Static Control of Nanoparticles in Stagnation Point Flow of Carreau Nanofluid Flow over an Exponentially Stretching Surface

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Abstract :

The present work is devoted to analyze the Stagnation point flow of a Carreau fluid in the presence of nanoparticles over an exponential stretching sheet. Additionally we studied condition of zero normal flux of nanoparticles is introduced at the surface along with convective boundary condition. Moreover consideration of nanoparticles which are passively control at the surface is physically more realistic condition. The modeled equations for Carreau nanofluid are converted into nonlinear ordinary differential equations by utilizing suitable similarity transformations. The resulting equations were numerically solved using Runge Kutta method with shooting technique for various values of governing parameters. Obtained numerical results have been compared with previously published data and found in good agreement. The effects of various non dimensional parameters on velocity, temperature and concentration are discussed in detail and presented through graphs. It is found that velocity profile increases with increasing the Weissenberg number We.

Keywords: Exponential stretching sheet, Stagnation point flow Carreau fluid, convective condition, passive control, nanoparticles, Brownian motion, thermophoresis.

Introduction:

In past few years the boundary layer flow of Newtonian and non-Newtonian fluids over stretching sheet has gained much interest because of their vast applications in engineering processes. Some examples of practical applications of moving stretching surfaces are extraction of polymer sheet, wire drawing, paper production, glass-fiber production, hot rolling, solidification of liquid crystals, petroleum production, continuous cooling and fibers spinning, exotic lubricants and suspension solutions.

Crane [1] was the first who studied the boundary layer flow induced by a stretching sheet problem which moves with a velocity varying linearly with the distance from a fixed point. Then Carragher and Crane [2] discussed the heat transfer on a continuous stretching sheet. Most of the existing literature deals with the study of boundary layer flow over a linear stretching surface. Gupta and Gupta [3] mentioned that stretching sheet may not necessarily be linear. Thereafter the pioneering work of Crane has been extended by several researchers [4-6] under different physical aspects. Flow and heat transfer characteristics due to an exponential stretching sheet have a wider application in technology. Magyari and Keller [7] considered heat and mass transfer characteristics of the boundary layer flow due to an exponentially stretching continuous surface and solved analytically and numerically. Then Elbashbeshy [8] studied the suction effects on heat transfer flow over an exponentially stretching continuous surface. Bidin and Nazar [9] analyzed the effect of thermal radiation on the steady laminar two-dimensional boundary layer flow and heat transfer over an exponentially Stretching sheet. The study of boundary layer flow under various physical aspects over an exponential stretching sheet was conducted by few researchers [10-12].

Conventional heat transfer fluids are inadequate to meet the present challenges in modern world since they are weak conductors. There are several methods to increase the heat transfer efficiency. Nanofluid is a new class of heat transfer fluid that contains small tiny particles and base fluid such as water, ethylene glycol, and propylene glycol. Thus such kind of tiny particle known as nanoparticle and which ranges from 1-100nm in diameter. The main vision of suspending the nanoparticles within base fluid is to enhance the thermal conductivity. The term nanofluid was introduced by Choi [13]. Nanofluid is homogeneous mixture of conventional base fluid and nanoparticles. Following the pioneering work of Choi, Buongiorno [14] proposed a model in which seven slip mechanism is considered namely, inertia, Brownian diffusion, thermophoresis, diffusionphoresis, Magnus effect, fluid drainage and gravity settling and according to him absolute velocity can be viewed as the sum of the base fluid velocity and a relative velocity, he revealed that massive increase in the thermal conductivity is due to the existence of two main velocity slip mechanisms namely Brownian motion and thermophoretic diffusion of nanoparticles. Using Buongiornos model Kuznetsov and Nield [15] have investigated the natural convective boundary layer flow of a nanofluid past a vertical plate analytically. Khan and Pop [16] firstly examined the classical problem of two dimensional boundary layer flow of nanofluid induced by stretching sheet.

Heat transfer on the surface is influenced by convective boundary condition and as a result the quality of the final outcome of the manufacturing industries. To define the linear convective heat exchange condition for algebraic entities, the convective boundary conditions are considered. The study of convective heat transfer in a magnetic field is important in processes, such as in gas turbine, nuclear plants, and thermal energy storage. It is agreed that the convective boundary conditions are more practical in various industrial and engineering processes, for instance, transpiration cooling process, material drying, etc. Aziz [17] introduced the idea of using convective surface boundary condition to investigate the boundary layer flow of the Blasius problem over a at surface. Makinde and Aziz [18] addressed the boundary layer flow induced in a nanofluid by imposing the convective condition induced by stretching sheet.

Nadeem and Lee [19] obtained analytic solutions of boundary layer flow of nanofluid over an exponentially stretching surface using homotopy analysis method (HAM). Later Bhattacharya and Layek [20] extended this problem by considering magnetic field effects on the boundary layer flow of nanofluid over an exponential stretching permeable stretching sheet. Several researchers [21]-[23] focused on the boundary layer flow of a nanofluid over an exponentially stretching sheet under different physical conditions.

In recent years non-Newtonian fluid flows are encountered in various engineering and industrial applications. The theory of non-Newtonian fluid mechanics has been developed due to the inadequacy of the Newtonian constitutive equations in predicting the flow behavior of many fluids particularly the fluids which have large molecular weight. One can know that all the rheological characteristics of non-Newtonian fluids cannot be constituted in a single equation due to the flow diversity in nature. Hence various constitutive equations [24]-[25] are constructed to understand such type of fluids. In general the modeled equations for the flows of non-Newtonian fluids are much complicated, highly nonlinear and of higher order than the Navier Stokes equations. Carreau fluid was first introduced by Carreau [26] in 1972. This model fits reasonably well with the suspensions of polymers behavior in many flow situations. Parti- cularly, the Carreau model is well-suited for certain dilute, aqueous, polymer solutions and melts. For example, the various polymeric solutions, such as 1% methylcellulose tylose in glycerol solution and 0.3% hydroxyethyl-cellulose Natrosol HHX in glycerol solution. It describes both shear thinning and shear thickening behaviors of many non-Newtonian fluids.

Unlike power-law fluid, the Carreau model is one of the non-Newtonian fluid models for which constitutive relationship valid for both low and high shear rates. Further, Khellaf and Lauriat [27] investigated the flow and heat transfer to Carreau fluid in the annular space between two concentric cylinders. Flow of a Carreau fluid down an inclined with a free surface was examined by Tshehla [28]. Hayat et al [29] studied the boundary layer flow of Carreau fluid over a stretching sheet with convective boundary condition. Olajuwon [30] addressed heat and mass transfer in hydromagnetic flow of Carreau fluid past a vertical porous plate with thermal radiation and thermal diffusion. Akbar et.al [31] examined MHD stagnation point flow of Carreau fluid toward a permeable shrinking sheet and obtained dual solutions. Nonlinear radiation effects on MHD flow of Carreau fluid over a nonlinear stretching sheet with convective boundary condition is studied by Masood et.al [32].

In all the aforementioned studies discussed the boundary layer flow when nanoparticle flux at a surface is non-zero. Recently, Kuznetsov and Nield [33] reinvestigated their existing model and revised a model, in their investigation they proposed a new enhanced boundary condition for the nanoparticle volume fraction at the boundary which is passively controlled rather than an active control and the nanoparticle flux at the wall is zero. In such condition, they argued this model is more physically realistic as compared to the earlier model. Recently Prabhakar *et al* [34] examined passive control of nanoparticles of tangent hyperbolic fluid past a stretching sheet with the effect of inclined magnetic field.

To the best of author's knowledge no study has been presented to analyze flow of a Carreau nanofluid over an exponentially stretching sheet with zero normal flux of nanoparticles, so we considered in this article. The prime objective of the present study is to discuss the analysis for the Carreau fluid model in the presence of nanoparticles over an exponential stretching sheet.

Mathematical formulation

Consider the steady two dimensional flow of a Carreau nanofluid over an exponential stretching sheet at the stagnation region. We considered Cartesian coordinate system and x-axis taken along the stretching sheet in the direction of the flow, y -axis normal to the surface and we assumed that the sheet is stretched exponentially in the direction of x, with the velocity $U_w = ae^{x/L}$ defined at $y = 0$. The effect of Brownian motion and thermophoresis has also considered. The velocity of the external flow is U_∞. Where as $T_w = T_\infty + ce^{\frac{x}{l}}$ and $C_W = C_\infty + de^{\frac{x}{l}}$ be the temperature and nanoparticles concentration at the sheet where T_∞ and C_∞ denote the ambient temperature and concentration respectively.

Extra stress tensor for Carreau fluid is

$$\tau_{ij} = \eta_0 \left[1 + \frac{n-1}{2}(\Gamma\bar{\dot{\gamma}})^2\right]\gamma_{ij}$$

Where τ_{ij} is the extra stress tensor and η_0 is the zero shear rate viscosity, Γ is the time constant, n is the power law index, $\bar{\dot{\gamma}}$ is defined as $\bar{\dot{\gamma}} = \sqrt{\frac{1}{2}\sum_i\sum_j \bar{\dot{\gamma}}_{ij}\bar{\dot{\gamma}}_{ji}}$

Flow equations for Carreau fluid model after applying the boundary layer approximations can be defined as follows

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U_\infty \frac{dU_\infty}{dx} + \upsilon\frac{\partial^2 u}{\partial y^2} + \upsilon\frac{3(n-1)\Gamma^2}{2}\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2(x)}{\rho_f}(u - U_\infty) \quad (2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \tau\left\{D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right\} \quad (3)$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \left(\frac{D_T}{T_\infty}\right)\frac{\partial^2 T}{\partial y^2} \quad (4)$$

Here u and v are the velocity components in the x- and y-direction respectively, ρ_f is the density of the base fluid, υ is the kinematic viscosity , σ is electrical conductivity, $\alpha = \frac{k}{(\rho C)_f}$ is thermal diffusivity T is the temperature, C is nanoparticle volume fraction, $(\rho c)_p$ is the effective heat capacity of nanoparticles, $(\rho c)_f$ is the heat capacity of the base fluid, $\tau = \frac{(\rho c)_p}{(\rho c)_f}$ is the ratio of the nanoparticle heat capacity and base fluid heat capacity, T is the temperature, T_∞ is the constant temperature of the fluid in the inviscid free stream, D_B is the Brownian diffusion coefficient and D_T is the thermophoresis diffusion coefficient. We consider that the magnetic field $B(x)$ is of the form $B(x) = B_0 e^{x/2L}$, B_0 being constant.

The boundary conditions corresponding to the problem are as follows.

$$u = U, v = 0, -k\frac{\partial T}{\partial y} = h_f(T_f - T), D_B\frac{\partial C}{\partial y} + \frac{D_T}{T_\infty}\frac{\partial T}{\partial y} = 0 \text{ at } y = 0$$

$$u \to 0, v \to 0, T = T_\infty, C = C_\infty \text{ as } y \to \infty \quad (5)$$

$$u = U_w = ae^{\frac{x}{L}}f'(\eta), v = -\sqrt{\frac{\upsilon a}{2L}}e^{x/2L}\{f(\eta) + \eta f'(\eta)\}, \eta = y\sqrt{\frac{a}{2\upsilon L}}\, e^{x/2L} \quad (6)$$

By employing the similarity transformations (6), the governing equations (2)-(4) reduced to the following ordinary differential equations.

$$f''' + ff'' - 2f'^2 + \frac{3(n-1)We^2}{2}f'''f''^2 - M(f' - \epsilon) + 2\epsilon^2 = 0 \quad (7)$$

$$\frac{1}{Pr}\theta'' + f\theta' - 2f'\theta + Nb\theta'\phi' + Nt\theta'^2 = 0 \quad (8)$$

$$\phi'' + Le(f\theta' - 2f'\phi) + \frac{Nt}{Nb}\theta'' = 0 \quad (9)$$

By using (5) the boundary conditions take the form

$$f(0) = 1, f'(0) = 1, \theta'(0) = -Bi\big(1 - \theta(0)\big), Nb\phi'(0) + Nt\theta'(0) = 0 \quad (10)$$

$$f'(\eta) \to \epsilon\, , \theta(\eta) \to 0, \phi(\eta) \to 0 \text{ as } \eta \to \infty \quad (11)$$

where $M = \frac{2\sigma B_0^2 L}{U\rho}$ is the magnetic parameter, the stretching parameter$\epsilon = \frac{b}{a}$ $We = \frac{a^3 e^{3x/L}\Gamma^2}{\upsilon L}$ is the Weissenberg number, $Pr = \frac{\upsilon}{\alpha}$ is Prandtl number, $Sc = \frac{\upsilon}{D_B}$ is Schmidt number, $Nt = \frac{D_T(T_f - T_\infty)(\rho c)_p}{T_\infty \upsilon(\rho c)_f}$ is the thermophoresis parameter, $Nb = \frac{D_B C_\infty(\rho c)_p}{\upsilon(\rho c)_f}$ is the Brownian motion is parameter. Physical quantity of interest are Skin friction coefficient and Nusselt number are given by

$$C_f = \frac{\tau_w}{\rho {U_w}^2}\text{where } \tau_w = \left(\frac{\partial u}{\partial x}\right) + \frac{(n-1)\Gamma^2}{2}\left(\frac{\partial u}{\partial y}\right)^3, Nu_x = \frac{xq_w}{k(T_f - T_\infty)} \quad (12)$$

where k is the thermal conductivity of the nanofluid and q_w, q_m are the heat and mass fluxes at the surface respectively given by

$$q_w = -k\left(\frac{\partial T}{\partial u}\right)_{y=0} \quad (13)$$

By substituting equation (6) into equations (12)-(13), we will get

$$\sqrt{Re}C_f = \left[f''(\eta) + \frac{(n-1)We^2}{2}(f'(\eta))^3\right]_{\eta=0}, \sqrt{\frac{2}{Re}}Nu_x = -\theta'(0)$$

Where $Re = \frac{U_w L}{\upsilon}$ local Reynold number

3. Results and discussion

Numerical solutions of the nonlinear ordinary differential equations (7)-(10) with corresponding boundary condition (11) have been obtained by using Runge-Kutta method with shooting technique. the values of local skin friction coefficient $f''(0)$ and Nusselt number$-\theta'(0)$ are presented in table 1 and table 2

The theme of this section is to discuss the effects of various physical parameters such as magnetic parameter M, Weissenberg number We, Prandtl number Pr, thermophoresis parameter Nt, Brownian motion parameter Nb, Biot number Bi, Lewis number Le on the velocity profile $f'(\eta)$, temperature profile $\theta(\eta)$ and concentration profile $\phi(\eta)$. In the present study we have considered the non-dimensional parameter values such as $M = 1.0, We = 1, n = 1.5,\ Nt = 0.1\ Nb = 0.3, Pr = 1, Le = 1.0, Bi = 1.0, Le = 1.0$ and these values are kept as common in the entire graphical illustration except for the varied values as shown in respective figures.

Table.1 Values of skin friction coefficient for different values of M and We when $n = 1.5$, $Nt = 0.1, Nb = 0.3, Pr = 1.0, Sc = 2.0, Bi = 1.0$

M	We	$f''(0)$
0.5	0.5	
1		-0.889985
2		-0.971383
3		-1.041963
	0.5	-0.970281
	1.0	-0.889985
	1.5	-0.813219

Table.2 Values of $-\theta'(0)$ for different values of We, Nt, Pr, Bi when $n = 1.5, Nb = 0.3$, $Sc = 2.0$

We	Nt	Pr	Bi	$-\theta'(0)$
0.5	0.1	1.0	1.0	
0.5				0.585062
1.0				0.586169
1.5				0.587463
	0.1			0.586169
	0.2			0.585201
	0.3			0.584230
		1.0		0.586169
		1.5		0.638199
		2.0		0.673123
			0.3	0.247665
			0.5	0.369692

			0.7	0.468603

Figures 2-12 depicts the graphical representations of the various controlling parameters on the velocity, temperature and concentration profiles. **Fig 2** Illustrates the effects of stretching parameter ϵ on velocity it is clear from the graph that whenever $\epsilon < 1$ the velocity boundary layer thickness increases and when $\epsilon > 1$ the velocity boundary layer thickness decreases.

and it is also clear that velocity profile increases on both the cases. Effect of magnetic parameter M on the velocity, is displayed in **Fig.3** by keeping the other parameters fixed. It is observed that an increase in magnetic field parameter decreases the velocity. Physically the magnetic field is associated with a force which is called Lorentz force and it is a retarded force which shows resistance to the motion of the fluid. Hence velocity of the fluid reduced but temperature and concentration enhances with strong magnetic field.

Fig.4 is plotted to observe the influence of Weissenberg number We on the velocity,. It is evident from that the velocity profile enhances by increasing Weissenberg numberWe. **Fig.5-Fig.6** explores the variations of power law index n on velocity. It is clear from the figures, the temperature distribution is reduced with increase in power law index but velocity distribution increases as we increase power law index n.

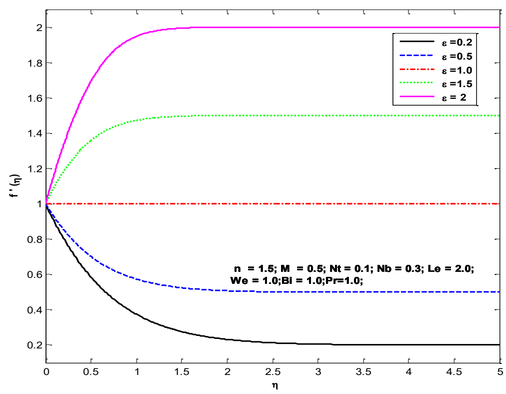


Fig2. Velocity profile for various values of ϵ

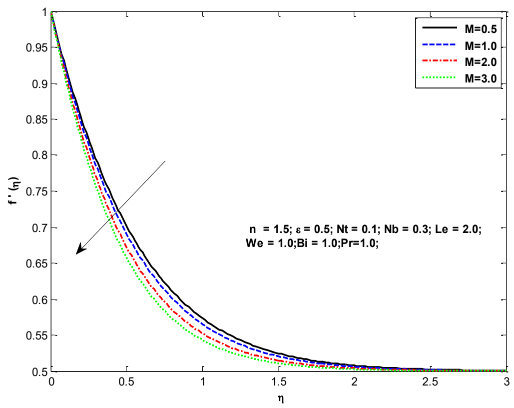
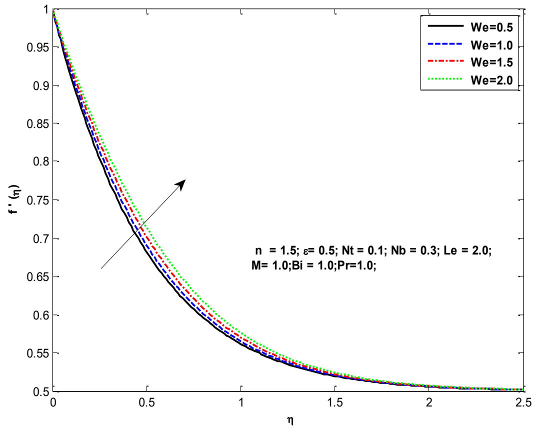


Fig3. Effect of M on velocity profile $f'(\eta)$



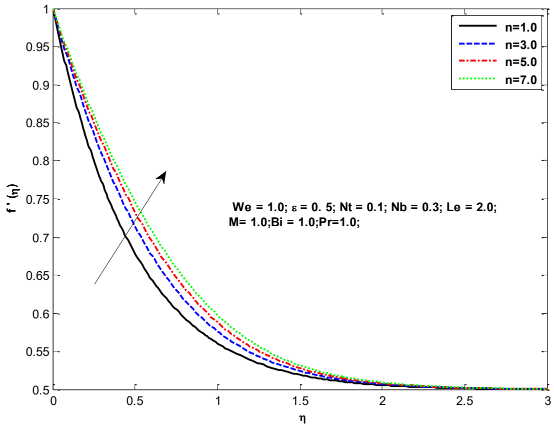


Fig4. Effect of W_e on velocity profile $f'(\eta)$ Fig5. Effects of n on velocity profile $f'(\eta)$

Impact of Biot number Bi on the temperature $\theta(\eta)$ is displayed in **Fig.7**. A rapid increase is found on temperature profile $\theta(\eta)$with rising the values of Biot numberBi. Physically Biot number Bi is expressed as the convection heat transfer at the surface of the body to conduction within the surface of a body. In general it depends on characteristic length of the surface, thermal conductivity of the surface and convective heat transfer coefficient of the hot fluid below the surface. Higher Biot number Bi represents less conductive substance such as plastic, paper, polymer etc, whereas smaller Biot number indicates higher conductive materials which include aluminum, iron, and steel etc. In addition an increase in Bi, results higher surface temperature which significantly enhances the temperature and thickness of the thermal boundary layer.

Fig.8is plotted to observe the influence of Biot number Bi on concentration profile. It is clear from the figure nanoparticle concentration enhances with increase in Biot number. **Fig.9** reveals the variation in concentration profile with the effect of Brownian motion parameter Nb. As Brownian motion effect increase, the concentration gradient increases as a result the Brownian force increases which boot the nanoparticles concentration at the surface. Hence the concentration profile $\phi(\eta)$ increases at the stretching sheet wall up to a certain value of η but after this point opposite trend is observed.

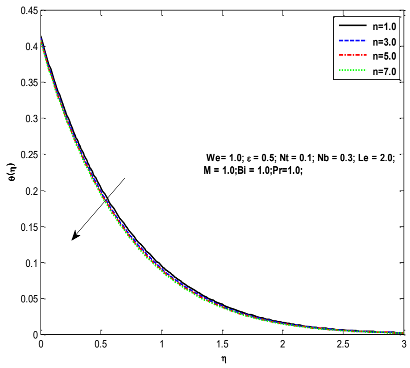


Fig 6. Effect of n on temperature profile $\theta(\eta)$

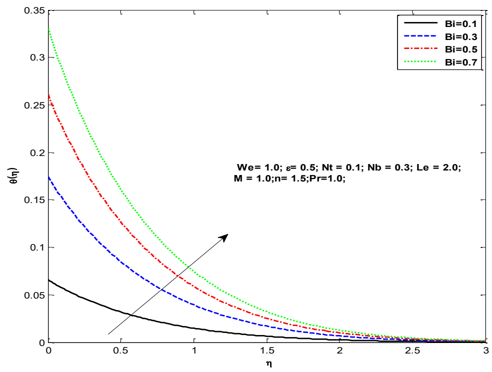


Fig7. Effects of Bi on temperature profile $\theta(\eta)$

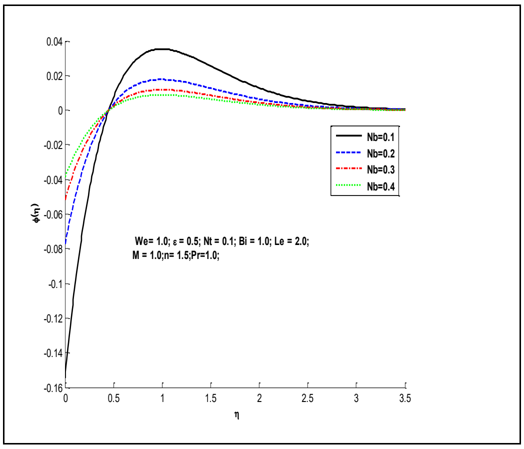


Fig8. Effects of Bi on concentration profile $\phi(\eta)$

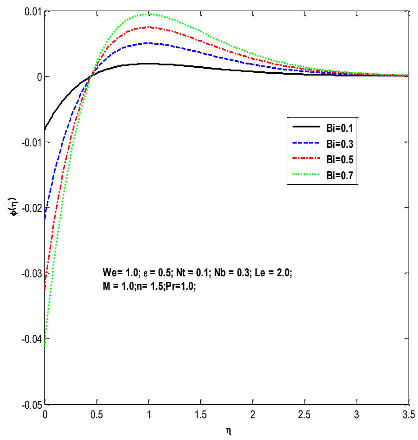


Fig9: Effects of Nb on concentration profile $\phi(\eta)$

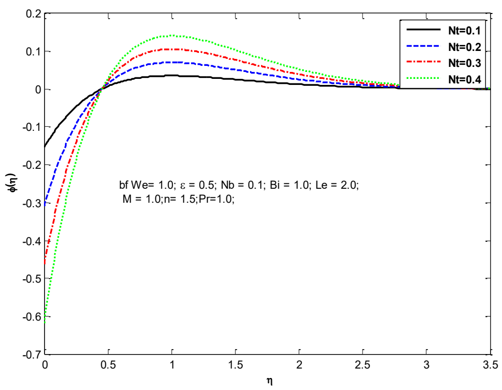


Fig10. Effects of Nt on concentration profile $\phi(\eta)$

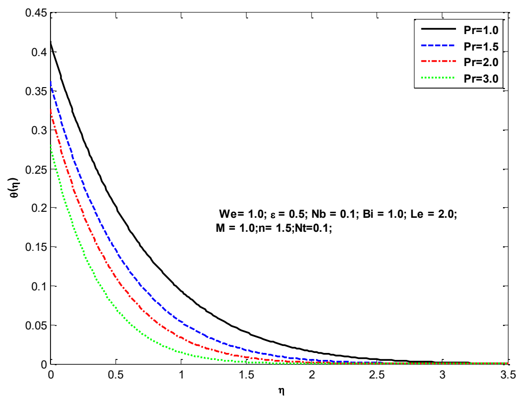


Fig11. Effects of Pr on temperature profile $\theta(\eta)$

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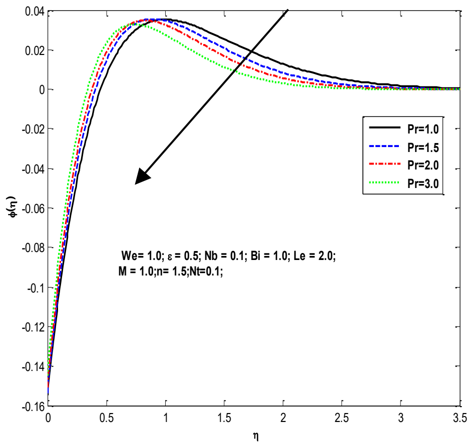

Fig12. Effects of Pr on concentration profile $\phi(\eta)$

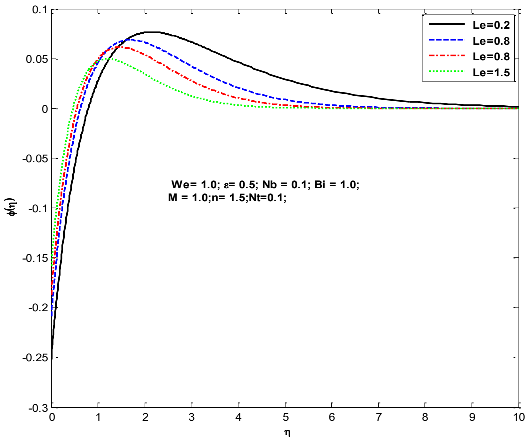

Fig 13. Effects of Le on $\phi(\eta)$

Fig.10 illustrates the influence of the thermophores is parameter Nt on the concentration profiles. Since the impact of Brownian force is to counter balance the influences of thermophoretic force, as the influence of thermophoretic force increases the concentration gradient at the surface decreases, as result the concentration profile decreases at the stretching sheet wall up to a certain value of η but after this point concentration profile increase which is opposite to Brownian motion effect. In both the above cases the nature of concentration profile may happened due to the condition of zero normal flux of nanoparticles at the wall for the stretched flow which is the revised nanoparticle concentration boundary condition.

Fig.11-Fig.12 are prepared to show the effects of Prandtl number Pr on the tempaerature and concentraton profile. It is observed from the **Fig.11** that temperature profile $\theta(\eta)$ is a decreasing function of Pr. In fact by raising the Prandtl number Pr thermal diffusivity decrease and thus the heat diffuses away slowly from the heated surface. It is clear from the **Fig.12** nanoparticle concentration profile decreases by increasing the Prandtl number Pr **Fig.13** Illustrate the concentrate profile with respect the various values of Lewis number. It is observed that there is increment near the wall and after that the concentration profile decreased.

Conclusions

The numerical study of the flow of a Carreau nanofluid over an exponential stretching sheet with convective boundary condition has been reported in this paper. The governing partial differential equations are converted into nonlinear ordinary differential equation by employing suitable similarity transformations and solved numerically by using Runge Kutta method with shooting technique for various values of governing parameters. It has been presented the influence of various governing parameters such as magnetic parameter$M$, Weissenberg number We, power law index n, Prandtl number Pr, thermophoresis parameter Nt, Brownian motion parameter Nb, Schmidt number Sc on velocity, temperature and concentration profiles illustrated graphically. The prime results of this numerical study are summarized as follows.

- Increase in magnetic field M, leads to occurrence of Lorentz force decelerates the velocity.
- Impact of Weissenberg number We and power law index n are similar on velocity.
- Temperature decreases with increasing power law index
- As the thermophoresis parameter increases concentration profile increases but opposite trend is observed in the case of Brownian motion parameter.
- It was found that heat transfer rate was accelerated with the enhancing values of the Biot number.
- Concentration profile is increases with Lewis number.

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