An Inventory Model with time Dependent,Quadratic Deterioration rate, Exponential Demand, and time Dependent Holding Cost

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Abstract:- In this study, we develop a deterministic model with exponential time-dependent demand. The deterioration rate follows a quadratic time dependence and shortages are not allowed to meet the demand. Additionally, the holding cost varies with time. The mathematical model is designed to find the optimal total inventory cost. A numerical example is included to validate the proposed model.

***Keywords**:* Inventory, Demand, Holding Cost Time Dependent, No Shortage, Deterioration.

1. Introduction

Inventory control is crucial in supply chain management. Recently, most inventory issues have concentrated on the integration between suppliers and retailers. The integrated inventory model has gained increasing importance as both suppliers and retailers aim to enhance their mutual benefits. Consequently, they strive to establish strategic alliances to minimize costs or maximize profits. Through collaboration and information sharing, trading partners can achieve better outcomes. This means that the optimal contract quantity and the number of deliveries need to be determined at the beginning of the contract based on their combined total cost function.

Inventory plays a crucial role in every organization, be it manufacturing or service-oriented. Maintaining some level of inventory is essential for the smooth operation of any business. Claims of not holding any inventory are simply unrealistic. The Economic Order Quantity (EOQ) model, one of the earliest models developed by Wilson [21]. Traditional inventory models typically assume a constant demand rate. In reality, the demand for physical goods can vary based on time, stock levels, and pricing. The selling price significantly impacts the inventory system. Burwell [22], developed an economic lot size model addressing price-dependent demand influenced by quantity and freight discounts. Mondal [23] and colleagues examined an inventory system for ameliorating items with a price-dependent demand rate.

Harris introduced the foundational Economic Order Quantity (EOQ) model [4] in 1915, which Wilson later refined in 1934 by developing the calculation method. In 1963, Ghare and Schrader created an inventory model for items experiencing exponential decay [3]. Dave and Patel, in 1981, were the first to investigate inventory with linearly increasing demand without allowing shortages. More recent advancements include studies by Chung and Ting in 1993 [2], and Wee in 1995, which focused on models for deteriorating inventory. In 1999, Chang and Dye proposed a model that incorporates time-varying demand and partial backlogging [1]. Goyal and Giri (2001) reviewed modern trends in modeling inventories of deteriorating items, categorizing the models based on demand variations and constraints. In 2005, Ouyang and Cheng introduced a model for items with exponentially declining demand and partial backlogging. In 2007 [10], Alamri and Balkhi investigated the impact of learning and for getting on the optimal production lot size for deteriorating items with time-varying demand and deterioration rates. That same year, Dye and colleagues established the optimal selling price and lotsize while considering variable deterioration rates and exponential partial backlogging, noting that the fraction of backlogged orders increases exponentially as the waiting time for replenishment decreases. In 2008, Roy developed a deterministic inventory model in which the deterioration rate increases proportionally overtime, with demand influenced by the selling price and holding costs that vary over time [12]. Additionally, Liao introduced an EOQ model incorporating non-immediate receipt and exponential deterioration of items, alongside a two-level trade credit system. In 2009, Pareek and colleagues introduced a deterministic inventory model for deteriorating items that incorporates salvage value and accounts for shortages [11]. That same year, Skouri proposed an inventory model characterized by ramp-type demand, partial backlogging, and Weibull's deterioration rate [13]. In 2010, Mishra and Singh developed a deteriorating inventory model featuring waiting time partial backlogging, along with constant demand and constant deterioration rates [8]. In 2001, Yang and Wee developed a joint inventory model for unit deterioration, tailored for scenarios involving multiple consumers and a single merchant. Wee also examined supply chain inventory models that year, emphasizing the role of information technologies in enhancing coordination and mechanization, thereby reducing ordering costs for a single merchant and multiple consumers. In 2009, Zavanella and Zanoni introduced an analytical model based on an industrial case featuring a single merchant and multiple consumers [14]. In 2011, Shah created ajoint inventory model within a supply chain system with quadratic demand, involving multiple consumers and a single merchant. In 2015, Ghiami and Williams analyzed inventory models with a two-stage creation process for decaying items, where the manufacturer dispatches fixed production rate items in predefined order quantities for a set period, with excess inventory used for future deliveries. In 2010,Mandal introduced an EOQ inventory model for deteriorating items with a Weibull distribution, incorporating ramp-type demand and shortages [6]. Mishra and Singh (2011) developed an inventory model considering ramp-type demand, time-dependent deterioration with salvage value, and shortages. They also created a model for deteriorating inventory that accounts fortime-dependent demand and holding cost, with partial backlogging. In 2011, Hung proposed an inventory model that includes generalized demand, deterioration, and backorder rates [5]. Tripathi RP developed an EOQ model focused on cash flow and quantity-dependent trade credits [15]. Tripathi RP and Shweta established an EOQ model with quadratic time-sensitive demand, parabolic-time linked holding cost, and salvage value [16]. Goyal and S.K. devised an EOQ model considering permissible delays in payments [17]. Gupta PN and Agarwal introduced an order level inventory model with time-dependent deterioration [18]. Hariga M and Ben-Daya explored stochastic inventory models with deterministic variable lead time [19]. Hollier R.H. and Mak K.L. researched an inventory model for deteriorating items featuring generalized exponential decreasing demand, constant holding cost, and time-varying deterioration rates [20].

The rest of the paper is structured as follows: The next section outlines the notation and assumptions utilized throughout the paper. Section 3 presents the mathematical formulation.

2. Assumptions and Notations

1. Deterioration rate is quadratic with time dependent.
2. Deterioration rate = $a + bt + ct^2$; $a, b, c > 0$.
3. Demand rate is exponential and time dependent, i.e., $D(t) = fe^{gt}$; $f,g > 0$ and f, g are constants.
4. Shortages are not allowed.
5. Holding cost is time dependent $h(t) = ht$; $h > 0$ and h is constants.
6. Q is the maximum inventory level during $(0, T)$.
7. T is the length of the cycle.
8. Replenishment is instantaneous; lead time is zero.
9. There is no repair or replenishment of deteriorating item during the period under consideration.
10. C_1 is purchase cost per unit per unit time.

11. A is ordering cost per unit per unit time.

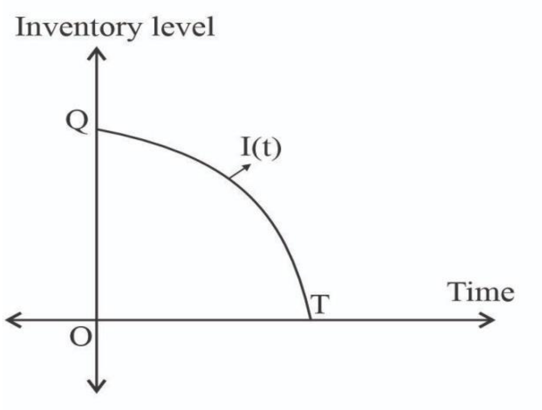

Figure 1: Graphical Representation of the Inventory System

3. Mathematical formulation and solution

The mathematical formulation, the rate of change of the inventory I(t) during period (0, T) by the following differential equation.

$$\frac{dI(t)}{dt} = -(a+bt+ct^2)I(t) - fe^{gt};\ 0 \le t \le T \qquad \ldots(1)$$

Where a, b, c, f and g are constants.

The initial conditions for the inventory level is Q when time t = 0, and also when t = T the inventory level Q becomes zero with deterioration rate. In cycle time inventory reaches maximum level and backlogged and again raises inventory level to Q.

Thus, boundary conditions are as follows:

$$I(0) = Q,\ I(T) = 0,$$

The solution of equation (1) with boundary conditions

Which is linear whose integrating factor is

$$e^{\int (a+bt+ct^2)dt} = e^{at+\frac{bt^2}{2}+\frac{ct^3}{3}}$$

Solution is $$I(t)\, e^{at+\frac{bt^2}{2}+\frac{ct^3}{3}} = -\int (fe^{gt})\, e^{at+\frac{bt^2}{2}+\frac{ct^3}{3}} + K \qquad \ldots(2)$$

Where K is constant of integration

Using initial condition I(0) = Q then we get K = Q and

$$Q = \int_0^T fe^{gt} e^{at+\frac{bt^2}{2}+\frac{ct^3}{3}}\, dt$$

$$Q = f\left[\frac{a^2T^3}{2} + \frac{5abT^4}{12} + \frac{agT^3}{2} + \frac{aT^2}{2} + \frac{bgT^4}{4} + \frac{bT^3}{3} + \frac{cT^4}{4} - \frac{gT^2}{2} - T\right] \quad \ldots(3)$$

Neglecting the higher power of t^4

Therefore solution of equation (1) is

$$I(t) = f\left[\frac{a^2(T^3+t^3)}{2} + \frac{5ab(T^4+t^4)}{12} + \frac{ag(T^3+t^3)}{2} + \frac{a(T^2+t^2)}{2} + \frac{bg(T^4+t^4)}{4} + \frac{b(T^3+t^3)}{3} + \frac{c(T^4+t^4)}{4} + \frac{g(T^2-t^2)}{2} - (T-t)\right] \quad \ldots(4)$$

(Neglecting higher powers of t greater than t^4)

3(a) Holding Cost:

Let h is the holding cost per unit time.

$$HC = \int_0^T h(t)I(t)dt$$

$$HC = h\left[\frac{7}{10}T^5\left(\frac{a^2}{2} + \frac{ag}{2} + \frac{b}{3}\right) + \frac{2}{3}T^6\left(\frac{5}{12}ab + \frac{1}{4}bg + \frac{c}{4}\right) + \frac{g}{8}T^4 + \frac{T^3}{6}\right] \quad \ldots(5)$$

(Neglecting higher powers of t greater than t^4)

3(b) Purchase Cost:

Let C_1 be the per unit cost i.e.

$$PC = C_1\left[Q + \int_0^T D(t)\, dt\right]$$

$$PC = C_1\left[Q + \frac{f}{g}e^{gt}\right] \quad \ldots(6)$$

4. Total cost

The total variable cost function for one cycle is given by

The total cost is TC i.e. TC = OC + PC + HC

$$TC = A + C_1\left[Q + \frac{f}{g}e^{gt}\right] + h\left[\frac{7}{10}T^5\left(\frac{a^2}{2} + \frac{ag}{2} + \frac{b}{3}\right) + \frac{2}{3}T^6\left(\frac{5}{12}ab + \frac{1}{4}bg + \frac{c}{4}\right) + \frac{g}{8}T^4 + \frac{T^3}{6}\right] \ldots(7)$$

Differentiating equation (7) with respect to T then we get the following

$$\frac{\partial TC}{\partial T}$$

To minimize the total cost TC(T) per unit time, the optimum value of T can be obtain by solving the following equation.

$$\frac{\partial TC}{\partial T} = 0$$

Providing that the equation (7) satisfies the following condition.

$$\left(\frac{\partial^2 TC}{\partial T^2}\right) > 0 \qquad \ldots(8)$$

It is clear in figure 2, the total cost obtained is convex, hence provide the optimal solution. Using the principle of maxima and minima the optimal cycle time and the economic order quantity can be obtained.

5. Results and Discussion

For the numerical and graphical analysis, we considered as inputs of the parameters in proper unit of the model, A = 100, C_1 = 2, f = 10, g = 1000, h = 5, a = 5, b = 10, c = 5. The output of the model by using a mathematical software (the optimal value of the total cost, the time when the inventory level reaches zero and the time when the maximum shortage occur) is TC = 54.07, at T = 0.11.

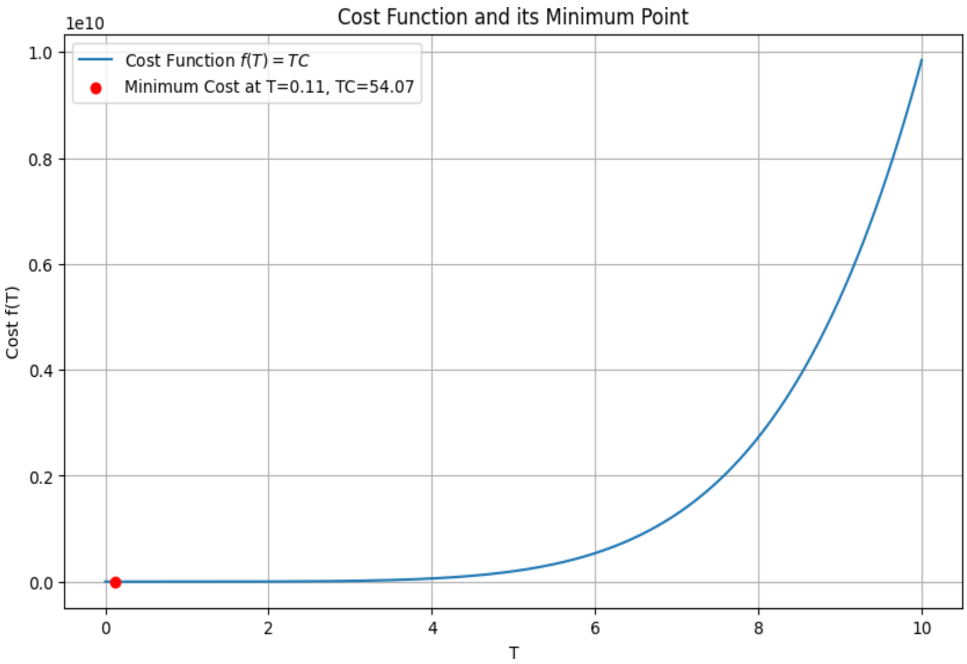

Figure 2 : Total cost function in 2D

If we plot the total cost function (7) with some values of T such that Tat 0.11 then we get the strictly convex graph of the total cost function (TC) which is given by figures 2.

From the figures 2 the observation is that the total cost function of the model gives that the total inventory cost per unit time of the inventory system is minimum.

Table 1 : Optimal cycle time for the model

Cycle Time (Years)	Total Cost ($)
0.11	54.07
0.12	96.56
0.22	279.33
0.33	1475.65
0.44	4580.12

6. Conclusion

This study of paper presents a deterministic inventory model tailored for business enterprises, considering that items degrade over time in storage. In this model, demand is exponential and varies with time, the deterioration rate is quadratic, and holding costs are also exponential and time-dependent. Moreover, the model does not account for shortages. To identify the optimal solution, the model aims to minimize the total inventory cost, which is illustrated through a numerical example.

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