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Experimental Study of the Ratio of the Conjugate Heights of the Hydraulic Jump Evolving in A Rectangular Channel of Composite Section with Rough Major Bed and Positive Slope

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Abstract: Hydraulic jump is a phenomenon commonly observed downstream of dams, serving to dissipate hydraulic energy or in water transport channels to raise the level of the watercourse. It is characterized by a rapid transition from a supercritical flow state to a subcritical state, leading to considerable energy dissipation. This paper aims to study and analyze the conjugate heights of the hydraulic jump in a rectangular channel with a composite section, characterized by a uniform rough major bed and a positive slope, under diverse flow conditions through experimental research. A series of experiments were conducted in a rectangular channel featuring artificially rough beds composed of homogeneous plastic granules. Hydraulic parameters were measured for different bed roughness and positive slopes, such as the two conjugate heights, initial h_1, final h_2, and flow rate. Analysis of the experimental data showed that the rough major bed and the slope of the channel have an effect on the ratio of the conjugate heights. With the availability of a large amount of experimental data on hydraulic jumps on rough channel beds, mathematical experimental approaches have been obtained to express the Y ratio of the conjugate heights of the hydraulic jumps as a function of roughness and slope parameters.

Keywords*:* The hydraulic jump, The Y ratio of conjugate heights, slope, compound rectangular channel, rough major bed.

1. Introduction

The study of hydraulic jump problems began to arise at the beginning of the 19th century. The hydraulic jump was linked to the principle of momentum by Belanger in 1828. Researchers conducted experiments, and the results showed excellent verification of the momentum principle of the hydraulic jump (Chen-Feng Li, 1995). When the flow profile transitions from supercritical to subcritical, a standing wave known as a hydraulic jump is created. This causes an abrupt rise in the water surface within the channel, leading to the formation of a turbulent zone. In this zone, surface rollers are generated, resulting in intense water mixing and air entrainment, and accompanied by the dissipation of a substantial amount of water energy (Alghwail, 2020).

Accelerating the flow rate or flow of water in open channels from supercritical to subcritical flow through the formation of a perfectly developed hydraulic jump is one of the most important measures used to form this hydraulic phenomenon near a dam, weir, or any hydraulic installation. This procedure greatly helps to reduce the flow velocity and dissipate the energy of the flowing water, thus reducing the distance to be protected to avoid the erosion problem at the bottom of the canal (Alghwail, 2021).

Numerous researchers have studied the hydraulic jump, including (Bradley and Peterka, 1957), (Hager and Bretz, 1987), (Hager and Li, 1992), and (Ead and Rajaratnam, 2002) The majority of these studies focused on hydraulic jumps in rectangular channels with zero slope. However, the first in-depth study of hydraulic jumps in rectangular channels with a positive slope was conducted by (Bakhmeteff and Matzke 1938), who examined the surface profile, jump length, and velocity distribution of the hydraulic jump.

Significant works by researchers such as Wilson (1970), (Ohashi et al. 1973), (Rajaratnam and Murahari 1974) and (Mikhalev and Hoang Ty An, 1976). (Debabeche et al. 2009) explored the hydraulic jump with a positive slope in triangular channels. (Cherhabil, S., 2010) in his doctoral thesis, extended the study of hydraulic jumps with positive slopes to triangular and U-shaped channels. Later, (Kateb, S., 2014) focused on hydraulic jumps in trapezoidal channels. The most recent research on hydraulic jumps in rectangular channels with varying positive slopes was conducted by (Nouacer, B., 2023).

The hydraulic jump evolving in rectangular channels composed of rough beds has recently been studied by (Djamaa, W. , 2022; Lacheheb, S. , 2023).

This study presents experimental research on the hydraulic jump in a rectangular channel with a composite section, featuring a rough major bed and a positive slope. The configuration used corresponds to the type D jump as classified by (Kindsvater, 1944). The objective is to experimentally determine the impact of the major bed's roughness and the channel's slope on the ratio of the conjugate heights of the jump (Y). Homogeneous rough mats made of plastic granules were created and tested for this type of hydraulic jump.

Materials And Methods

All experiments on the hydraulic jump evolving in a rectangular channel of compound section with a rough major bed and a positive slope were carried out at the Laboratory for the Exploitation and Development of Natural Resources in Arid Zones (EVRNZA) of the Civil Engineering Department. and Hydraulics of the University of Ouargla.

All tests were conducted in a free-surface channel, as shown in Figure 4. The length of this channel is 10 m, its height is 0.5 m, and its side walls are made of transparent plexiglass. The experimental channel is horizontal with a compound rectangular section, length of 4 m, and height h=15.5 cm with the height of the minor bed b = 14.4 cm and major bed B = 25 cm. The channel's bottom is completely horizontal (zero slope). It is connected to the supply basin via a circular pipe with a 150 mm diameter, which leads into a closed metal box. An opening is placed in a flat metal wall of a specified width, creating an entry into the channel. This wall is designed to generate a torrential flow with a variable output section. The height of this flow will correspond to the initial height h_1 of the ledge. The volume flow rates are controlled by adjusting the valve. An axial pump supplies the channel, maintaining a flow rate of 55.55 l/s. the flow rate has been measured with a rectangular weir placed at the end, the specific flow rate is obtained by injecting the height h into equation (1) of the weir (Hachemi Rachedi, L., 2005)

$$Q = 0{,}3794B\sqrt{2g}\beta(1 + 0{,}16496\beta^{2{,}0716})^{3/2}\, h_{dev}^{3/2} \qquad (1)$$

The experiment was carried out under four initial heights h1 (cm) = 2.5, 3, 3.5, and 4. The tests started under four different roughness tested ε(mm) = 6; 8; 10 ; 12. The upstream part of the channel is placed on a screw-nut system which gives the possibility to lower or raise it by means of a metal arm that rotates the screw, and consequently, the slope of the channel varies value: tang (α) = 0; 0,005 ; 0,01 ; 0,015.

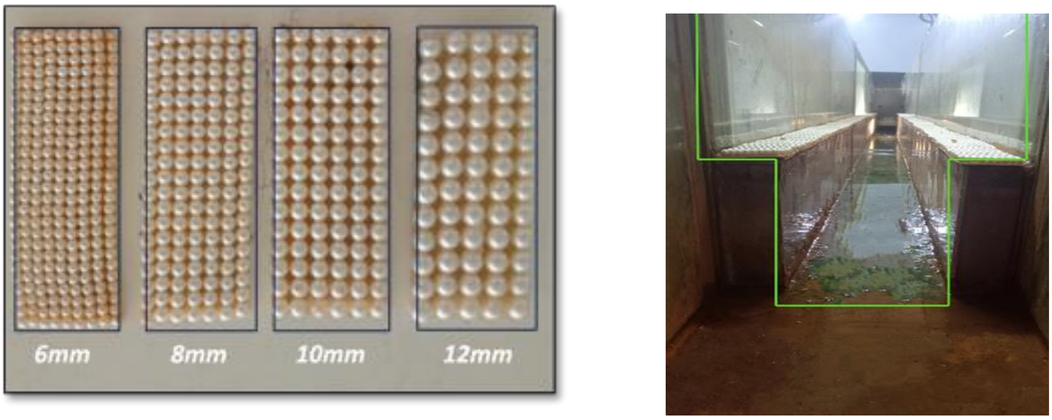

Figure 1: The mats for differ diameters of the tested roughness

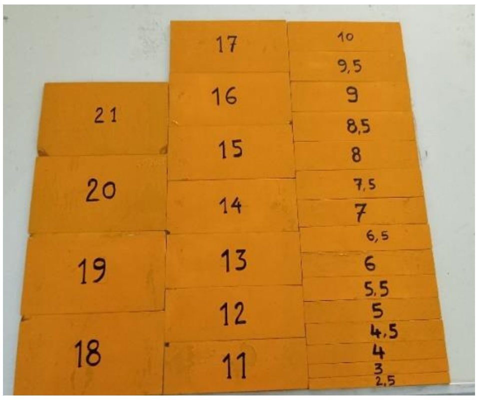

Figure 2: Series of thin thresholds used to control the jump

Figure 3: Rough compound section

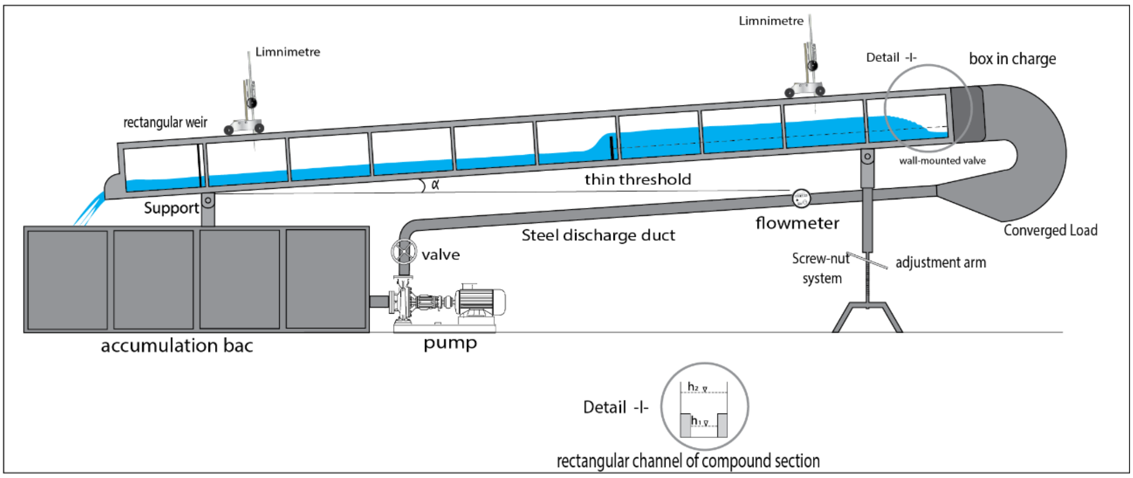

Figure 4: Simplified schema of the compound rectangular section measuring channel with variable positive slope, used for the experiment

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2. Results And Discussion

Variation of the Y ratio of the conjugate heights of the jump as a function of the Froude number F_1 and the channel's inclination angle (α) :

Figure (5) show the adjustment of the ratio of the conjugate pitches $Y= h_2/h_1$ as a function of the Froude number F_1 for four α distinct angles of inclination, such as: tang (α) = 0; 0,005 ; 0,01 ; 0.015, and absolute roughness: (ε = mm) = 0; 06 ; 08 ; 10 and 12

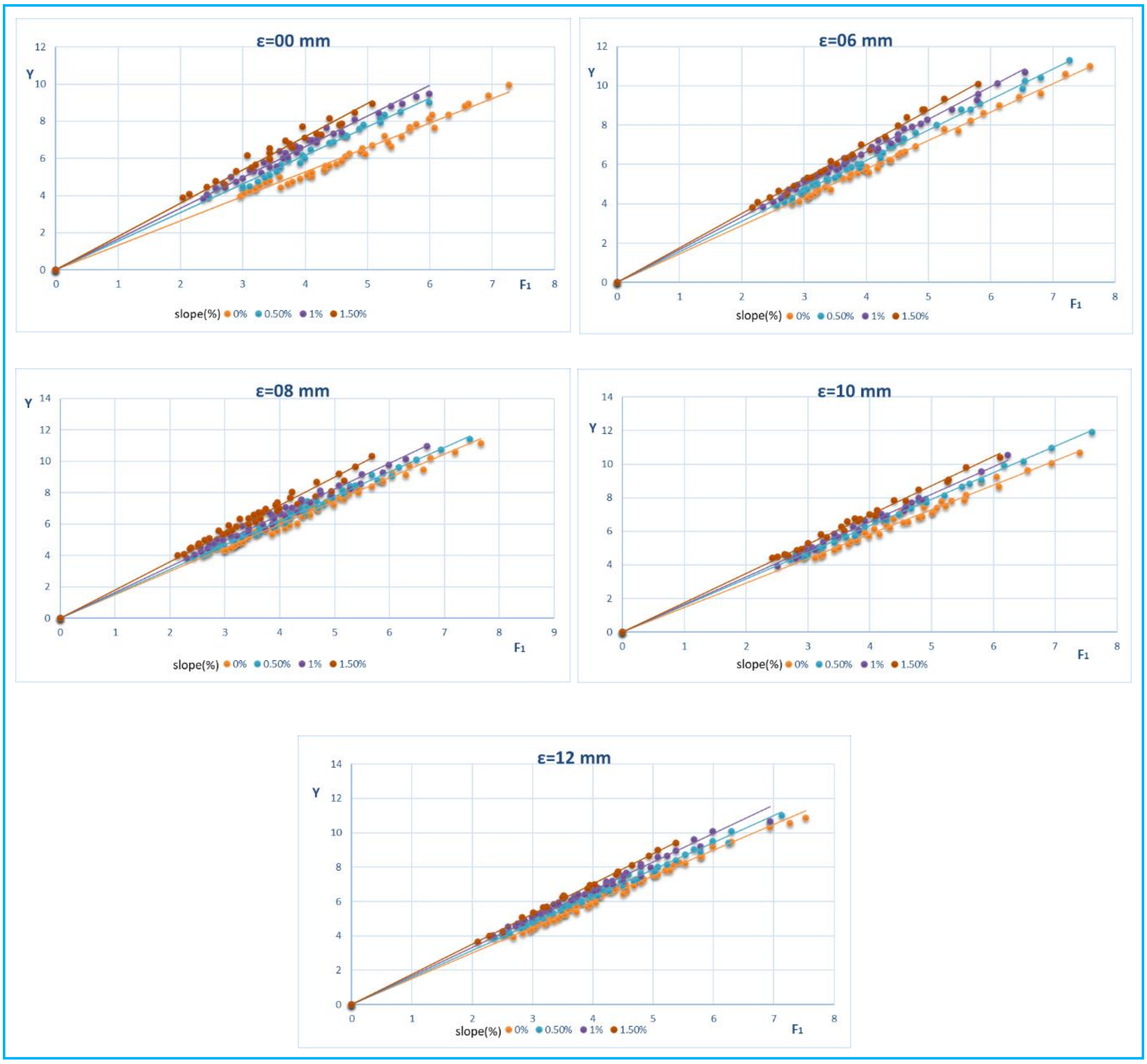


Figure 5 : Variation of the ratio of the conjugate heights of the jump as a function of the Froude number for roughness ε (mm) = 0, 6 , 8 , 10 and 12. tang (α) = 0 (•); tang (α) = 0.005 (•); tang (α) = 0.01(•);

tang (α) = 0.015 (•) experimental points (—) Adjustment curves

The analysis of the experimental measurements shows that each scatter plot follows the shape of a single curve. An adjustment by the linear least squares method shows that this curve is of the form:

$Y= a F_1$

Tables (1) to (5) show the experimental data. Figures (6) to (10) show the relation adjustment for the four inclinations of the rectangular channel of the compound section.

Table 1: Values of the parameters a_1

roughness 0mm		
tang(α)	**a_1**	**R^2**
0	1.3177	0.9987
0.005	1.5416	0.9992
0.010	1.6598	0.9989
0.015	1.7930	0.9981

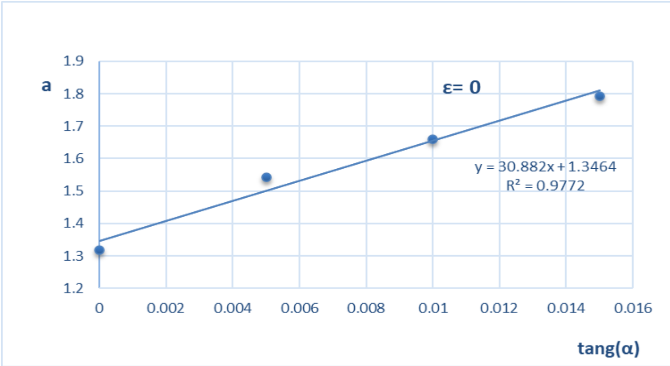


Figure 6: Variation of the parameter a_1 as a function of

tang (α) and roughness (ε=0 mm)

Table 2: Values of the parameters a_2

roughness 6mm		
tang(α)	**a_2**	**R^2**
0	1.4410	0.9997
0.005	1.5509	0.9997
0.010	1.6593	0.9996
0.015	1.7496	0.9994

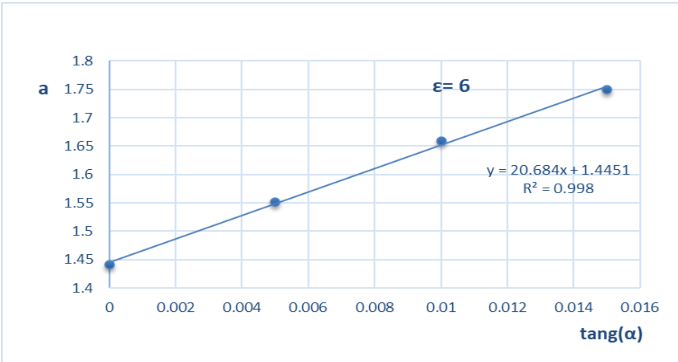


Figure 7: Variation of the parameter a_2 as a function of

tang (α) and roughness (ε = 6 mm)

Table 3: Values of the parameters a_3

roughness 8mm		
tang(α)	**a3**	**R^2**
0	1.4908	0.9991
0.005	1.5552	0.9992
0.010	1.6483	0.9984
0.015	1.7995	0.9981

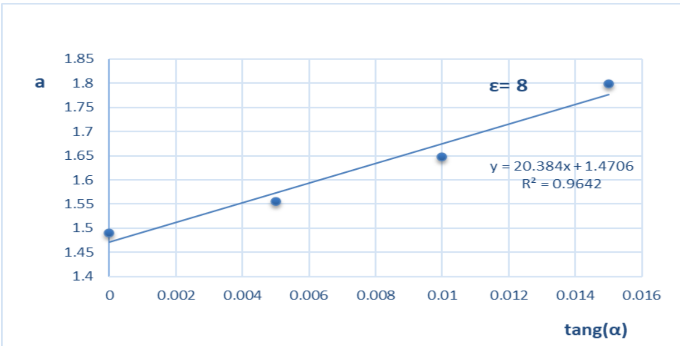


Figure 8: Variation of the parameter a_3 as a function of

of tang (α) and roughness (ε = 8 mm)

Table 4: Values of the parameters a_4

roughness 10 mm		
tang(α)	**a_4**	**R^2**
0	1.4600	0.9991
0.005	1.5804	0.9997
0.010	1.6413	0.9994
0.015	1.7407	0.9995

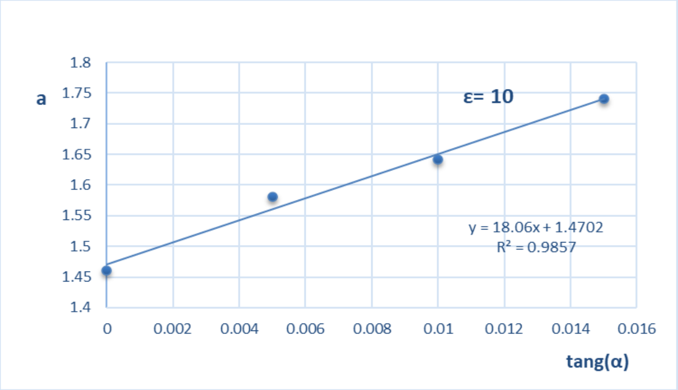


Figure 9: Variation of the parameter a_4 as a function of

tang (α) and roughness (ε = 10 mm)

Table 5: Values of the parameters a_5

roughness 12mm		
tang(α)	**a_5**	**R^2**
0	1.4988	0.9994
0.005	1.5720	0.9995
0.010	1.6596	0.9990
0.015	1.7557	0.9998

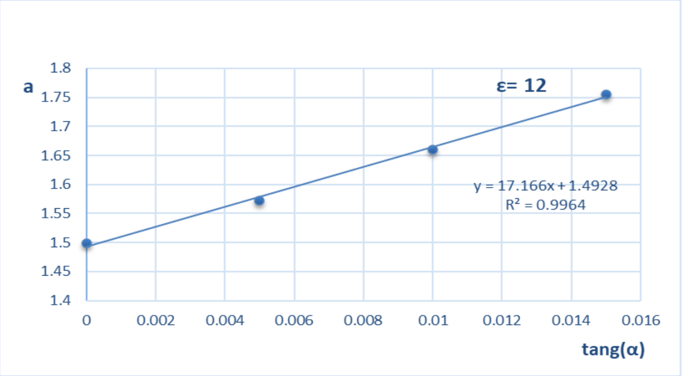


Figure 10: Variation of the parameter a_5 as a function

tang (α) and roughness (ε = 12 mm)

Figures (6 to 10) clearly show that the variation of the parameter **'a'** as a function of the slope of the tang(α) channel.

The Tang slope (α) also follows a linear distribution according to the following relationship:

$a_1 = 30.882 \text{ tang}(\alpha) + 1.3464$ $R^2 = 0.9772$; ε = 00mm

$a_2 = 20.684 \text{ tang}(\alpha) + 1.4451$ $R^2 = 0.9980$; ε = 06mm

$a_3 = 20.384 \text{ tang}(\alpha) + 1.4706$ $R^2 = 0.9642$; ε = 08mm

$a_4 = 18.060 \text{ tang}(\alpha) + 1.4702$ $R^2 = 0.9857$; ε = 10mm

$a_5 = 17.166 \text{ tang}(\alpha) + 1.4928$ $R^2 = 0.9964$; ε = 12mm

Replacing the parameters a with their respective expressions in the relation $Y = a F_1$, we obtain the following:

$Y = [30.882 \text{ tang}(\alpha) + 1.3464] F_1$; ε = 00mm

$Y = [20.684 \text{ tang}(\alpha) + 1.4451] F_1$; ε = 06mm

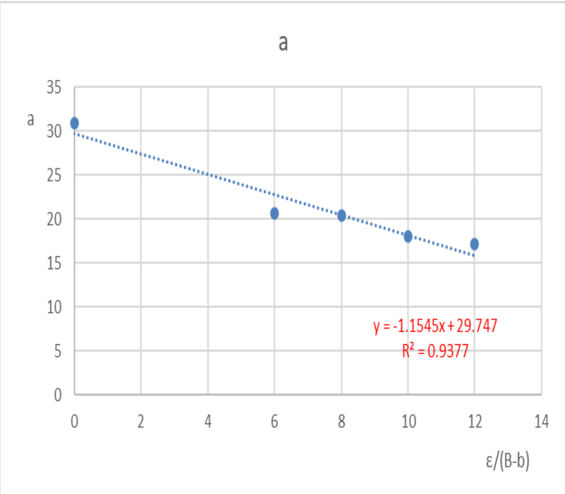
$Y = [20.384 \text{ tang}(\alpha) + 1.4706] F_1$; ε = 08mm

$Y = [18.060\ tang(\alpha) + 1.4702]\ F_1$; $\varepsilon = 10mm$

$Y = [17.166\ tang(\alpha) + 1.4928]\ F_1$; $\varepsilon = 12mm$

The table below shows the values a and b of the roughness coefficient correlated with the channel's inclination angle.

tang(α)	**0 ; 0.005 ; 0,01 ; 0,015**		
ε /(B-b) (mm)	**a**	**b**	**R^2**
0	30.882	1.3464	0.9772
6	20.684	1.4451	0.998
8	20.384	1.4706	0.9642
10	18.060	1.4702	0.9857
12	17.166	1.4928	0.9964



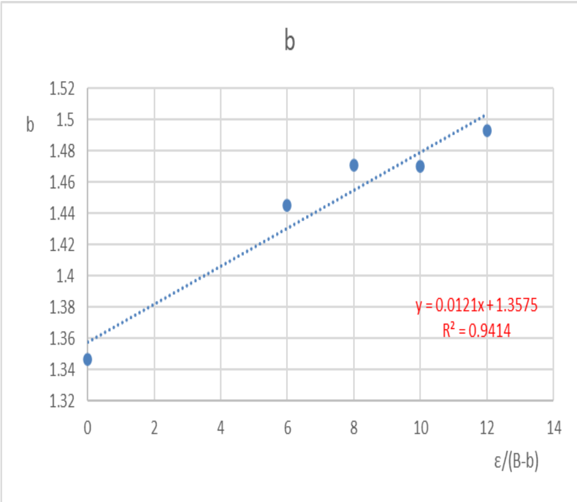


Figure 11: Variation of the parameter 'a' and 'b' as a function of Roughness (ε)

$a^* = -1.1545\varepsilon/(B-b) + 29.747$ $R^2 = 0.9377$

$b^* = 0.0121\varepsilon/(B-b) + 1.3575$ $R^2 = 0.9414$

It would be convenient to reformulate the five relationships into an overall equation in the following form:

$$Y^* = [(-1.1545\ \varepsilon/(B-b) + 29.747)\ tang(\alpha) + 0.0121\ \varepsilon/(B-b) + 1.3575]\ F_1 \quad (2)$$

The global relation (2), being explicit and dimensionless in (Y^*), provides us with a simple and practical means for the determination of the ratio (Y) of the conjugate heights, knowing the incident Froude number F_1 and the slope of the tang channel (α) and the absolute roughness (ε).

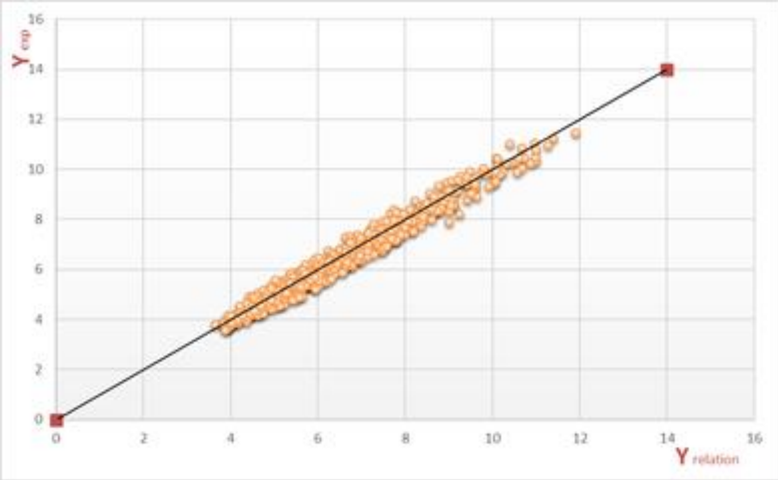


Figure 12: Variation in the ratio of the experimental conjugate heights Y_{exp} as a function of the ratio of the theoretical conjugate heights $Y_{relationship}$ calculated according to the relation (2). (—) First bisector of the equation: $Y_{exp} = Y_{relation}$

This figure clearly shows that most of the experimental measurements revolve around the first justifying bisector, that the latter is reliable, and that the overall relationship obtained applies to the design of hydraulic structures, particularly damping basins.

3. Conclusion:

The hydraulic jump in a rectangular channel of compound section with rough major bed and positive slope has been studied experimentally. In this scientific research work, we have experimentally examined the effect of the major bed's roughness and the channel's positive slope on the hydraulic jump evolving in a compound rectangular channel. A dimensionless global empirical relationship was obtained expressing the variation of the ratio of the conjugate heights Y as a function of the Froude number F1 of the incident flow and the relative roughness (ε/(B-b)) and the slope (tang(α)). It was deduced that this characteristic Y is reduced as a function of the importance of the absolute roughness tested.

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