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Viscous Dissipation, Radiation, Pressure Work Effects on MHD Mixed Convection Flow from A Vertical Flat Plate with Internal Heat Generation (Absorption)

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Abstract: - In this study, the impact of internal heat generation (or absorption) into steady mixed convection MHD boundary layer flow over a vertical flat plate has been investigated. The analysis took into account the impacts of viscous dissipation, pressure work and radiation. The magnetic field is applied transversally to the stream direction, and the fluid is considered to be viscous, incompressible, and electrically conducting. By using appropriate coordinate transformations, the governing boundary region equations are translated into a non dimensional form. The numerical solution of coupled nonlinear ODE's using an implicit finite difference method in association with the quasi-linearization method yields nonsimilar solutions. To demonstrate the velocity and temperature distributions, numerical outcomes are obtained for various values of parameters. Comparisons with formerly published works are made, and excellent agreement is discovered. The analysis reveals that relevant parameters have a considerable impact on flow fields.

Keywords: *Convection, Internal heat generation, MHD, Radiation, Viscous dissipation.*

1. Introduction

The flow composition of mixed convection, one of the transport phenomena, is made up of free and forced convection. An exterior pushing mechanism and internal volumetric forces work together to find these flow patterns. It's employed in a variety of environmental, geophysical and energy related engineering applications, including as wind-free solar panels, electrical devices known as fans, nuclear reactors cooled at various stages of emergency and low-velocity heat exchangers.

The consequence of both forced and natural convection is present in equal order in a mixed convective flow. We discovered large temperature differences between both the heat body surface and the free stream in various realistic fields.

Temperature variations cause density gradients in the dynamics of the fluid, which get more articulated when gravitational dual-mode convection effects are present. The two-dimensional mixed laminar convective flow across a vertical level plate is the most basic actual model of this type of flow, and it has been extensively studied [1–8]. The overseeing factor for the laminar boundary hybrid convection stream, which measures the ratio of buoyancy powers to inertial forces within the boundary layer, is generally understood to be ($\xi = Gr_x / \mathrm{Re}_x^2$, in which Gr_x - Grashof number & Re_x - Reynolds number). The free convection cutoff can be reached if ξ turns out to be very large, but forced convection occurs when the ξ maximum drops to zero, which occurs at the main edge.

It is essential to consider the affects of heat generation on those travelling fluids in numerous specialized and actual problems combining exothermic or endothermic reactions of fluids, as noted by Vajravelu [9]. It has been shown by Westphalia et al. [10] that natural thermally driven convection can effectively simulate mimic combustion. While most physical situations make correct modeling of internal heat generation extremely difficult, certain straight forward mathematical models may represent typical behavior.

One kind of energy that comes from a source and travels through materials or space is radiation. Heat, sound, and light are all forms of radiation. This study considers radiation because it may have a significant influence on how heat transfer processes are regulated in the polymer manufacturing sector. Numerous new technical processes, such as solar power technologies, gas turbines, combustion-based energy generation from fossil fuels, and various propulsion mechanisms for spacecraft reentry, aircraft, and missiles, occur at high temperatures. Understanding radiation is so essential, and its effects cannot be understated.

Exploration of MHD dynamics and heat exchange in various environments has been sparked by the impact of an applied magnetic field on the management of boundary layer flow and the output of numerous systems employing electrically conductive fluids. Following it, numerous researchers [11-19] have spent the last few years researching the affects of internal heat generation, MHD and Radiation.

The previously mentioned published studies disregard the influence of pressure work and viscous dissipation. Heat conduction is significantly impacted by viscous dissipation even at modest velocities, especially for highly viscous flows. Due to viscosity, it converts kinetic energy into internal energy, which warms the fluid and creates silky movement. This rationale leads to the invention of numerous tools for use in riverbeds to lessen the kinetic energy of flowing water, so lowering the likelihood of erosion on the bottoms and banks of rivers. Because of this, the controlling parameter for dynamics of the fluid is called the dimensionless parameter Eckert number.

In this case, however, we have considered the effect of pressure work on a mixed convection flow alongside a vertical flat plate. Generally, the discussion and study of mixed convection streams and pressure work consequences are ignored.

The object of the current study is to determine how mixed-mode flow from a vertical flat plate along with dissipation through viscosity is affected by MHD, pressure work, radiation, and heat generation (absorption).

2. Objectives

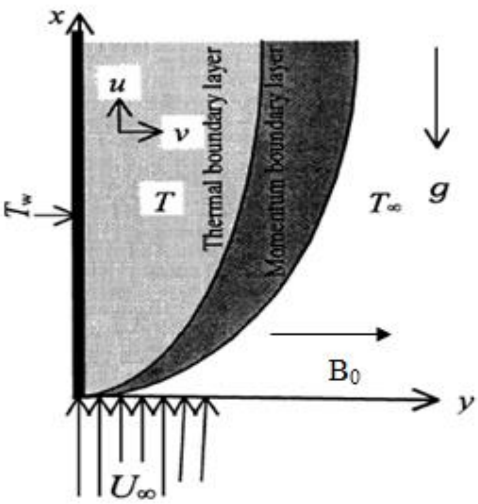

Figure 1. The Flow Arrangement And Coordinate Reference System.

We study the incompressible fluid's constant two-dimensional laminar mixed convective flow along a semi-infinite vertical flat plate. The location of the plate is in the x-y plane. The uniform temperature T_w of the plate is higher than that of the free stream T_w. Additionally, a constant free stream velocity U_∞ parallel to the vertical plate is assumed.

We also assume that the lone temperature-responsive attributes are density and viscosity, with density being unique only to the degree that it affects the momentum equation's buoyancy term (Boussinesq approximation). A diagrammatic representation of the coordinate system and flow domain is presented in Figure 1.

In accordance with the aforementioned assumptions, the two-dimensional boundary region equations for the hybrid convective fluid motion through the vertically aligned, infinitely extending plate are provided below.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta(T-T_\infty)+\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{T\beta u}{\rho C_P}\frac{\partial p}{\partial x}+\frac{Q_0}{\rho C_p}(T-T_\infty)+\frac{1}{\rho C_P}\frac{\partial q_r}{\partial y}+\frac{K}{C_P}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

The boundary conditions to be fulfilled by the above conditions are,

$$u=0,\quad v=0,\quad T=T_w \quad \text{at} \quad y=0$$

$$u=U_\infty,\quad T=T_\infty \qquad \text{as} \quad y\to\infty \tag{4}$$

In this context, v and u represent the fluid components parallel to the y-axis and x-axis, respectively, adjacent to the plate, g – gravitational acceleration, ρ -fluid density, C_p - Constant pressure specific heat, β - coefficient of thermal expansion, Q_0 - internal heat generation, T- temperature inside the layer of boundary, and α - thermal diffusivity. $\frac{\partial p}{\partial x}$ is the pressure of hydrostatic.

The radiative heat flux is simplified by using the Rosseland definition for radiation [20]

$$q_r=-\frac{4\sigma}{3\alpha *}\frac{\partial T^4}{\partial y} \tag{5}$$

The term T^4 could be imparted as a temperature linear function by accepting that the temperature in the flows contrasts. So, we enter a Taylor sequence with respect to T_∞ expanding T^4 and ignoring higher-level terms.

$$T^4\cong 4T_\infty^3 T-3T_\infty^4 \tag{6}$$

Utilizing eqn. (5) and (6), (3) becomes

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{T\beta u}{\rho C_P}\frac{\partial p}{\partial x}+\frac{Q_0}{\rho C_p}(T-T_\infty)+\frac{16\sigma T_\infty^3}{3\alpha^*\rho C_P}\frac{\partial^2 T}{\partial y^2}+\frac{K}{C_P}\left(\frac{\partial u}{\partial y}\right)^2 \tag{7}$$

Further,

$$\psi(x,y)=\upsilon_\infty \mathrm{Re}_x^{1/2}(1+\xi)^{1/4}f(\xi,\eta),\qquad \eta=\frac{y}{x}\mathrm{Re}_x^{1/2}(1+\xi)^{1/4},\qquad \xi=\frac{Gr_x}{\mathrm{Re}_x^2}$$

$$\mathrm{Re}_x=\frac{U_\infty x}{\upsilon_\infty},\qquad \theta(\xi,\eta)=\frac{T-T_\infty}{T_w-T_\infty},\qquad Gr_x=\frac{g\beta(T_w-T_\infty)x^3}{\upsilon_\infty^2}. \tag{8}$$

Where ψ is the definition of the stream function that complies with the continuity equation (1) & is defined by,

$$u = \frac{\partial \psi}{\partial y} \text{ .and } v = -\frac{\partial \psi}{\partial x} \tag{9}$$

By transforming equations (1)-(4) and (7) as described above:

$$F'' + G\left(\frac{\xi}{1+\xi}\right) + F' f\left(\frac{2+3\xi}{4(1+\xi)}\right) - F^2\left(\frac{\xi}{2(1+\xi)}\right) - (1+\xi)^{-1/2} MF = \xi[FF_\xi - F' f_\xi] \tag{10}$$

$$(1+N_R)G'' + \Pr QG + \Pr E_C F'^2 - \varepsilon GF \Pr + \left(\frac{2+3\xi}{4(1+\xi)}\right) G' f \Pr = \Pr \xi[FG_\xi - G' f_\xi] \tag{11}$$

The boundary conditions are

$$f(\xi,0) = F(\xi,0) = 0 \qquad G(\xi,0) = 1$$

$$F(\xi,\infty) = (1+\xi)^{-1/2} \qquad G(\xi,\infty) = 0 \tag{12}$$

Correspondingly, the local shear stress & the local surface, Heat flux can be corresponded, as

$$\tau_x = \xi^{-1/2}(1+\xi)^{3/4} f''(\xi,0) \quad \text{and}$$

$$q_x = (1+N_R)\xi^{-1/2}(1+\xi)^{1/4} G'(\xi,0) \tag{13}$$

3. Methods

Using an implicit finite difference approach, the coupled nonlinear ODEs (10) and (11) have been cracked along with the boundary setting (12). This is agreed upon in arrangement with a quasilinearization process [21, 22]. For brevity, the process is not included here since it is described in Inoue and Tate [20]. To assess the dependability, the present study's findings were compared to Raju et.al [6] earlier inquiry when skin friction (F') and heat transmission (G') had Q = M = ε = N_R = Ec = 0.0 for a range of values of ξ, for Pr = 0.7, as shown in Table 1. Our outcomes are considered to be in remarkable agreement with [6] results for the right 4 decimal places of accuracy.

TABLE 1. Numerical Values Of $F'(\xi,0)$ And $G'(\xi,0)$ Taking.Pr = 0.72 While Q = ε = M = N_R = Ec = 0.0

ξ	$F'(\xi,0)$		$-G'(\xi,0)$	
	Present	**Raju et.al[6]**	**Present**	**Raju et.al[6]**
0.0000	0.3323	0.3321	0.2931	0.2928
0.29011	0.5742	0.5919	0.3333	0.3373
0.42115	0.6709	0.6889	0.3462	0.3505
0.52992	0.7348	0.7536	0.3547	0.3584
0.62920	0.7897	0.8020	0.3605	0.3639
0.72399	0.8230	0.8404	0.3651	0.3680
1.20059	0.9357	0.9743	0.3765	0.3823
1.77903	0.9888	1.0171	0.3802	0.3828
2.59153	1.0099	1.0282	0.3796	0.3794
3.87298	1.0140	1.0251	0.3770	0.3751
6.16948	1.0079	1.0145	0.3728	0.3706
11.06602	0.9959	1.0002	0.3685	0.3662
24.97999	0.9826	0.9848	0.3635	0.3619
99.99500	0.9693	0.9699	0.3585	0.3576
∞	0.9567	0.9570	0.3536	0.3531

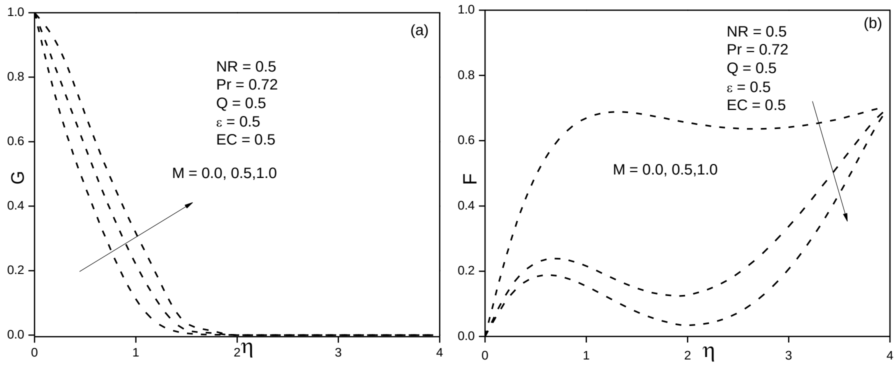

Figure 2.A) The Temperature Along With B) The Velocity Flow Curve For Several Of M

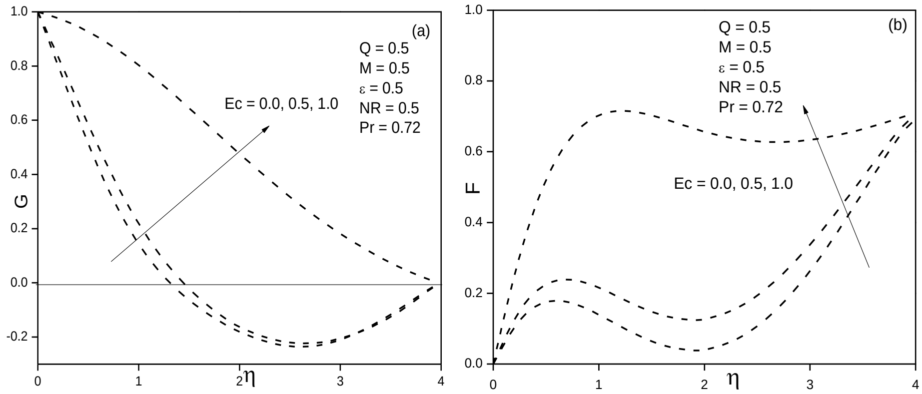

Figure 3.A) The Temperature Along With B) The Velocity Profiles For Several Of Ec

The varying velocity (F) and temperature (G) profiles for various M values are depicted in Fig. 2. It is very likely that the velocity increases as it approaches the magnetic field boundary. This is because the Lorentz force, a type of resistive force, is amplified when orthogonal magnetic influence is introduced to a conductive fluid. The fluid's axial movement tends to be slowed down by this force. Additionally temperature pattern increments along η bearing. The magnetic field elevates the fluid temperature within the region of boundary due to intense heating. As a result, we state that the boundary layer's deterioration is brought on by a magnetic field.

Fig.3 represents the velocity and temperature distributions for multiple values of Eckert number (Ec). Velocity and heat transfer distributions accelerate at the point when Ec estimations are extended. The viscous dissipation, as a heat generation inside the fluid, increases the bulk fluid temperature. This can be attributed to the additional heating in the flow system due to viscous dissipation.

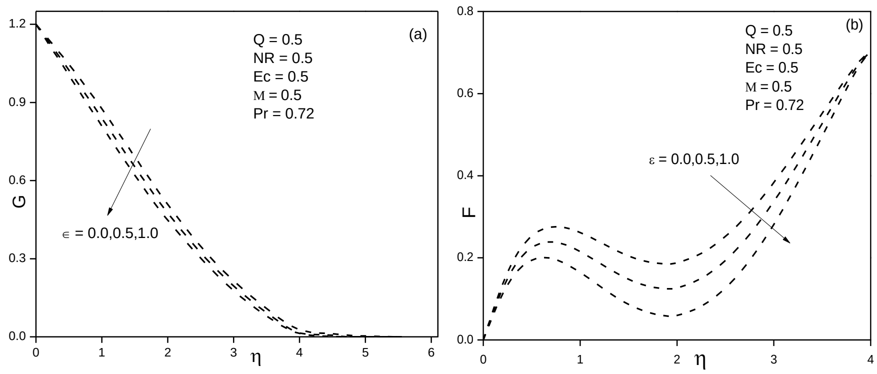

Figure 4.A) The Temperature Along With B) The Velocity Pattern For Several Of *ε*

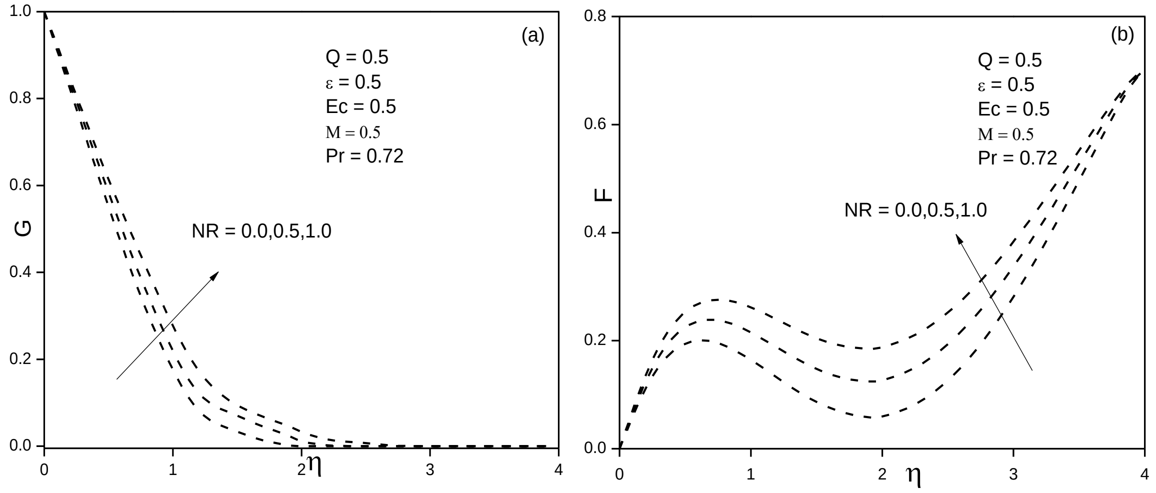

Figure 5. A) The Temperature Along With B) The Velocity Profiles For Several Of N_r

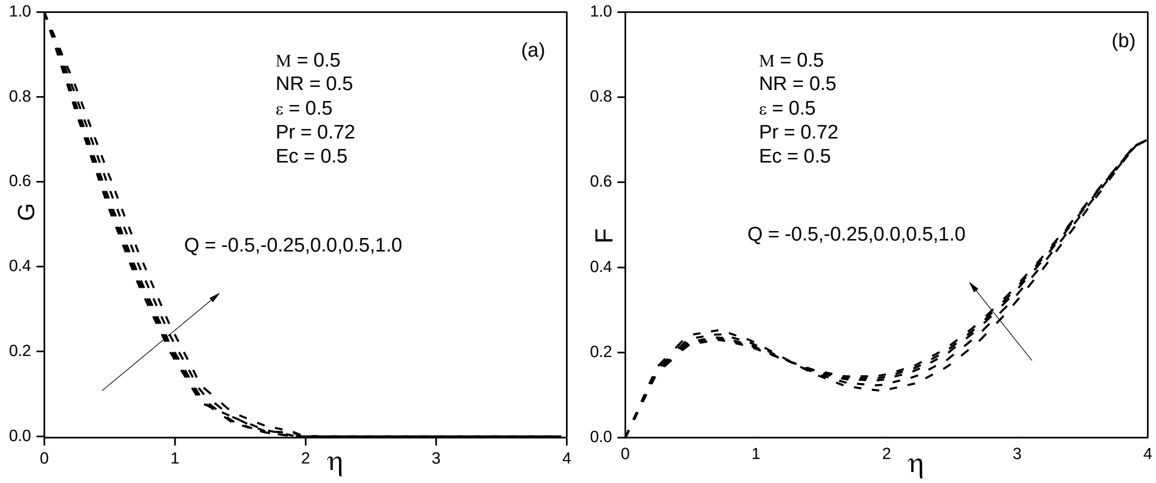

Figure 6.A) The Temperature Along With B) The Velocity Pattern For Multiple Values Of Q

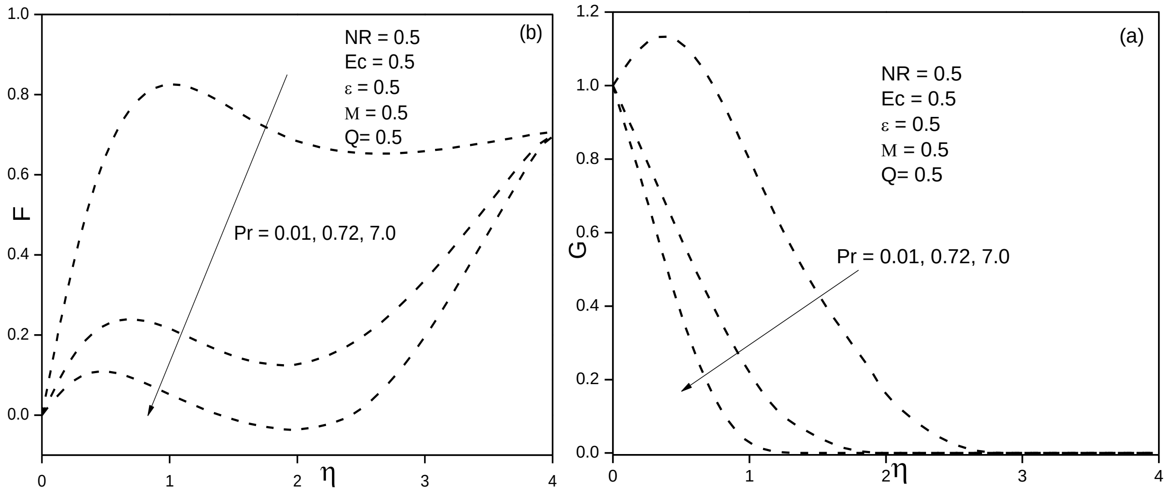

Figure 7.A) The Temperature Along With B) The Velocity Pattern For Multiple Values Of Pr

Fig.4 demonstrated the temperature as well as flow velocity pattern for multiple values of pressure work parameters ($\in$). From Fig. 4, it can be realised that as $\in$ grows, both the velocity and temperature profile declines with $\in$. Aligned with vertical flat plate, the velocity and temperature profiles are compressed and suppressed. Hence, it demonstrates that both thermal and momentum boundary layer thicknesses are observed to be decreasing.

Fig.5 presents the results, which take into account how radiation (N_R) affects, temperature (G) along with velocity (F) profiles. More streams enter the region of boundary as a result of the fluid's temperature rising, raising the fluid's velocity there. Moreover, expanding F causes the momentum region depth to increase. The temperature profile rises in tandem with NR. This enables the fluid to cool the device by transferring heat energy from the flow locale. This is accurate given that the Roseland computation raises the temperature.

The effects of the internal heat generation (absorption) (Q) parameter on temperature and velocity distributions are displayed in Fig. 6. As can be seen, increasing Q (i.e.,-0.5,-0.25,0.0.0.25,0.5) increases both the velocity and temperature pattern. The fluid's velocity and temperature in the boundary layer region will rise as an outcome of Q's increased heat content near the plate zone, according to the redaction. Additionally, as the internal heat generation (absorption) parameter rises, so does the depth of the momentum as well as thermal boundary layers.

Fig. 7(a) illustrates how the Prandtl number (Pr = 0.01, 0.72, 7.0) affects velocity profiles, demonstrating how velocity distributions are significantly smaller close up to the wall than they are when Pr is larger. The extra shooting magnitude falls with increasing Pr. In low Prandtl numbers (Pr = 0.7), the influence is significant since the obvious low fluid viscosity, which raises the velocity inside the boundary layer. In contrast, Fig. 7 depicts the outcome of Pr on the temperature pattern (b). This is because a fluid with a greater Prandtl number has a lower thermal conductivity, which reduces conduction and hence the depth of the thermal boundary layer, lowering temperature.

4. CONCLUSIONS

The dominance of viscous dissipation, radiation, pressure work and internal thermal generation on MHD hybrid convection flow from a vertical flat plate is investigated in this paper. The following is a summary of the current investigation's findings.

➢ The velocity profile decreases and temperature profile heightens with an ascent in qualities of magnetic field parameter.

➢ The momentum and thermal boundary region thicknesses expand with rising Eckert number, radiation and heat generation parameters, bringing about an expansion in both temperature as well as velocity profiles.

➢ At the time when the pressure work boundary increases, the temperature and velocity distributions both lessening.

➢ When the pressure work factor rises, the temperature as well as velocity profiles decrease.

➢ The velocity alongwith temperature distributions decrease when the Prandtl number values are increased.

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