|U\|},a_1)$	$f(u_{\|U\|},a_2)$	………	$f(u_{\|U\|},a_k)$	………	$f(u_{\|U\|},a_{\|A\|})$

2.2 Information system

An information system can be represented as a triple $S = (X, A, V)$, where

i) X is a non-empty, finite set of objects

ii) A is a nonempty, finite set of attributes, i.e., $a : X \rightarrow V_a$ is a function for any $a \in A$, where V_a is called the domain of 'a' and

iii) $V = U\ \{V_a : a \in A\}$.

For example, Table 2 shows an information system S with a set of eight objects, say $X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$, set of four attributes, say $A = \{a, b, c, d\}$, and a set of their values $V = \{a_1, a_2, b_1, b_2, c_1, c_2, d_1, d_2\}$.

Table 2: Information System S

	A	b	c	d
x_1	a_1	b_1	c_1	d_1
x_2	a_2	b_1	c_1	d_1
x_3	a_2	b_2	c_1	d_2
x_4	a_2	b_2	c_2	d_2
x_5	a_2	b_1	c_1	d_1
x_6	a_2	b_2	c_1	d_2
x_7	a_2	b_1	c_2	d_2
x_8	a_1	b_2	c_2	d_1

Additionally, assume that $A = A_{St}\ U\ A_{Fl}$,, where attributes in A_{St} are called *stable* and attributes in A_{Fl} are called *flexible*. For example, "Date of birth" is a stable attribute, where as "Interest rate" for each customer account is a flexible attribute. The action rules are constructed from classification rules. In the other words, pre-existing classification rules are used or generated using a rule discovery algorithm and then action rules are constructed either from certain pairs of these rules or from a single classification rule. For instance, algorithm *ARAS* [19] generates sets of terms (built from values of attributes) around classification rules and constructs action rules directly from them. We aim to achieve the following objectives.

i). To extract action rules directly from a decision system without using pre-existing classification rules.

ii). To extract action rules that have minimal attribute involvement.

To meet these two goals, we introduce the notion of *expert sets*, *frequent expert sets*, and show how to build action rules from them. The symbols and notations used to define and describe the information are given in Table 3.

2.3 Action Rules

Let $S = (X, A, V)$ be an information system, where $V = U\ \{V_a : a \in A\}$. The notion of an atomic action set. By an *atomic action set* we mean an expression $(a, a_1 \rightarrow a_2)$, where a is an attribute and $a_1, a_2 \in V_a$. If $a_1 = a_2$, then a is called *stable* on a_1. Instead of $(a, a_1 \rightarrow a_1)$, we often write (a, a_1) for any $a_1 \in V_a$. By *action sets* we mean a smallest collection of sets such that:

i) If t is an atomic action set, then t is an action set.

ii) If t_1, t_2 are action sets and '$\bullet$' is a 2-argument function called *composition*, then $t_1 \bullet t_2$ is a candidate action set.

iii) If t is a candidate action set and for any two atomic action sets $(a, a_1 \rightarrow a_2)$, $(b, b_1 \rightarrow b_2)$ contained in t we have $a \neq b$, then t is an action set.

By the *domain* of an action set t, denoted by $Dom(t)$, we mean the set of all attribute names listed in t. So, by an *action rule* we mean any expression $r = [\, t_1 \Rightarrow t_2 \,]$, where t_1 and t_2 are action sets. Additionally, we assume that $Dom(t_2) \cup Dom(t_1) \subseteq A$ and $Dom(t_2) \cap Dom(t_1) = \Phi$. The domain of action rule r is defined as $Dom(t_1) \cup Dom(t_2)$.

2.4 Soft Expert Set

Let U be an initial universe set and let E be a set of parameters.

***Definition* 1:** A pair (F, E) is called a soft set (over U) if and only if F is a mapping of E into the set of all subsets of the set U.

In other words, a soft set is a parameterized family of subsets of the set U. Every set $F(\varepsilon)$, $\varepsilon \in A$, from this family may be considered as the set of ε-elements of the soft set (F, E) or as the set of ε-approximate elements of the soft set.

***Definition* 2:** Let U be a universe set, E be a set of parameters and X be a set of experts (agents). Let $O=\{0, 1\}$ be a set of opinions, where *'0'* represents *disagree* and *'1'* represents *agree*. Let $Z= E \times X \times O$ and $A \subseteq Z$. A pair (F, A) is called a *soft expert set* over U, where F is a mapping given by $F: A \rightarrow P(U)$, where $P(U)$ denotes the power set of U.

Table 3: Symbols used their meaning

Symbols used	Meaning	Symbols used	Meaning
S	Information system	t	Expert set
X	Set of objects	$t_1 \bullet t_2$	Composition of expert sets
A or E	Set of attributes or parameters	N_S	Standard interpretation
V	Domain	$Dom(t)$	Domain of an expert set t
U	Set of universe	$t_1 \Rightarrow t_2$	If t_1 then t_2
$P(U)$	Power set of U	$r = [\, t_1 \Rightarrow t_2 \,]$	Action rule
(F, A)	Soft expert set	$Coo(u)$	Co-occurrence set of u
F	Mapping	$Sup(X)$	Support of a set X
O	Set of opinions	$Sup(r)$	Support of an action r
$Z=E\times X\times O$	Triple of attribute, object, opinion	$Conf(r)$	Confidence of an action r
$\{a, 0 \rightarrow 1\}$	Atomic expert set	*Card*	Cardinality
$\{a, 1 \rightarrow 0\}$	Atomic expert set	*Min_sup*	Minimum support
$\{a, 0\}$	Attribute '*a*' is stable	*Min_conf*	Minimum confidence

Now we define information system in terms of soft expert set system and establish its properties.

3. Information System into a Soft Expert Sets

In this section, we introduce two prepositions to provide theoretical basis of our work.

Proposition 1: If (F, E) is a soft expert set over the universe U then (F, E) is an information system $S = (U, A, V)$.

Proof: Let (F, E) be a soft expert set over the universe U, we define a collections of mappings

$F = \{f_1, f_2 \ldots ; f_n\}$ where $f_1 : U \rightarrow V_1$ and $f_1 = \begin{cases} 1, & x \in f(e_1) \\ 0, & x \notin f(e_1) \end{cases}$

$$f_2 : U \rightarrow V_2 \quad and \quad f_2 = \begin{cases} 1, & x \in f(e_2) \\ 0, & x \notin f(e_2) \end{cases}$$

$$\vdots$$

$$f_n : U \rightarrow V_n \quad and \quad f_n = \begin{cases} 1, & x \in f(e_n) \\ 0, & x \notin f(e_n) \end{cases}$$

Thus, if $A=E$, $V=\bigcup_{e_i \in A} V_{e_i}$, where $V_{e_i}=\{0, 1\}$, then a soft expert set *(F, E)* can be considered as an information system *S= (U, A,V)*.

Proposition 2: If *S = (U, A, V)* is an information system then *S = (U, A, V)* is a soft expert set over the universe *U.*

Proof: Proof is trivial.

The following example will help understand the ability of a soft expert set and the relationship an information system based on expert opinions

***Example* 1:** Let there are eight candidates who form the universe $U=\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$. Consider a set of attributes or parameters $E=\{a, b, c, d\}$, Let $X=\{p, q, r, s\}$ be a set of experts or committee members and $O=\{0=stable, 1=flexible\}=\{0, 1\}$ be a set of opinions for the experts and $Z=E\times X\times O$ and $A\subseteq Z$.

Considering the soft expert set *(F, A)* which describes the opinions of the four experts about the candidates, we get the following:

(F, A) = {

expert *p* has an opinion that the attribute *a* is stable for the candidates x_1, x_8,
expert *q* has an opinion that the attribute *b* is stable for the candidates x_1, x_2, x_5, x_7,
expert *r* has an opinion that the attribute *c* is stable for the candidates x_1, x_2, x_3, x_5, x_6,
expert *s* has an opinion that the attribute *d* is stable for the candidates x_1, x_2, x_5, x_8,
experts *q, r, s* have no opinions for the attribute *a,*
experts *p, r, s* have no opinions for the attribute *b,*

expert *p* has an opinion that the attribute *a* is flexible for the candidates $x_2, x_3, x_4, x_5, x_6, x_7$,
expert *q* has an opinion that the attribute *b* is flexible for the candidates x_3, x_4, x_6, x_8,
expert *r* has an opinion that the attribute *c* is flexible for the candidates x_4, x_7, x_8,
expert *s* has an opinion that the attribute *d* is flexible for the candidates x_3, x_4, x_6, x_7,
experts p, *q, s* have no opinions for the attribute *c,*
experts *q, q, r* have no opinions for the attribute *d* }

In other words

(a, p, 0) { x_1, x_8}, (b, q, 0){ x_1, x_2, x_5, x_7}
(c, r, 0){ x_1, x_2, x_3, x_5, x_6 } (d, s, 0){ x_1, x_2, x_5, x_8}
(a, p, 1) {$x_2, x_3, x_4, x_5, x_6, x_7$}
(b, q, 1) { x_3, x_4, x_6, x_8}
(c, r, 1) { x_4, x_7, x_8 }
(d, s, 1) { x_3, x_4, x_6, x_7}

Note *that (a, q, 0), (a, r, 0), (a, s, 0), (b, p, 0), (b, r, 0), (b, s, 0), (c, p, 0), (c, q, 0), (c, d, 0), (d, p, 0), (d, q, 0), (d, r, 0), (a, q, 1), (a, r, 1), (a, s, 1), (b, p, 1), (b, r, 1), (b, s, 1), (c, p, 1), (c, q, 1), (c, d, 1), (d, p, 1), (d, q, 1), (d, r, 1)* are empty sets, since the experts gave no opinions.

Thus we can view the soft expert set *(F, A)* as consisting of the following collection of approximations:

(F, A)={(a, p, 0) { x_1, x_8*},(b, q, 0){* x_1, x_2, x_5, x_7*},*
(c, r, 0){ x_1, x_2, x_3, x_5, x_6 *}, (d, s, 0){* x_1, x_2, x_5, x_8*},*
(a, p, 1){ $x_2, x_3, x_4, x_5, x_6, x_7$*},*
(b, q, 1){ x_3, x_4, x_6, x_8*},*
(c, r, 1){ x_4, x_7, x_8 *},*
(d, s, 1){ x_3, x_4, x_6, x_7*}}.*

Obviously we can write as

(F, A)={(a, 0) { x_1, x_8*},(b, 0){* x_1, x_2, x_5, x_7*},*
(c, 0){ x_1, x_2, x_3, x_5, x_6 *}, (d, 0){* x_1, x_2, x_5, x_8*},*
(a, 1){ $x_2, x_3, x_4, x_5, x_6, x_7$*},*
(b, 1){x_3, x_4, x_6, x_8*},*
(c,1){ x_4, x_7, x_8 *},*
(d, 1){ x_3, x_4, x_6, x_7*}}.*

It means that

i) Expert opinions for the attribute "a" is stable for the candidates x_1, x_8 and flexible for the candidates $x_2, x_3, x_4, x_5, x_6, x_7$.

ii) Expert opinions for the attribute “b” is stable for the candidates x_1, x_2, x_5, x_7 and flexible for the candidates x_3, x_4, x_6, x_8.

iii) Expert opinions for the attribute “c” is stable for the candidates x_1, x_2, x_3, x_5, x_6 and flexible for the candidates x_4, x_7, x_8.

iv) Expert opinions for the attribute “d” is stable for the candidates x_1, x_2, x_5, x_8 and flexible for the candidates x_3, x_4, x_6, x_7.

Table 4: Representation of the soft expert set from Example 1

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	*Count*
(a, o)	*1*	*0*	*0*	*0*	*0*	*0*	*0*	*1*	*2*
(b, o)	*1*	*1*	*0*	*0*	*1*	*0*	*1*	*0*	*4*
(c, o)	*1*	*1*	*1*	*0*	*1*	*1*	*0*	*0*	*5*
(d, o)	*1*	*1*	*0*	*0*	*1*	*0*	*0*	*1*	*4*
(a, 1)	*0*	*1*	*1*	*1*	*1*	*1*	*1*	*0*	*6*
(b, 1)	*0*	*0*	*1*	*1*	*0*	*1*	*0*	*1*	*4*
(c, 1)	*0*	*0*	*0*	*1*	*0*	*0*	*1*	*1*	*3*
(d, 1)	*0*	*0*	*1*	*1*	*0*	*1*	*1*	*0*	*4*

Table 5: Stable soft expert set

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
(a, o)	*1*	*0*	*0*	*0*	*0*	*0*	*0*	*1*
(b, o)	*1*	*1*	*0*	*0*	*1*	*0*	*1*	*0*
(c, o)	*1*	*1*	*1*	*0*	*1*	*1*	*0*	*0*
(d, o)	*1*	*1*	*0*	*0*	*1*	*0*	*0*	*1*
$c_j=\sum_i x_{i,j}$	c_1=4	c_2=3	c_3=1	c_4=0	c_5=3	c_6=1	c_7=1	c_8=2

Table 6: Flexible soft expert set

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
(a, 1)	*0*	*1*	*1*	*1*	*1*	*1*	*1*	*0*
(b, 1)	*0*	*0*	*1*	*1*	*0*	*1*	*0*	*1*
(c, 1)	*0*	*0*	*0*	*1*	*0*	*0*	*1*	*1*
(d, 1)	*0*	*0*	*1*	*1*	*0*	*1*	*1*	*0*
$k_j=\sum_i x_{i,j}$	k_1= 0	k_2= 1	k_3=3	k_4= 4	k_5=1	k_6=3	k_7=3	k_8=2

Table 7: Evaluate stable and flexible attributes

$c_j=\sum_i x_{i,j}$	$k_j=\sum_i x_{i,j}$	$s_j=c_j-k_j$
$c_1 = 4$	$k_1 = 0$	$s_1 = 4$
$c_2 = 3$	$k_2 = 1$	$s_2 = 2$
$c_3 = 1$	$k_3 = 3$	$s_3 = -2$
$c_4 = 0$	$k_4 = 4$	$s_4 = -4$
$c_5 = 3$	$k_5 = 1$	$s_5 = 2$
$c_6 = 1$	$k_6 = 3$	$s_6 = -2$
$c_7 = 1$	$k_7 = 3$	$s_7 = -2$
$c_8 = 2$	$K_8 = 2$	$s_8 = -1$

Then, $\max\{s_j\}=s_1=4$ means that candidate ‘x_1’ is more stable and $\min\{s_j\}=s_4=-4$ means that candidate ‘x_4’ is more flexible out of them.

Table 8: Representing the soft expert set as information system

	A	*b*	*c*	*d*
x_1	*0*	*0*	*0*	*0*

x_2	1	0	0	0
x_3	1	1	0	1
x_4	1	1	1	1
x_5	1	0	0	0
x_6	1	1	0	1
x_7	1	0	1	1
x_8	0	1	1	0

That is

i) Expert opinions in second column of the attribute "*a*" is stable for the candidates x_1, x_8 and flexible for the candidates $x_2 x_3, x_4, x_5, x_6, x_7$.

ii) Expert opinions in third column of the attribute "*b*" is stable for the candidates x_1, x_2, x_5, x_7 and flexible for the candidates x_3, x_4, x_6, x_8.

iii) Expert opinions in forth column of the attribute "*c*" is stable for the candidates x_1, x_2, x_3, x_5, x_6 and flexible for the candidates x_4, x_7, x_8.

iv) Expert opinions in fifth column of the attribute "*d*" is stable for the candidates x_1, x_2, x_5, x_8 and flexible for the candidates x_3, x_4, x_6, x_7.

Next we define and describe action rules in terms of soft expert set.

4. Action Rules for Soft Expert Set

Let *(F, E)* be a soft expert set over *U*, expert set *X* and its opinion set $O = \{stable = 0,\ flexible = 1\} = \{0, 1\}$. So, an atomic expert set can be defined as follows.

Definition 3: Atomic expert set and domain

It can be defined as the expression $(a, 0 \rightarrow 1)$ where '*a'* is an attribute in *A* and *0, 1* are expert opinion values of '*a'*. If the attribute is stable or dies not change its value then the atomic expert set is expressed as $(a, 0)$. The domain of an atomic expert set is its attribute. $Dom((a, 0 \rightarrow 1)) = a$

By an ***atomic expert set,*** we represent an expression $\{a, 0 \rightarrow 1\}$, where '*a*' is an attribute and $0, 1 \in O$. The '*a*' is called *stable* or *flexible*, instead of

if $\{a, 0 \rightarrow 0\}$ often write $(a, 0)$ for $0 \in O$, then the attribute '*a*' is *stable* or

if $\{a, 1 \rightarrow 1\}$ often write $(a, 1)$ for $1 \in O$, then the attribute '*a*' is *flexible*.

The ***domain*** of an expert set *t*, denoted by *Dom(t)* means the set of all attribute names listed in *t*. The concept of atomic expert set can be extended to define expert set.

Definition 4: Expert sets

Constructed as the conjunction of atomic expert sets with the composition operator '●'. If t_1, t_2 are two atomic expert sets with different attributes then $t = t_1 \bullet t_2$ is an expert set. The domain of the expert set *t* is the set of attributes from all its atomic expert sets. Here, $Dom(t) = Dom(t_1) \cup Dom(t_2)$.

By ***expert sets*** we mean a smallest collection of sets such that

i) If '*t'* is an atomic expert set then '*t'* is also an expert set.

ii) If t_1, t_2 are expert sets and '●' is a 2-argument function called *composition*, then $t_1 \bullet t_2$ is a *candidate expert set*.

iii) If *t* is a candidate expert set and for any two atomic expert sets $(a, 0 \rightarrow 1), (b, 0 \rightarrow 1)$ contained in *t* we have $a \neq b$, then *t* is an expert set.

Definition 5: Action rule

An action rules *r* is expressed as $r = [t_1 \rightarrow t_2]$, where t_1 and t_2 are two expert sets. Typically t_2 is the action comprising only the expert opinion of the attribute.

The support is calculated similarly to expert sets by considering the $t_1 \bullet t_2$ as an expert set itself. By an ***action rule*** we mean any expression $r = [\, t_1 \Rightarrow t_2 \,]$, where t_1 and t_2 are expert sets. Additionally, we assume that $Dom(t_2) \cup Dom(t_1) \subseteq A$ and $Dom(t_2) \cap Dom(t_1) = \Phi$. The domain of action rule *r* is defined as $Dom(t_1) \cup Dom(t_2)$.

Definition 6: Standard interpretation (noted Ns)

The introduction of the *Standard Interpretation* is the basis of measures like support and confidence. In association mining, the support of an itemset is simply the count of objects. For action rules, we need to consider two sets. The first set is all the objects with attributes value equal to the initial state of the action; the second set, respectively, is all the objects having attributes values equal to the values of the final state of the action.

Definition 7: Support of an expert set

Assume t an expert set with standard interpretation $N_S(t) = [Y_1, Y_2]$. The support *Sup* of t defined as $\sup(t) = \min\{card(Y_1),\ card(Y_2)\}$, for the two states, the support is concerned only with the state having the lowest occurrences, it was only in terms of number of occurrences of the initial state.

Next we define co-occurrence set, support and confidence in terms of expert set to build the framework for association action rules.

***Definition 8: Co-occurrence* set**

Let *(F, A)* be a soft expert set over the inverse U and $u \in U$. An item *co-occurrence* set in a transaction *'x'* can be defined as

$Coo(u) = \{z \in Z : (u, z) = 1\}$, where $Z = E \times X \times O$ and $A \subseteq Z$.

List of the co-occurrence of items in transaction of Example 1, Table 4:

$Coo(x_1) = \{(a, 0), (b, 0), (c, 0), (d, 0)\}$
$Coo(x_2) = \{(a, 1), (b, 0), (c, 0), (d, 0)\}$
$Coo(x_3) = \{(a, 1), (b, 1), (c, 0), (d, 1)\}$
$Coo(x_4) = \{(a, 1), (b, 1), (c, 1), (d, 1)\}$
$Coo(x_5) = \{(a, 1), (b, 0), (c, 0), (d, 0)\}$
$Coo(x_6) = \{(a, 1), (b, 1), (c, 0), (d, 1)\}$
$Coo(x_7) = \{(a, 1), (b, 0), (c, 1), (d, 1)\}$
$Coo(x_8) = \{(a, 0), (b, 1), (c, 1), (d, 0)\}$

Let *(F, A)* be a soft expert set over the universe U and $X \subseteq A$. A set of attributes X is said to be *supported* by transaction $u \in U$, if $X \subseteq Coo(u)$.

Definition 9: Support of a set

Let *(F, A)* be a soft expert set over the universe U and $X \subseteq A$. The *support of a set* of attributes or parameters X, denoted by *sup(X)* is defined by the number of transaction U supporting X.

$\sup(X) = |\{u : X \subseteq Coo(u)\}|$, where $|X|$ is the cardinality of X.

***Definition 10: Support* and *confidence* of an action rule**

The support and confidence of an action rule $r = [t_1 \rightarrow t_2]$, considering $N_S(t_1) = [Y_1, Y_2]$ and $N_S(t_2) = [Z_1, Z_2]$, with Y_1, Y_2 are not empty. *Support* and *confidence* of r are defined as follows:

$$\sup(r) = \min\{card(Y_1 \cap Z_1),\ card(Y_2 \cap Z_2)\} ------> (i)$$

$$conf(r) = \left[\left\{\frac{card(Y_1 \cap Z_1)}{card(Y_1)}\right\} \bullet \left\{\frac{card(Y_2 \cap Z_2)}{card(Y_2)}\right\}\right] ----------> (ii)$$

The generation of an action rules is similar to association rule mining where frequent item sets are first extracted. The algorithm, which is any algorithm based on Apriori U, generates expert sets with support that exceeds a users specified threshold value called *minimum support* (mins_sup). Such expert sets which meet this criterion is also called frequent expert set.

An action rule is constructed as follows.

i) If t is a frequent expert set and t_1 *is* a subset of t then $r = [t_1 \rightarrow t_2]$,

ii) If $conf(r) \geq min\_conf$, where min_conf is the minimum confidence specified, r is a valid rule.

Let us assume that a soft expert set *(F, A)* in the information system as given in Table 8, with *{a, c}* as stable attributes and *{b, d}* as *flexible* attributes. We take minimum support $\lambda_1 = 2$ and minimum confidence $\lambda_2 = 4/9$. Thus the following *frequent expert sets* can be constructed.

$sup\{(a, o)\} = \{x_1, x_8\} = 2$
$sup\{(b, o)\} = \{x_1, x_2, x_5, x_7\} = 4$
$sup\{(c, o)\} = \{x_1, x_2, x_3, x_5, x_6\} = 5$
$sup\{(d, o)\} = \{x_1, x_2, x_5, x_8\} = 4$
$sup\{(a, 1)\} = \{x_2 x_3, x_4, x_5, x_6, x_7\} = 6$
$sup\{(b, 1)\} = \{x_3, x_4, x_6, x_8\} = 4$
$sup\{(c, 1)\} = \{x_4, x_7, x_8\} = 3$
$sup\{(d, 1)\} = \{x_3, x_4, x_6, x_7\} = 4$

Association action rules can be constructed from frequent expert sets. For instance, we can generate association action rule $[(a, 1) \bullet (b, 0 \to 1)] \to [(c, 0) \bullet (d, 0 \to 1)]$

Now action rules assuming that the soft expert set in the form of information system, we assume that *a, c* are *stable* and *b, d* are *flexible* attributes. For the expressions

i) $(a, 0)$ means that the value '*0*' of attribute '*a*' remains unchanged.
ii) $(c, 0)$ means that the value '*0*' of attribute '*c*' remains unchanged.
iii) $(a, 1)$ means that the value '*1*' of attribute '*a*' remains unchanged.
iv) $(c, 1)$ means that the value '*1*' of attribute '*c*' remains unchanged.
v) $(b, 0 \to 1)$ means that the value of the attribute '*b*' is changed from *0* to *1*
vi) $(d, 0 \to 1)$ means that the value of the attribute '*d*' is changed from *0* to *1*.
vii) $(b, 1 \to 0)$ means that the value of the attribute '*b*' is changed from *1* to *0*
viii) $(d, 1 \to 0)$ means that the value of the attribute '*d*' is changed from *1* to *0*.

The expression $r = [\{(a, 0) \bullet (b, 0 \to 1)\} \Rightarrow (d, 0 \to 1)]$, is an example of an action rule.

The rule says that if value '*a*' remains unchanged and value '*b*' will change from *0* to *1*. Then it is expected that the value'*d*' will change from *0* to *1*. The domain *Dom(r)* of action rule *r* is equal to $\{a, b, d\}$

Standard interpretation N_S of soft expert set *(F, A)* is defined as follows.

i) If $(a, 0 \to 1)$ is an atomic expert set then $N_S \{(a, 0 \to 1)\}=[\{x \in X : a(x)=0\}, \{x \in X : a(x)=1\}]$

ii) If $t_1=[(a, 0 \to 1) \bullet t]$ and $N_S(t)=[Y_1, Y_2]$, where $Y_1=\{x \in X : a(x)=0\}$, $Y_2=\{x \in X : a(x)=1\}$, then

$N_S (t_1) =[Y_1 \cap \{x \in X : a(x)=0\},\ Y_2 \cap \{x \in X : a(x)=1\}]$

Let us define $[Y_1, Y_2] \cap [Z_1, Z_2] = [Y_1 \cap Z_1, Y_2 \cap Z_2]$ and we assume that $N_S (t_1) =[Y_1 , Y_2]$ and $N_S (t_2) =[Z_1 , Z_2]$. If *t* is an expert set and $N_S (t)=[Y_1 , Y_2]$ then the ***support of t*** in soft expert set *(F, A)* is defined as $\sup(t) = \min\{card(Y_1),\ card(Y_2)\}$

Let $r = [t_1 \Rightarrow t_2]$ be an action rule, where $N_S(t_1) = [Y_1, Y_2]$ and $N_S(t_2)=[Z_1, Z_2]$. ***Support*** and ***confidence*** of *r* can be defined as follows.

$$\sup(r) = \min\{card(Y_1 \cap Z_1),\ card(Y_2 \cap Z_2)\}, \quad conf(r) = \left[\left\{\frac{card(Y_1 \cap Z_1)}{card(Y_1)}\right\} \bullet \left\{\frac{card(Y_2 \cap Z_2)}{card(Y_2)}\right\}\right]$$

5. An Application Soft Expert Set Approach to the Association Action Rules.

Consider is an approximation that the UGC is to decide the grades to be awarded to colleges based on their infrastructure and other quality teachers. Meanwhile, UGC selects a four member expert committee and give suggestion to give their assessment based on four points such as *location* (good or bad), *strength of the faculty* members (good or bad), *infrastructure* (good or bad) and *strength of the student's* (good or bad),

Assume that there are eight colleges, which form the universe $U=\{c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8\}$. So the four member expert committee considers a set of attributes or parameters, $E= \{a, b, c, d\}$ where the parameters *a, b, c, and d* stand for *location*, *faculty strength*, *infrastructure* and *student's strength* of the colleges. Expert committee decides the opinion for each attribute as bad or good.

Let $O = \{bad = 0,\ good = 1\} = \{0, 1\}$ be a set of opinions for the experts. $Z = E \times X \times O$ and $A \subseteq Z$. Considering the soft expert set *(F, A)* which describes the four experts to make their opinions for the colleges, we get the following.

$(F, A)=\{(a, 0) \{ c_1, c_8\}, (b, 0)\{ c_1, c_2, c_5, c_7\}, (c, 0)\{ c_1, c_2, c_3, c_5, c_6 \}, (d, 0)\{ c_1, c_2, c_5, c_8\}, (a, 1)\{ c_2, c_3, c_4, c_5, c_6, c_7\}, (b, 1)\{c_3, c_4, c_6, c_8\},(c,1)\{ c_4, c_7, c_8 \}, (d, 1)\{ c_3, c_4, c_6, c_7\}\}$.

We assume that, attributes *location* and *infrastructure* are called *stable* and attributes *faculty strength* and *student's strength* are called *flexible*.

Table 9: Representing the soft expert set as information system

	A	b	c	d
c_1	0	0	0	0
c_2	1	0	0	0
c_3	1	1	0	1
c_4	1	1	1	1
c_5	1	0	0	0
c_6	1	1	0	1
c_7	1	0	1	1
c_8	0	1	1	0

A) *Atomic expert set*

The expression $(a,\ 0 \rightarrow 1)$ where '*a'* is an attribute in *A* and *0, 1* are expert opinion values of '*a'*. If the attribute is stable or did not change its value then the atomic expert set is expressed as (*a*, 0). The domain of an atomic expert set is its attribute. $Dom(a,\ 0 \rightarrow 1)=a$

Example: Consider *student's strength* a flexible attribute with values V_{ss}={30%, 10%, 50%}. The atomic expert set (*student's strength*, 30%→10%) means changing the value of student strength from 30% to 10%.

B) *Expert sets*

The conjunction of atomic expert sets with the composition operator '●'. If t_1, t_2 are two atomic expert sets with different attributes, then $t=t_1 \bullet t_2$ is an expert set. The domain of the expert set *t* is the set of attributes from all its atomic expert sets, here, $Dom(t)= Dom(t_1) \cup Dom(t_2)$.

Example: Consider *Infrastructure as* a stable attribute and *Student's strength* a flexible attribute. An expert set could be the composition [(*Infrastructure*, good)●(*Faculty strength*, 30%→10%)●(*Student's strength*, bad → good)], which could be read as follows, for the college infrastructure is good, change faculty strength from 30% → 10% and student strength, good → bad.

C) *Standard interpretation (N_S)*

The *Standard Interpretation* is the basis of measures like support and confidence. In association mining, the support of an itemset is simply the count of objects. For action rules, we need to consider two sets. The first set is all the objects with attributes value equal to the initial state of the action; the second set, respectively, is all the objects having attributes values equal to the values of the final state of the action.

Example: The *Standard interpretation* of the expert set N_S, [(*Location*, good)●(*Faculty strength*, 60% → 80%)●(*Student's strength*, bad→ good)]=[Y_1,Y_2], Where: Y_1= {x ∈X: Location (x) = good ∧ Faculty strength(x) = 60% ∧ Student's strength(x) = bad}. Y_2= {x ∈X: Location (x) = good ∧ Faculty strength(x) = 80% ∧ Student's strength(x) = good}.

D) *Action rule*

Assume t an expert set with standard interpretation $N_S(t)=[Y_1,\ Y_2]$. The support *Supp* of *t* defined as $sup(t) = min\{card(Y_1),\ card(Y_2)\}$, for the two states, the support is concerned only with the state having the lowest occurrences, it was only in terms of number of occurrences of the initial state. An action rules *r* is expressed as $r = [t_1 \rightarrow t_2]$, where t_1 and t_2 are two expert sets. Typically t_2 is the action comprising only the expert opinion of the attribute.

Example: [(*Location, good*) ● (*Faculty strength, 40% → 65%*) ● (*Infrastructure, bad*) ● (*Student's strength, 70 % → 85%*)]

The support is calculated similarly is expert sets by considering the $t_1 \bullet t_2$ as an expert set itself.

E) *Support* and *confidence*

The support and confidence of an action rule $r = [t_1 \rightarrow t_2]$, considering $N_S(t_1) = [Y_1,\ Y_2]$ and $N_S(t_2)=[Z_1,\ Z_2]$, with Y_1, Y_2 are not empty. *Support* and *confidence* of *r* are defined as follows:

$$\sup(r) = \min\{card(Y_1 \cap Z_1),\ card(Y_2 \cap Z_2)\}\ ,\ conf(r) = \left[\left\{\frac{card(Y_1 \cap Z_1)}{card(Y_1)}\right\} \bullet \left\{\frac{card(Y_2 \cap Z_2)}{card(Y_2)}\right\}\right]$$

An action rule is constructed as following:

i) If *t* is a frequent expert set and t_1 *is* a subset of *t* then $r = [t_1 \rightarrow t_2]$,

ii) If $Conf(r) \geq min\_Conf$, where *min_Conf* is the minimum confidence specified, r is a valid rule.

In Table 9, we can find many action rules associated with soft expert set in the form of information system. Let us take

$r = [\{(a, 1) \bullet (b, 0 \rightarrow 1)\} \Rightarrow (d, 0 \rightarrow 1)]$ as an example of action rule then,

$N_S\{(a, 1)\} = [\{x_2, x_3, x_4, x_5, x_6, x_7\}, \{x_2, x_3, x_4, x_5, x_6, x_7\}]$

$N_S\{(b, 0 \rightarrow 1)\} = [\{x_1, x_2, x_5, x_7\}, \{x_3, x_4, x_6, x_8\}]$,

$N_S\{(a, 1) \bullet (b, 0 \rightarrow 1)\} = [\{x_2, x_5, x_7\}, \{x_3, x_4, x_6,\}]$

$N_S\{(d, 0 \rightarrow 1)\} = [\{x_1, x_2, x_5, x_8\}, \{x_3, x_4, x_6, x_7\}]$

Clearly $\sup(r) = \min\{2, 3\} = 2$ and $conf(r) = \{(2/3) \bullet (3/3)\} = \{(2/3) \bullet 1\} = 2/3$

To generate the expert opinion tables:

Table 10: Good opinion for expert table

	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8
(a, 1)	0	1	1	1	1	1	1	0
(b, 1)	0	0	1	1	0	1	0	1
(c, 1)	0	0	0	1	0	0	1	1
(d, 1)	0	0	1	1	0	1	1	0
$m_j = \sum_i x_{i,j}$	$m_1 = 0$	$m_2 = 1$	$m_3 = 3$	$m_4 = 4$	$m_5 = 1$	$m_6 = 3$	$m_7 = 3$	$m_8 = 2$

Table 11: Bad opinion for expert table

	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8
(a, o)	1	0	0	0	0	0	0	1
(b, o)	1	1	0	0	1	0	1	0
(c, o)	1	1	1	0	1	1	0	0
(d, o)	1	1	0	0	1	0	0	1
$n_j = \sum_i x_{i,j}$	$n_1 = 4$	$n_2 = 3$	$n_3 = 1$	$n_4 = 0$	$n_5 = 3$	$n_6 = 1$	$n_7 = 1$	$n_8 = 2$

Table 12: Evaluate grades

$m_j = \sum_i x_{i,j}$	$n_j = \sum_i x_{i,j}$	$s_j = m_j - n_j$
$m_1 = 0$	$n_1 = 4$	$s_1 = -4$
$m_2 = 1$	$n_2 = 3$	$s_2 = -2$
$m_3 = 3$	$n_3 = 1$	$s_3 = 2$
$m_4 = 4$	$n_4 = 0$	$s_4 = 4$
$m_5 = 1$	$n_5 = 3$	$s_5 = -2$
$m_6 = 3$	$n_6 = 1$	$s_6 = 2$
$m_7 = 3$	$n_7 = 1$	$s_7 = 2$
$m_8 = 2$	$n_8 = 2$	$s_8 = 0$

Then, $\max\{s_j\} = s_4 = 4$ means that the college 'c_4' is eligible for A^+ grade and $\min\{s_j\} = s_1 = -4$ means that college 'c_1' is not eligible for the any grade.

6. Advantages of Soft Expert Set Approach over Traditional Approach

From the above discussion, one can observe that

i) The support –confidence framework of the traditional ARM well represented using soft expert set.

ii) Although the framework attempts the define market basket association mining using two states (i.e. stable and flexible) soft expert set approach. However, it can be extended to multi-state association mining without ambiguity.

7. Conclusion

In this paper, we discuss the relationship between soft expert set and information system. We show that, soft expert set can be represented as an information system.

Our results show that the soft expert set is an information system in which the attributes only take two values stable and flexible, i.e., 0 and 1. We have proposed soft expert set approach for association action rule mining from the expert's opinions which have been successfully demonstrated using suitable example. We assume that attributes are divided into two groups: stable and flexible. By stable attributes we mean attributes whose values cannot be changed (for instance, location and infrastructure of the college). On the other hand attributes (like, faculty strength and student's strength of the college) whose values can be changed are called flexible. Rules are extracted from the expert's opinions.

We have introduced and defined support-confidence framework of association action rules using soft expert set approach. To establish the approach we have used approximate examples using the expert opinions in terms of stable and flexible values for a given attribute sets. We have also introduced the concept of attributes co-occurrence in expert's opinions for the given universal set. Finally, we establish the applicability of soft expert set approach on the dependency of expert's opinions for the attributes and the concept of an information system for soft expert set approach for the action rule mining.

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