Tuijin Jishu/Journal of Propulsion Technology
ISSN: 1001-4055
Vol. 46 No. 1 (2025)

NRGβ - Continuous and Irresolute Mappings in Neutrosophic Topological Spaces

Eswari R[1] and Nirmala Irudayam F [2]

[1]*Research Scholar, Department of Mathematics, Nirmala College for Women, Coimbatore, Tamil Nadu, India*

[2]*Assistant Professor, Department of Mathematics, Nirmala College for Women, Coimbatore, Tamil Nadu, India.*

eswari.mar5@.gmail.com

nirmalairudayam@ymail.com

***Abstract*:-** In this research work, we introduce the ideas of NRGβ continuous mappings and NRGβ irresolute mappings in Neutrosophic topological spaces. We examine and establish numerous characteristics and characterizations related these mappings in Neutrosophic topological spaces. Additionally, the relationships between the Neutrosophic irresolute mappings and the Neutrosophic continuous mappings are examined.

Keywords:- Neutrosophic topological space; NRGβ – closed set; NRGβ continuous mappings, NRGβ irresolute mappings.

1. INTRODUCTION

Over the years, other types of topological spaces were introduced as a generalization of topological spaces, making topology a traditional science. The Neutrosophic set was defined by Smarandache [2] using three components: T stands for Truth, F for Falsehood, and I for Indeterminacy. Numerous real-world issues in the fields of business, finance, medicine, engineering, and social sciences involve uncertainty that neutrosophic topology addresses. Saied Jafari introduced Neutrosophic generalized closed sets, whereas Salama et al. [3] and R. Dhavaseelan [6] introduced Neutrosophic topological spaces. After this, C. Maheswari[11] et al. introduced the Neutrosophic b closed set. Renu Thomas established the concept of Neutrosophic β open sets and β closed sets [9]. In 2024, R. Eswari and F. Nirmala Irudayam presented the NRGβ closed set [13]. In this research, we introduce and investigate NRGβ continuous mappings and NRGβ irresolute mappings in neutrosophic topological spaces.

2. PRELIMINARIES

We review the basic definitions, operations, and fundamental results of Neutrosophic Sets in this section.

Definition 2.1[3] Consider $\mathbb{R}$ to be a fixed, non-empty set. An object with the following form is a NS,

$\mathrm{D} = \{< r, \mu_{\mathrm{D}}(r), \sigma_{\mathrm{D}}(r), \gamma_{\mathrm{D}}(r) >: r \epsilon \mathbb{R}\}$, where $\mu_{\mathrm{D}}, \sigma_{\mathrm{D}}, \gamma_{\mathrm{D}} : \mathrm{D} \to [0,1]$ and $0 \leq \mu_{\mathrm{D}}(r) + \sigma_{\mathrm{D}}(r) + \gamma_{\mathrm{D}}(r) \leq 3$ and

$\mu_{\mathrm{D}}(r)$ – signifies the degree of membership function

$\sigma_{\mathrm{D}}(r)$ - signifies the degree of indeterminacy function

$\gamma_{\mathrm{D}}(r)$ - signifies the degree of non-membership function.

N(X) is the set of all Neutrosophic sets over X.

Definition 2.2[3] If $\mathrm{D} = \{< r, \mu_{\mathrm{D}}(r), \sigma_{\mathrm{D}}(r), \gamma_{\mathrm{D}}(r) >\}$ is a NS on X, then $\mathrm{D}^c = \{< r, \gamma_{\mathrm{D}}(r), 1 - \sigma_{\mathrm{D}}(r), \mu_{\mathrm{D}}(r) >: r \in \mathbb{R}\}$ is the complement of D.

Definition 2.3[3] If $\mathrm{D} = \{< r, \mu_{\mathrm{D}}(r), \sigma_{\mathrm{D}}(r), \gamma_{\mathrm{D}}(r) >\}$ and $\mathrm{E} = \{< r, \mu_{\mathrm{E}}(r), \sigma_{\mathrm{E}}(r), \gamma_{\mathrm{E}}(r) >\}$ are any two Neutrosophic sets, then $\mathrm{D} \subseteq \mathrm{E} \Leftrightarrow \{ \mu_{\mathrm{D}}(r) \leq \mu_{\mathrm{E}}(r), \sigma_{\mathrm{D}}(r) \leq \sigma_{\mathrm{E}}(r), \gamma_{\mathrm{D}}(r) \geq \gamma_{\mathrm{E}}(r)\}$.

Definition 2.4[3] Let ℝ be a non- empty fixed set with two Neutrosophic sets $D = \{< r, \mu_D(r), \sigma_D(r), \gamma_D(r) > \}$ and $E = \{< r, \mu_E(r), \sigma_E(r), \gamma_E(r) >\}$, then

1. $D \vee E = \{r, \max\{\mu_D(r), \mu_E(r)\}, \max\{\sigma_D(r), \sigma_E(r)\}, \min\{\gamma_D(r), \gamma_E(r)\} : r \epsilon \mathbb{R}\}$.
2. $D \wedge E = \{r, \min\{\mu_D(r), \mu_E(r)\}, \min\{\sigma_D(r), \sigma_E(r)\}, \mathrm{mar}\{\gamma_D(r), \gamma_E(r)\} : r \epsilon \mathbb{R}\}$.

Definition 2.5[3] If τ_N is the set of Neutrosophic subsets of R that meet the following criteria, and ℝ is a non-empty set, then

1. $\mathbb{0}_N, \mathbb{1}_N \in \mathfrak{I}_{\mathbb{N}}$
2. $\vee T_i \in \mathfrak{I}_{\mathbb{N}}$ for every $\{T_i : i \in j\} \subseteq \mathfrak{I}_{\mathbb{N}}$
3. $T_1 \wedge T_2 \in \mathfrak{I}_{\mathbb{N}}$ for any $T_1, T_2 \in \mathfrak{I}_{\mathbb{N}}$

The space (ℝ, $\mathfrak{I}_{\mathbb{N}}$) is referred to as a Neutrosophic topological space (ṄṪṠ) when $\mathfrak{I}_{\mathbb{N}}$ is a Neutrosophic topology. A NOS is the element of $\mathfrak{I}_{\mathbb{N}}$, while a NCS is its complement.

Definition 2.6 [3] Let (ℝ, $\mathfrak{I}_{\mathbb{N}}$) be a ṄṪṠ and $D = \{< r, \mu_D(r), \sigma_D(r), \gamma_D(r) >\}$ be a NS in ℝ. Then Neutrosophic closure of D is *Ncl(D)* = ∧{Ḣ: Ḣ is a NCS in ℝ and D ⊆ Ḣ } and Neutrosophic interior of D is *Nint*(D) = ∨{ Ṁ: Ṁ is a NCS in ℝ and D ⊆ Ṁ }.

Definition 2.7[2] Let (ℝ, $\mathfrak{I}_{\mathbb{N}}$) be a N-T-S and $D = \{< r, \mu_D(r), \sigma_D(r), \gamma_D(r) >\}$ be a NS in ℝ. Then Neutrosophic **β**closure of D is *N**β**cl(D)* = ∧ {Ḣ: Ḣ *is a N**β**CS in* ℝ *and D* ⊆ Ḣ *}* and Neutrosophic **β** interior of D is *N**β**int(D)* = ∨{Ṁ: Ṁ *is a N**β**OS in* ℝ *and* Ṁ⊆ *D}.*

Definition 2.8[13] Let (ℝ, $\mathfrak{I}_{\mathbb{N}}$) be a ṄṪṠ and $D = \{< r, \mu_D(r), \sigma_D(r), \gamma_D(r) >\}$ be a NS in ℝ. Then D is said to be *Neutrosophic regular generalized **β** closed set*[13] (NRG**β**CS) if *N**β**cl(D)* ⊆ $\hat{U}$ whenever $D \subseteq \hat{U}$ and $\hat{U}$ is a NROS in ℝ.

3. NRGβ CONTINUOUS MAPPINGS

The notions of NRGβ continuous mappings in Neutrosophic topological spaces are presented in this section.

Definition 3.1: Let ɟ : (ℝ, $\mathfrak{I}_{\mathbb{N}}$) → (𝕊, $\gamma_{\mathbb{N}}$) be a mapping. If $ɟ^{-1}(V)$ is a NRGβ closed set in ℝ for every NCS V in 𝕊, then ɟ is referred to as an NRGβ continuous (NRGβ CTS) mapping.

Example 3.2 : Let L={<r,(0.6,0.5,0.4), (0.7,0.5,0.3)>} and M={<s,(0.9,0.5,0.1),(0.8,0.5,0.2)>} where

ℝ ={k_1,k_2}and 𝕊 ={k_3,k_4}. Then on ℝ and 𝕊, respectively, Neutrosophic Topologies are $\mathfrak{I}_{\mathbb{N}}$ ={0_N,L,1_N} and

$\gamma_{\mathbb{N}}$ ={0_N,M,1_N}. Using ɟ (k_1)= k_3 and ɟ (k_2)= k_4 , define a mapping ɟ : (ℝ, $\mathfrak{I}_{\mathbb{N}}$) → (𝕊, $\gamma_{\mathbb{N}}$). Then ɟ is NRGβ CTS.

Theorem 3.3 : Every mapping that is Neutrosophic continuous (N CTS), Neutrosophic α continuous (Nα CTS), Neutrosophic generalised continuous (NG CTS), Neutrosophic α generalised continuous (NαG CTS), Neutrosophic generalized pre continuous (NGP CTS), Neutrosophic semi continuous (NS CTS), Neutrosophic generalized semi continuous (NGS CTS), Neutrosophic b continuous (Nb CTS), Neutrosophic β continuous (Nβ CTS) and Neutrosophic generalized β continuous (NGβ CTS) is also an NRGβ continuous.

Remark 3.4: The converse of above theorem is not true as we can shown by the countable examples.

Example 3.5 : Let L={<r,(0.5,0.5,0.5), (0.7,0.5,0.3)>} and M={<s,(0.3,0.5,0.4),(0.9,0.5,0.1)>} where ℝ ={k_1,k_2},

𝕊 ={k_3,k_4}. Then on ℝ and 𝕊, respectively, Neutrosophic Topologies are $\mathfrak{I}_{\mathbb{N}}$ ={0_N,L,1_N} and $\gamma_{\mathbb{N}}$ ={0_N,M,1_N}.

Using ɟ (k_1)= k_3 and ɟ (k_2)= k_4 , define a mapping ɟ : (ℝ, $\mathfrak{I}_{\mathbb{N}}$) → (𝕊, $\gamma_{\mathbb{N}}$) by ɟ (k_1)= k_3 and ɟ (k_2)= k_4 . Since the NS J={<y,(0.4,0.5,0.3), (0.1,0.5,0.9)>} is NCS in 𝕊, $ɟ^{-1}$(J) is a NRGβCS but not NCS in ℝ. As a result f is not a N CTS mapping but rather than an NRGβ continuous mapping.

Example 3.6 : Let L={<r,(0.5,0.5,0.6), (0.7,0.5,0.6)>} and M={<s,(0.3,0.5,0.9),(0.5,0.5,0.7)>} where ℝ ={k_1,k_2},

---

$\mathbb{S}=\{k_3,k_4\}$. Then on $\mathbb{R}$ and $\mathbb{S}$, respectively, Neutrosophic Topologies are $\Im_N=\{0_N,L,1_N\}$ and $\gamma_N=\{0_N,M,1_N\}$. Using

ɉ $(k_1)=k_3$ and ɉ $(k_2)=k_4$, define a mapping ɉ : $(\mathbb{R}, \Im_N) \to (\mathbb{S}, \gamma_N)$ by ɉ$(k_1)=k_3$ and ɉ $(k_2)=k_4$ .Since the NS J={<s,(0.9,0.5,0.3), (0.7,0.5,0.5)>} is NCS in $\mathbb{S}$, ɉ$^{-1}$(J) is a NRGβCS but not NαCS in $\mathbb{R}$. As a result ɉ is not

Nα CTS mapping but rather than an NRGβ continuous mapping.

Example 3.7 : Let L={<r,(0.5,0.5,0.6), (0.8,0.5,0.4)>} and M={<s,(0.7,0.5,0.3),(0.2,0.5,0.8)>} where $\mathbb{R}=\{k_1,k_2\}$,

$\mathbb{S}=\{k_3,k_4\}$. Then on $\mathbb{R}$ and $\mathbb{S}$, respectively, Neutrosophic Topologies are $\Im_N=\{0_N,L,1_N\}$ and $\gamma_N=\{0_N,M,1_N\}$. Using

ɉ $(k_1)=k_3$ and ɉ $(k_2)=k_4$, define a mapping ɉ: $(\mathbb{R}, \Im_N) \to (\mathbb{S}, \gamma_N)$. Since the NS J={<s,(0.3,0.5,0.7), (0.8,0.5,0.2)>} is NCS in $\mathbb{S}$, ɉ$^{-1}$(J) is a NRGβCS but not NGCS in $\mathbb{R}$. As a result ɉ is not NG CTS mapping but rather than an NRGβ continuous mapping.

Example 3.8 : Let L={<r,(0.7,0.5,0.3), (0.4,0.5,0.8)>} and M={<s,(0.3,0.5,0.9),(0.5,0.5,0.7)>} where $\mathbb{R}=\{k_1,k_2\}$,

$\mathbb{S}=\{k_3,k_4\}$. Then on $\mathbb{R}$ and $\mathbb{S}$, respectively, Neutrosophic Topologies are $\Im_N=\{0_N,L,1_N\}$ and $\gamma_N=\{0_N,M,1_N\}$. Using

ɉ $(k_1)=k_3$ and ɉ $(k_2)=k_4$, define a mapping ɉ : $(\mathbb{R}, \Im_N) \to (\mathbb{S}, \gamma_N)$. Since the NS J={<s,(0.9,0.5,0.3), (0.7,0.5,0.5)>} is NCS in $\mathbb{S}$, ɉ$^{-1}$(J) is a NRGβCS but not NαGCS in $\mathbb{R}$. As a result ɉ is not NαG CTS mapping but rather than an NRGβ continuous mapping.

Example 3.9 : Let L={<r,(0.4,0.5,0.6), (0.7,0.5,0.3)>} and M={<s,(0.3,0.5,0.9),(0.5,0.5,0.7)>} where $\mathbb{R}=\{k_1,k_2\}$,

$\mathbb{S}=\{k_3,k_4\}$. Then on $\mathbb{R}$ and $\mathbb{S}$, respectively, Neutrosophic Topologies are $\Im_N=\{0_N,L,1_N\}$ and $\gamma_N=\{0_N,M,1_N\}$. Using

ɉ $(k_1)=k_3$ and ɉ $(k_2)=k_4$, define a mapping ɉ : $(\mathbb{R}, \Im_N) \to (\mathbb{S}, \gamma_N)$ .Since the NS J={<s,(0.9,0.5,0.3), (0.7,0.5,0.5)>} is NCS in $\mathbb{S}$, ɉ$^{-1}$(J) is a NRGβCS but not NGPCS in $\mathbb{R}$. As a result ɉ is not NGP CTS mapping but rather than an NRGβ continuous mapping.

Example 3.10 : Let L={<r,(0.7,0.5,0.3), (0.4,0.5,0.8)>} and M={<s, (0.6,0.5,0.4), (0.6,0.5,0.2>} where $\mathbb{R}=\{k_1,k_2\}$, $\mathbb{S}=\{k_3,k_4\}$. Then on $\mathbb{R}$ and $\mathbb{S}$, respectively, Neutrosophic Topologies are $\Im_N=\{0_N,L,1_N\}$ and $\gamma_N=\{0_N,M,1_N\}$. Using ɉ $(k_1)=k_3$ and ɉ $(k_2)=k_4$, define a mapping ɉ : $(\mathbb{R}, \Im_N) \to (\mathbb{S}, \gamma_N)$.Since the NS J={<s,(0.4,0.5,0.6), (0.2,0.5,0.6)>} is NCS in $\mathbb{S}$, ɉ$^{-1}$(J) is a NRGβCS but not NSCS in $\mathbb{R}$. As a result ɉ is not NS CTS mapping but rather than an NRGβ continuous mapping.

Example 3.11: Let L={<r, (0.7, 0.5, 0.4), (0.3, 0.5, 0.6)>}and M={<s,(0.9,0.5,0.6),(0.0,0.5,0.1)>}where $\mathbb{R}=\{k_1,k_2\}$, $\mathbb{S}=\{k_3,k_4\}$. Then on $\mathbb{R}$ and $\mathbb{S}$, respectively, Neutrosophic Topologies are $\Im_N=\{0_N,L,1_N\}$ and $\gamma_N=\{0_N,M,1_N\}$. Using

ɉ $(k_1)=k_3$ and ɉ $(k_2)=k_4$, define a mapping ɉ: $(\mathbb{R}, \Im_N) \to (\mathbb{S}, \gamma_N)$. Since the NS J={<s,(0.6,0.5,0.9), (0.1,0.5,0.0)>} is NCS in $\mathbb{S}$, ɉ$^{-1}$(J) is a NRGβCS but not NGSCS in $\mathbb{R}$. As a result ɉ is not NGS CTS mapping but rather than an NRGβ continuous mapping.

Example 3.12 : Let L={<r, (0.9,0.5,0.3),(0.8,0.5,0.2)>} and M={<s, (0.7,0.5,0.4),(0.6,0.5,0.3)>} where $\mathbb{R}=\{k_1,k_2\}$, $\mathbb{S}=\{k_3,k_4\}$. Then on $\mathbb{R}$ and $\mathbb{S}$, respectively, Neutrosophic Topologies are $\Im_N=\{0_N,L,1_N\}$ and $\gamma_N=\{0_N,M,1_N\}$. Using

ɉ $(k_1)=k_3$ and ɉ $(k_2)=k_4$, define a mapping ɉ : $(\mathbb{R}, \Im_N) \to (\mathbb{S}, \gamma_N)$ by ɉ$(k_1)=k_3$ and ɉ $(k_2)=k_4$. Since the NS J={<s,(0.4,0.5,0.7), (0.3,0.5,0.6)>} is NCS in $\mathbb{S}$, ɉ$^{-1}$(J) is a NRGβCS but not NbCS in $\mathbb{R}$. As a result ɉ is not

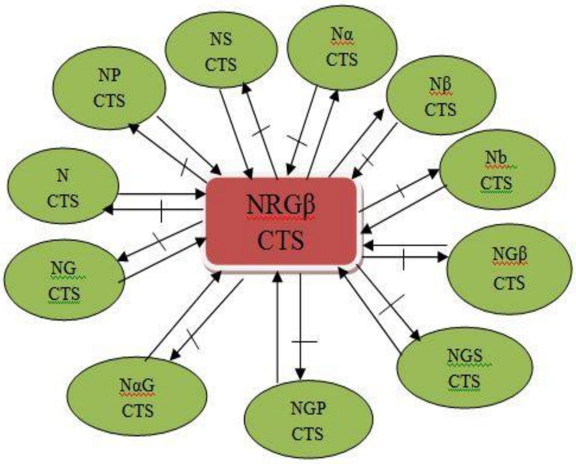
Nb CTS mapping but rather than an NRGβ continuous mapping.

Example 3.13 : Let L={<r, (0.6,0.5,0.4),(0.7,0.5,0.4)>} and M={<s, (0.2,0.5,0.8),(0.4,0.5,0.9)>} where $\mathbb{R}$ ={k_1,k_2}, $\mathbb{S}$ ={k_3,k_4}. Then on $\mathbb{R}$ and $\mathbb{S}$, respectively, Neutrosophic Topologies are $\Im_N$ ={0_N,L,1_N} and γ_N ={0_N,M,1_N}. Using

ɟ (k_1)= k_3 and ɟ (k_2)= k_4 , define a mapping ɟ : ($\mathbb{R}$, $\Im_N$) → ($\mathbb{S}$, γ_N).Since the NS J={<s,(0.8,0.5,0.2), (0.9,0.5,0.4)>} is NCS in $\mathbb{S}$, ɟ $^{-1}$(J) is a NRGβCS but not NβCS in $\mathbb{R}$. As a result ɟ is not Nβ CTS mapping but rather than an NRGβ continuous mapping.

Example 3.14 : Let L={<r, (0.5,0.5,0.5),(0.3,0.7,0.7)>} and M={<s, (0.6,0.4,0.4),(0.7,0.3,0.3)>} where $\mathbb{R}$ ={k_1,k_2}, $\mathbb{S}$ ={k_3,k_4}. Then on $\mathbb{R}$ and $\mathbb{S}$, respectively, Neutrosophic Topologies are $\Im_N$ ={0_N,L,1_N} and γ_N ={0_N,M,1_N}. Using ɟ (k_1)= k_3 and ɟ (k_2)= k_4 , define a mapping ɟ : ($\mathbb{R}$, $\Im_N$) → ($\mathbb{S}$, γ_N).Since the NS J={<s,(0.4,0.4,0.6), (0.3,0.3,0.7)>} is NCS in $\mathbb{S}$, ɟ $^{-1}$(J) is a NRGβCS but not NGβCS in $\mathbb{R}$. As a result ɟ is not NGβ CTS mapping but rather than an NRGβ continuous mapping.

Therefore, the following conclusions can be drawn from the aforementioned theorem and examples:



Theorem 3.15 : Consider Neutrosophic topological spaces ($\mathbb{R}$, $\Im_N$), ($\mathbb{S}$, γ_N) and ($\mathbb{T}$, σ_N). Then ɠ ∘ ɟ : ($\mathbb{R}$, $\Im_N$) →($\mathbb{T}$, σ_N) is a NRGβ continuous mapping if ɟ: ($\mathbb{R}$, $\Im_N$) → ($\mathbb{S}$, γ_N) is a NRGβ continuous mapping and ɠ : ($\mathbb{S}$, γ_N) → ($\mathbb{T}$, σ_N) is a N CTS.

Proof: Suppose that V is a NCS in $\mathbb{T}$. Since ɠ : ($\mathbb{S}$, γ_N) → ($\mathbb{T}$, σ_N) is a N CTS mapping, $ɠ^{-1}(V)$ is NCS in $\mathbb{S}$. Then $ɟ^{-1}[ɠ^{-1}(V)]$ is NRGβCS in $\mathbb{R}$ since ɟ is a NRGβ continuous mapping. However $ɟ^{-1}[ɠ^{-1}(V)] = (ɠ \circ ɟ)^{-1}(V)$. Then $(ɠ \circ ɟ)^{-1}(V)$ is NRGβCS in $\mathbb{R}$. Therefore ɠ ∘ɟ is a NRGβ continuous mapping.

Theorem 3.16 : A mapping ɟ : ($\mathbb{R}$, $\Im_N$) → ($\mathbb{S}$, γ_N) is NRGβ continuous mapping if and only if each NOS in $\mathbb{S}$ has an inverse image that is an NRGβ OS in $\mathbb{R}$.

Proof:

Part 1 : If V is a NOS in $\mathbb{S}$, then V^C is a NCS in $\mathbb{S}$. Since ɟ is an NRGβ continuous mapping, $ɟ^{-1}(V^C)$ is NRGβCS in $\mathbb{R}$. Since $ɟ^{-1}(V^C) = (ɟ^{-1}(V))^C$, $ɟ^{-1}(V)$ is a NRGβ OS in $\mathbb{R}$.

Part 2 : Suppose V is a NCS in $\mathbb{S}$. Then V^C is a NOS in $\mathbb{S}$. $ɟ^{-1}(V^C)$ is NRGβ OS in $\mathbb{R}$, according to the hypothesis. As $ɟ^{-1}(V^C) = (ɟ^{-1}(V))^C$, $(ɟ^{-1}(V))^C$ is a NRGβ OS in $\mathbb{R}$. Thus $ɟ^{-1}(V)$ is NRGβ CS in $\mathbb{R}$. Hence ɟ is NRGβ continuous mapping.

Theorem 3.17: For every NCS V in $\mathbb{S}$, let $ɟ^{-1}(V)$ be a NRCS in $\mathbb{R}$ and ɟ : ($\mathbb{R}$, $\Im_N$) → ($\mathbb{S}$, γ_N) be a mapping. Then ɟ is a NRGβ CTS.

Proof: Assume that $ɟ^{-1}(V)$ is a NRCS in ℝ and that V is a NCS in 𝕊. Since every NRCS is closed and every closed set is NRGβCS, $ɟ^{-1}(V)$ is a NRGβCS in ℝ. This means that ɟ is is a NRGβ CTS.

Definition 3.18 : A Neutrosophic topology (ℝ, $\Im_N$) is referred to as a

1. If every NRGβCS in ℝ is a NCS in ℝ, then the space is Neutrosophic RGβa$T_{1/2}$ (or simply NRGβa$T_{1/2}$).
2. If every NRGβCS in ℝ is a NGCS in ℝ, then the space is Neutrosophic RGβb$T_{1/2}$ (or simply NRGβb$T_{1/2}$).
3. If every NRGβCS in ℝ is a NGβCS in ℝ, then the space is Neutrosophic RGβc$T_{1/2}$ (or simply NRGβc$T_{1/2}$).

Theorem 3.19 : Given an NRGβ CTS mapping ɟ : (ℝ, $\Im_N$) ⟶ (𝕊, γ_N),

(i) if ℝ is an NRGβa$T_{1/2}$ space, then ɟ is an NCTS.
(ii) if ℝ is a NRGβb$T_{1/2}$ space, then ɟ is a NG CTS.
(iii) if ℝ is a NRGβc$T_{1/2}$ space, then ɟ is a NGβ CTS.

Theorem 3.20 : The mapping from Neutrosophic topology in ℝ to a Neutrosophic topology in 𝕊 is represented by the notation ɟ : (ℝ, $\Im_N$) ⟶ (𝕊, γ_N). be a mapping from Neutrosophic topology in ℝ into a Neutrosophic topology in 𝕊. If ℝ is a NRGβa$T_{1/2}$ space, then the following conditions are equal.

(i) ɟ is a mapping of NRGβ CTS.
(ii) $ɟ^{-1}(\mu)$ is a NRGβOS in ℝ if $\boldsymbol{\mu}$ is a NOS in 𝕊.
(iii) For every NOS $\boldsymbol{\mu}$ in 𝕊, $ɟ^{-1}$ (N int($\boldsymbol{\mu}$)) ⊆ N int(N cl(N int($ɟ^{-1}$(N int(μ))))).

Proof: It is evident that (i) ⇒ (ii) is true.

(ii) ⇒ (iii) In 𝕊, let $\boldsymbol{\mu}$ be a NOS. N int($\boldsymbol{\mu}$) is a NOS in 𝕊 accordingly. In that case $ɟ^{-1}(\text{N int}(\mu))$ is a NRGβOS in ℝ. $ɟ^{-1}(\text{N int}(\mu))$ is a NOS in 𝕊 since ℝ is a NRGβa$T_{1/2}$ space. Consequently $ɟ^{-1}(\text{N int}(\mu)) = N\ int(ɟ^{-1}(\text{N int}(\mu))) \subseteq N\ int(N\ cl(\text{N int}\ (ɟ^{-1}(\mu))))$.

(iii) ⇒ (i) In 𝕊, let $\boldsymbol{\mu}$ be a NCS. Then in 𝕊, $\boldsymbol{\mu}^c$ is a NOS. $ɟ^{-1}(\boldsymbol{\mu}^c)$ ⊆ N int(N cl(N int($ɟ^{-1}$(N int(μ^c))))) by hypothesis. This suggests that $ɟ^{-1}$ (N int($\boldsymbol{\mu}^c$)) ⊆ N int(N cl(N int($ɟ^{-1}$(N int(μ^c))))). Therefore in ℝ, $ɟ^{-1}(\boldsymbol{\mu}^c)$ is a N$\boldsymbol{\alpha}$OS is a NRGβOS. Hence in ℝ, $ɟ^{-1}(\mu)$ is a NRGβCS. Therefore ɟ is a NRGβ CTS mapping.

Theorem 3.21 : Let f be an NRGβ CTS mapping, such that f : (ℝ, $\Im_N$) ⟶ (𝕊, γ_N). Then the following conditions are hold.

(i) In ℝ for each NS A, ɟ (NRGβ cl(A))⊆Ncl(ɟ (A)).
(ii) In 𝕊, for every NS μ, NRGβ cl($ɟ^{-1}(\mu)$) ⊆ $ɟ^{-1}$(Ncl((μ))).

Proof:

(i) $ɟ^{-1}$(Ncl((ɟ (A))) is NRGβCS in ℝ since ɟ is a NRGβ CTS mapping and Ncl(ɟ (A)) is a NCS in 𝕊. In other words, NRGβ cl(A)⊆ $ɟ^{-1}$(Ncl((ɟ (A))). Therefore for every NS A in ℝ, ɟ (NRGβ cl(A)⊆ Ncl((ɟ (A)).

(ii)When $ɟ^{-1}(\mu)$ is substituted for A in (i), we obtain ɟ (NRGβ cl($ɟ^{-1}(\mu)$))⊆Ncl(ɟ ($ɟ^{-1}(\mu)$)) ⊆ Ncl(μ)). Thus, for every NS μ in 𝕊, NRGβ cl($ɟ^{-1}(\mu)$) ⊆ $ɟ^{-1}$(Ncl((μ))).

4.NRGβ IRRESOLUTE MAPPINGS

Definition 4.1: If for each NRGβCS V in 𝕊, $ɟ^{-1}(V)$ is a NRGβCS in ℝ, then a mapping ɟ : (ℝ, $\Im_N$) ⟶ (𝕊, γ_N) is a Neutrosophic Regular Generalized β irresolute (NRGβ irresolute).

Theorem 4.2: Given an NRGβ irresolute mapping ɟ : (ℝ, $\Im_N$) ⟶ (𝕊, γ_N), ɟ is a NRGβ CTS mapping.

Proof: Consider ɟ: (ℝ, $\Im_N$) ⟶ (𝕊, γ_N) to be a NRGβ irresolute mapping. Assume that V is any NCS in 𝕊. Since every NCS is NRGβCS, V is NRGβCS in 𝕊. $ɟ^{-1}(V)$ is a NRGβCS in ℝ, by hypothesis. Hence ɟ is a NRGβ CTS mapping.

Theorem 4.3: Given an NRGβ irresolute ɟ : (ℝ, $\Im_N$) ⟶ (𝕊, γ_N), if ℝ is a NRGβa$T_{1/2}$ space, then ɟ is a Neutrosophic irresolute mapping.

Proof: Suppose that V is a NCS in 𝕊. Then V is a NRGβCS in 𝕊. According to the hypothesis, $ɟ^{-1}(V)$ is a NRGβCS in ℝ. Since ℝ is a NRGβa$T_{1/2}$ space, $ɟ^{-1}(V)$ is a NCS in ℝ. Hence ɉ is a Neutrosophic irresolute mapping.

Theorem 4.4 : If ɉ : (ℝ, $\Im_N$) → (𝕊, γ_N) and ʛ : (𝕊, γ_N) → (𝕋, σ_N) are both NRGβ irresolute mappings, then ʛ ∘ɉ : (ℝ, $\Im_N$) →(𝕋, σ_N) is a NRGβ irresolute mapping.

Proof: In 𝕋, let V be a NRGβCS. Then $ʛ^{-1}(V)$ is a NRGβCS in 𝕊. Since ɉ is a NRGβ irresolute mapping, $ɟ^{-1}[ʛ^{-1}(V)]$ is NRGβCS in ℝ and so ʛ ∘ɉ is also an NRGβ irresolute mapping.

Theorem 4.5: Assuming that ɉ : (ℝ, $\Im_N$) → (𝕊, γ_N) and ʛ : (𝕊, γ_N) → (𝕋, σ_N) are NRGβ CTS mappings, then ʛ ∘ɉ : (ℝ, $\Im_N$) →(𝕋, σ_N) is a NRGβ CTS mapping.

Proof: Suppose that V is a NCS in 𝕋. Then $ʛ^{-1}(V)$ is a NRGβCS in 𝕊. Since ɉ is a NRGβ irresolute, $ɟ^{-1}[ʛ^{-1}(V)]$ is NRGβCS in ℝ. Hence ʛ ∘ɉ is a NRGβ CTS mapping.

Theorem 4.6: Given an NRGβ irresolute ɉ: (ℝ, $\Im_N$) → (𝕊, γ_N), if ℝ is an NRGβb$T_{1/2}$ space, then ɉ is a NG irresolute mapping.

Proof: In 𝕊, let V be an NRGβCS. By hypothesis, $ɟ^{-1}(V)$ is a NRGβCS in ℝ. Since ℝ is a NRGβb$T_{1/2}$ space, $ɟ^{-1}(V)$ is a NGCS in ℝ. Hence ɉ is a NG irresolute mapping.

Theorem 4.7: Suppose ɉ : (ℝ, $\Im_N$) → (𝕊, γ_N) is a mapping from a Neutrosophic topology ℝ into a Neutrosophic topology 𝕊. Then the following conditions are equivalent if ℝ and 𝕊 are NRGβa$T_{1/2}$ spaces.

(i) ɉ is a NRGβ irresolute mapping.
(ii) For each NRGβOS μ in , $ɟ^{-1}(\mu)$ is a NRGβOS in ℝ.
(iii) For every NS μ in 𝕊, Ncl($ɉ^{-1}(\mu)$) ⊆ $ɟ^{-1}$(N cl(μ)).

Proof: (i) ⇒ (ii): In 𝕊, let μ be any NRGβOS in 𝕊. Then in 𝕊, μ^c is a NRGβCS. $ɟ^{-1}(\mu^c)$ is a NRGβCS in ℝ, since ɉ is a NRGβ irresolute mapping. However, $ɟ^{-1}(\mu^c) = (ɟ^{-1}(\mu))^c$. Hence $ɟ^{-1}(\mu)$ is a NRGβOS in 𝕊.

(ii)⇒ (iii): Suppose μ is any NS in 𝕊 and μ⊆Ncl(μ). Then $ɉ^{-1}(\mu)$⊆ $ɉ^{-1}$(Ncl(μ)). Since Ncl(μ) is a NCS in 𝕊, Ncl(μ) is a NRGβCS in 𝕊. Therefore $(\text{Ncl}(\mu))^c$ is a NRGβOS in 𝕊. According to hypothesis $ɟ^{-1}((\text{Ncl}(\mu))^c)$ is a NRGβOS in ℝ. Since $ɟ^{-1}((\text{Ncl}(\mu))^c) = (ɟ^{-1}(\text{Ncl}(\mu)))^c)$, $ɟ^{-1}(Ncl(\mu))$ is a NRGβCS in ℝ. $ɟ^{-1}(Ncl(\mu))$ is a NCS in ℝ, since ℝ is a NRGβa$T_{1/2}$ space. Hence Ncl($ɉ^{-1}(\mu)$) ⊆ Ncl($ɉ^{-1}$(Ncl(μ)))= $ɟ^{-1}$(N cl(μ). That is Ncl($ɉ^{-1}(\mu)$) ⊆ $ɟ^{-1}$(N cl(μ)).

(iii) ⇒ (i): Assume μ is any NRGβCS in 𝕊. Then μ is a NCS in 𝕊 and Ncl(μ) = μ, since 𝕊 is a NRGβa$T_{1/2}$ space . Hence $ɉ^{-1}(\mu)$= $ɉ^{-1}$(Ncl(μ))⊇ Ncl($ɉ^{-1}(\mu)$). But clearly $ɉ^{-1}(\mu)$⊆ Ncl($ɉ^{-1}(\mu)$). Therefore Ncl($ɉ^{-1}(\mu)$)= $ɉ^{-1}(\mu)$ which implies $ɉ^{-1}(\mu)$ is a NCS and hence it is a NRGβCS in ℝ. Thus ɉ is a NRGβ irresolute mapping.

5.CONCLUSION

The characteristics of NRGβ irresolute and continuous mappings in Neutrosophic topological spaces have been introduced, examined, and their relationships developed in this research. We anticipate that the findings in this chapter will serve as the foundation for more mapping applications in Neutrosophic topological spaces.

REFERENCES:

1. R. Devi, K. Balachandaran and H. Maki : On generalized **α**-continuous functions and **α**- generalized continuous functions, Far East J. Math. Sci.,Special Volume, Part I, 1997, 1 - 15.
2. Florentin Smarandache.:,Neutrosophic and NeutrosophicLogic,*First International Conference* On Neutrosophic,Neutrosophic Logic, Set, Probability, and Statistics University of New Merico, Gallup, NM87301, USA , *smarand@unm.edu* ,(2002).
3. A.A.Salama and S.A. Alblowi., Generalized Neutrosophic Set and Generalized Neutrosophic Topological Spaces, *Journal computer Sci. Engineering*, Vol.(ii),No.(7)(2012).

4. A. A.Salama , F.Smarandache and V. Kromov, Neutrosophic closed set and neutrosophic continuous functions, *Neutrosophic Sets and Systems*,4(2014),4–8.

5. Iswarya P and Bageerathi K 2016 On Neutrosophic semi-open sets in Neutrosophic topological spaces, *International Journal of Mathematics Trends and Technology* **37**3 pp 24-33.

6. I. Arokiarani I, Dhavaseelan R, Jafari S and Parimala M, "On some new notions and functions in neutrosophic topological spaces", *Neutrosophic Sets Syst.* 2017, 16, 1619.

7. Al-Omeri, W.; Smarandache, F New Neutrosophic Sets via Neutrosophic Topological Spaces. In Neutrosophic Operational Research; Smarandache, F., Pramanik, S., Eds.; Pons Editions: Brussels, Belgium, 2017; Volume I, pp. 189–209.

8. R.Dhavaseelan, and S.Jafari., Generalized Neutrosophic closed sets, New trends in Neutrosophic theoryand applications ,Volume II,261-273,(2018).

9. Renu Thomas and Anila.S(2018), "On Neutrosophic semi-preopen sets and semi-preclosed sets in Neutrosophic Topological Spaces", International Journal of Scientific Research in Mathematical & Statistical Sciences, vol.5, pp.138-143.

10. V. K. Shanthi, S. Chandrasekar, K. SafinaBegam : Neutrosophic Generalized Semi- closed Sets In Neutrosophic topological spaces, International Journal of Research in Advent Technology, 6(7), 2018, 1739 - 1743.

11. C. Maheswari, S. Chandrasekar : Neutrosophic gb-closed Sets and Neutrosophic gb-Continuity, Neutrosophic Sets and Systems , Vol. 29, 2019, 89 - 99.

12. Merlin Kuriakose, A. Francina Shalini 2022 On Neutrosophic generalized **β** closed set in Neutrosophic Topological Spaces, International Journal of All Research Education and Scientific Methods (IJARESM), Volume 10, Issue 4.

13. R. Eswari and F. Nirmala Irudayam, "NRG**β** Closed Sets in Neutrosophic Topological Spaces" , Indian Journal of Natural Sciences, Vol.15 / Issue 85 / Aug / 2024.[15]