Tuijin Jishu/Journal of Propulsion Technology
ISSN: 1001-4055
Vol. 45 No. 4 (2024)

Sustainable Inventory System for Decaying Products with Expiration Date, Carbon Emission, and Price-Sensitive Exponentially Decreasing Demand

Sushil Kumar, Manish Kumar, Shivani Singh and Deeksha Jaiswal

Department of Mathematics and Astronomy,
University of Lucknow-Lucknow U.P.(India)

Abstract: In today's era, the carbon emissions of the manufacturing industries have become a sensitive issue worldwide. Also, controlling of carbon emissions becomes a major problem for developing countries. Every country faces climate change and global warming on account of carbon emissions. In the direction of reducing carbon emissions, the present study proposes an economically, socially, and environmentally savvy sustainable inventory system. It is generally assumed that the demand is price-sensitive exponentially decreasing and constant deterioration rate together with the expiration date, shortages, and carbon tax policy. Most of the products have a fixed life span for maintaining their quality in their original condition. Therefore, products with a maximum life span and more sustainability are preferred by the customers. The model is formulated with a sustainable goal that the incurred total cost is optimized together with the determination of optimal cycle length. The practical utility and a better understanding of this model are shown with the help of a numerical example. A sensitivity analysis of some crucial parameters is also conducted.

Keywords: Inventory, Carbon emission, Carbon Tax, Expiration Date, Deterioration, Partial Backlogging and Price-Sensitive Demand

1. Introduction:

In recent years, several inventory models have been developed by the researchers keeping in mind sustainable development. Sustainable development is defined as equity among social, environmental, and economic dimensions. In case of social sustainable the manufacturing industries know their duties about the society and keeping in mind they do work for the betterment of their amenities. The manufacturing companies protect the nature and conserve the natural resources in case of environmental sustainable. The carbon emissions, excess utilization of natural resources and greenhouse gases affect the environment and also the quality level of human life. In the present, global warming and climate change are the main factors affected by carbon emissions. Therefore, in the present time control of carbon emissions becomes a big challenge for the policymakers as well as the production managers.

In traditional inventory systems, deterioration cannot be ignored. Deterioration occurs after the arrival of new products in existence. The presence of deterioration affects both the revenue of the products and the total costs of the products. The deteriorating products can be classified into three categories. First of all are the products that maintain their constant utility throughout the lifespan such as medicines and blood (blood has a fixed lifespan of 21 days). The second types are the products that exponentially mold their utility throughout the lifespan such as fruits, vegetables, milk, bakery products, meat, and fish. And the third types are the periodical products such as newspapers and magazines. Generally, several scholars considered the constant deterioration rate of the products throughout the lifespan. But in reality, it is observed that the products deplete gradually and the products deplete completely when they attain their expiration dates. Therefore, the deterioration rate is assumed as a variable and it depends on the expiration date.

On analyzing the relevant literature review, some researchers and academicians paid their attention in this direction, Ghare and Schrader [1] proposed an inventory model for exponentially decaying products. Fujiwara and Perera [2] studied an EOQ (economic order quantity) model for continuously decaying products with linear trends in demand and exponential-type penalty cost. Abad [3] focused on an optimal pricing and lot sizing inventory system of perishable products and allowing shortages. Polatoglu and Sahin [4] determined optimal procurement policies for an inventory system of decaying products with a price-dependent demand rate. You [5] analyzed an inventory policy for an inventory system having price and time-dependent demand rates. Hsu et al. [6] proposed an optimal lot-sizing inventory system of decaying products with expiration dates. Mo. J. et al. [7] presented a note on an EOQ model for perishable products with stock and price-sensitive demand. Metcalf [8] market-based policy options to control US greenhouse gas emission. Sarkar [10] developed an EOQ model with a time-varying deterioration rate and assuming delay in payments. Cheng et al. [11] presented a carbon-constrained EOQ model. Horban [12] proposed an inventory system with dynamic pricing vs. acquiring information on consumers and suggested heterogeneous sensitivity to product freshness.

Several policies, rules, and regulations were implemented by the national and international governments or regulating authorities to mitigate carbon emissions. The common policy implemented by national and international governments or regulating authorities to control carbon emissions is known as Carbon Tax. A carbon tax is an additional charge paid by the manufacturing firms to the regulating authorities or governments on carbon discharge. This charge is used for the betterment or cleanness of the environment.

He et al. [13] focused on a lot sizing production inventory system with carbon emissions under cap and trade and carbon tax regulations. Teng et al. [14] developed an inventory system and presented the inventory lot-sizing policies for decaying products with expiration dates and offering advance payments. Chen et al. [15] focused on inventory and shelf space optimization systems with freshness and stock-dependent demand rates for fresh products along with expiration dates. Ahmad et al. [16] analyzed the effect of carbon tax and uncertainty for an inventory system and considered an economic policy for a second-generation biofuel supply chain. Ghosh et al. [17] presented a collaborative model for a two-echelon supply chain with uncertain demand and carbon tax policy. Wu et al. [18] studied the inventory policies for deteriorating items with expiration dates and advance cash credit payment schemes. Ullah et al. [19] proposed an inventory system with stochastic price-dependent demand and dynamic pricing for a multi-period newsvendor. Dey et al. [20] analyzed an integrated inventory system having discrete setup cost reduction and variable safety factors. They were considered the selling price dependent demand and investment in their inventory system. Iqbal and Sarkar [21] focused on an inventory system of deteriorating items and considered the recycling of lifetime-dependent deteriorated products through different supply chains. Gautam et al. [22] developed an inventory system for deteriorating items with price-dependent demand, variable deterioration rate, expiration dates, and preservation technology investment. Khanna and Yadav [23] considered an inventory system with price-sensitive demand, preservation technology investment, and the effect of carbon tax along with the cap and trade mechanism.

Sustainable development is an economic development favorable to the environment. It protects the environment from contaminated factors. Sustainable development was first time implemented by the United Nations and later the national and international governments implemented it. Bonney and Jaber [9] first time implemented sustainable development in their fundamental inventory model. They were shown that the inventory systems help to reduce environmental pollution. Mishra et al. [24] presented a sustainable production inventory system for decaying products assuming shortages and controllable carbon emission rate. Yadav and Khanna [25] focused on a sustainable inventory model for perishable products with price-reliant demand and expiration date under a carbon tax policy. Mishra and Mishra [26] focused on a sustainable inventory model for non-instantaneous deteriorating items with quality assessment under carbon emissions and allowing shortages. Sarkar et al. [27] analyzed an environmental and economic sustainability through innovative green products by remanufacturing of products.

Sindhuja and Arathi [28] studied an inventory model for deteriorating products under preservation technology with time-dependent quality demand. Magfura et al. [29] analyzed a sustainable inventory model with environmental impact for non-instantaneous deteriorating items with composite demand. Kumar et al. [30] constructed a profit optimization sustainable fuzzy inventory model for deteriorating items with partial backlogging incorporating social and environmental responsibility under the effect of learning. They were also considered the preservation technology investment, advertisement, carbon tax and green technology in their inventory model. Luis et al. [31] proposed a sustainable inventory model for deteriorating items with power demand and full backlogging under carbon emission tax.

1.1 Contribution: In the present study a sustainable inventory system is constructed with the goal that the incurred total cost is optimized together with the determination of optimal cycle length. The following factors are considered,

a. Regarding the financial sustainability, price-sensitive demand is considered. Partial backlogging is assumed and it depends upon the waiting time of the arrival of next orders to satisfy the customers demand.
b. Regarding the environmental sustainability, carbon emissions is proposed and it takes place due to the operational activities linked with inventory. The causes of carbon emissions are placing the orders, purchasing, holding and not the proper management of products. The carbon tax policy is introduced and it is helpful to reducing the carbon emissions.
c. Regarding the social sustainability, expiration dates of products is introduced. The products with large expiration dates are preferred by the customers. The proper management of wastage is necessary and it is the responsibility of the manufacturing firm/business.

1.2 Objective: The followings are the objective of this study,

a. What is the impact of backlogging parameter on the total cost?
b. What is the impact of expiration dates of the products on the total cost?
c. What is the impact of demand parameter a and price s of the products on the total cost?

2. Notations and Assumptions: The proposed model consists of the following notations and assumptions,

1. The Price-sensitive demand is $D(s) = e^{-as}$, where s is the price and $a > 0$.
2. The decaying products degrade over time and cannot be sold when approach the expiration date. The expiration date varying deterioration rate is $\theta(t) = \left(\frac{1}{1+m-t}\right)$, $0 \le t \le T \le m$, where m is an expiration dates. The deterioration rate is zero as $t \to 0$, and the deterioration rate is 1 as $t \to m$ i.e. all the products deteriorate at their expiration dates (as Wu et al. 2018). Hence, the products cannot be sold when time approaches the expiration dates.
3. The products are neither repaired nor replaced.
4. The lead time is zero.
5. The planning horizon is T.
6. The backlogging rate is $B(t) = \frac{1}{1+\delta(T-t)}$, where δ is a backlogging parameter and t is the waiting time.
7. The ordering cost per order is c_1.
8. The holding cost per unit per unit time is c_2.
9. The shortage cost per unit is c_3.
10. The purchasing cost per unit is p_c.
11. The lost sales cost per unit is l_c.
12. The carbon emission cost per unit is c_e.
13. The carbon tax per unit is c_t.
14. The shipping price per unit is s_p.
15. Time of zero inventory level is T_1.
16. The total inventory cost per cycle is $TC(T_1, T)$.

3. Mathematical Derivation of Model: Here, the inventory system under the aforementioned assumptions is given by figure 1. The cycle starts at a time $t = 0$ with an inventory level Q. The inventory level Q gradually depletes due to demand and deterioration in the time interval $[0, T_1]$ and reaches zero at time $t = T_1$. Just after the time $t = T_1$ shortages start and in the time interval $[T_1, T]$ shortage amount of inventory is partially backlogged at a rate of $B(t) = \frac{1}{1+\delta(T-t)}$, where δ $(0 < \delta < 1)$ is a backlogging parameter and t is a waiting time.

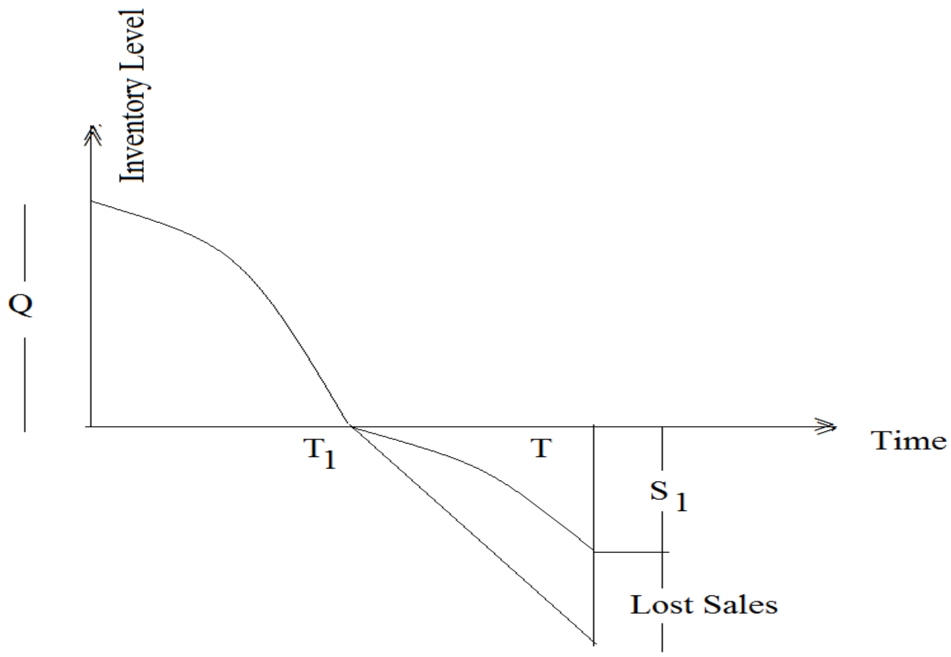

Figure 1, Graphical Representation of Model

The instantaneous inventory level at any time t satisfies the differential equations

$$\frac{dI}{dt}+\left(\frac{1}{1+m-t}\right)I=-e^{-as}, \qquad 0\le t\le T_1 \tag{1}$$

With boundary a condition $I(T_1)=0$

$$\frac{dI}{dt}=-\frac{e^{-as}}{1+\delta(T-t)}, \qquad T_1\le t\le T \tag{2}$$

With boundary a condition $I(T_1)=0$

The solutions of equations (1) and (2) are given by the equations (3) and (4) respectively,

$$I=e^{-as}\left[(1-m^2)T_1-(1-m^2)t-(1-m)tT_1+\left(\frac{1+m}{2}\right)T_1^2-\left(\frac{3m-1}{2}\right)t^2-\left(\frac{1}{2}\right)T_1^2t+\left(\frac{1}{2}\right)t^3\right], \qquad 0\le t\le T_1 \tag{3}$$

$$I=e^{-as}\left[T_1-t+\delta T^2+\left(\frac{\delta}{2}\right)t^2-\delta tT+\left(\frac{\delta}{2}\right)T_1^2-\delta TT_1\right], \qquad T_1\le t\le T \tag{4}$$

The starting order quantity Q is obtained by $I(0)=Q$ so

$$Q=\frac{e^{-as}(1-m)}{2}\left[2(1-m)T_1+T_1^2\right] \tag{5}$$

The backorder quantity S_1 is obtained by $I(T)=S_1$ so

$$S_1=e^{-as}\left[T_1-T+\left(\frac{\delta}{2}\right)T^2+\left(\frac{\delta}{2}\right)T_1^2-\delta TT_1\right] \tag{6}$$

The total inventory is $S_2 = Q - S_1$

or

$$S_2 = \left(\frac{e^{-as}}{2}\right)\left[(1+m)\left\{2(1-m)T_1 + T_1^2\right\} - \left(2T_1 - 2T + \delta T^2 + \delta T_1^2 - \delta TT_1\right)\right] \quad (7)$$

The associated inventory costs are determined as follows:

The ordering cost per cycle is $O_C = \frac{c_1}{T}$ (8)

The holding cost per cycle is $H_C = \left(\frac{c_2}{T}\right)\int_0^{T_1} I(t)dt$

Or

$$H_C = \left(\frac{c_2 e^{-as}}{T}\right)\left[\frac{(1-m^2)}{2}T_1^2 + \frac{(3m+1)}{6}T_1^3 + \frac{1}{4}T_1^2T^2 + \frac{1}{8}T_1^4\right], \quad (9)$$

The shortage cost per cycle is $S_C = \left(\frac{c_3}{T}\right)\int_{T_1}^{T} I(t)dt$

Or

$$S_C = \left(\frac{c_3 e^{-as}}{T}\right)\left[TT_1 - \frac{1}{2}T_1^2 - \frac{1}{2}T^2 + \frac{2\delta}{3}T^3 - \frac{2\delta}{3}T_1^3 - 2\delta T^2 T_1 + 2\delta TT_1^2\right], \quad (10)$$

The purchasing cost per cycle is $P_C = \left(\frac{p_c}{T}\right)S_2$

Or

$$P_C = \left(\frac{p_c e^{-as}}{T}\right)\left[-m^2T_1 + T + \frac{(1+m-\delta)}{2}T_1^2 - \frac{\delta}{2}T^2 + \delta TT_1\right], \quad (11)$$

The lost sales cost per cycle is $LS_C = \left(\frac{l_c}{T}\right)\int_{T_1}^{T}\left[1 - \frac{1}{1+\delta(T-t)}\right]e^{-as}dt$

Or

$$LS_C = \left(\frac{l_c e^{-as}}{T}\right)\left[\frac{1}{2}T^2 - TT_1 + \frac{1}{2}T_1^2\right], \quad (12)$$

The shipping cost per cycle is $SH_C = \left(\frac{s_p}{T}\right)[Q + S_1]$

Or

$$SH_C = \left(\frac{s_p e^{-as}}{T}\right)\left[(2-m^2)T_1 - T + \frac{\delta}{2}T^2 + \frac{(1+m+\delta)}{2}T_1^2 - \delta TT_1\right], \quad (13)$$

The carbon emission in shipping and holding is $CE_C = c_e(Q + S_1) + c_e\int_0^{T_1} I(t)dt$

Or

$$CE_C = c_e e^{-as}\left[(2-m^2)T_1 - T + \frac{\delta}{2}T^2 + \frac{(1+m-m^2+\delta)}{2}T_1^2 - \delta TT_1 + \frac{(3m+1)}{6}T_1^3\right.$$

$$+\frac{1}{4}T^2T_1^2+\frac{1}{8}T_1^4\Bigg], \tag{14}$$

The tax on carbon emissions is $C_T = c_t \times CE_C$

Or

$$C_T = c_e c_t e^{-as}\Bigg[(2-m^2)T_1 - T + \frac{\delta}{2}T^2 + \frac{(1+m-m^2+\delta)}{2}T_1^2 - \delta T T_1 + \frac{(3m+1)}{6}T_1^3$$

$$+\frac{1}{4}T^2T_1^2+\frac{1}{8}T_1^4\Bigg], \tag{15}$$

The total inventory cost per cycle is

$$TC(T_1,T) = \left(\frac{1}{T}\right)\left[O_C + H_C + S_C + P_C + LS_C + SH_C + C_T\right] \tag{16}$$

Where O_C, H_C, S_C, P_C, LS_C, SH_C, *and* C_T, are the ordering cost, holding cost, shortage cost, purchasing cost, lost sales cost, shipping cost, and carbon tax cost respectively.

Putting the values of the above-calculated costs in equation (16), we obtain the equation (17)

$$TC(T_1,T) = \left(\frac{e^{-as}}{T}\right)\Big[c_1 e^{as} + \{-m^2 p_c + (s_p + c_e c_t)(2-m^2)\}T_1 + (p_c - s_p - c_e c_t)T + \left(\frac{1}{2}\right)\{c_2(1-m^2)$$

$$-c_3 + p_c(1+m-\delta) + l_c\delta + s_p(1+m-\delta) + c_e c_t(1+m-m^2+\delta)\}T_1^2 + \frac{1}{2}\{-c_3$$

$$+\delta(-p_c + l_c + s_p + c_e c_t)\}T^2 + \{c_3 + \delta(p_c - l_c - s_p - c_e c_t)\}TT_1 + \frac{1}{6}\{(3m+1)(c_2 + c_e c_t)$$

$$-4\delta c_3\}T_1^3 + \frac{2\delta c_3}{3}T^3 - 2\delta c_3 T_1 T^2 + 2\delta c_3 T T_1^2 + \frac{1}{8}(c_2 + c_e c_t)T_1^4 + \frac{1}{4}(c_2 + c_e c_t)T^2T_1^2\Big] \tag{17}$$

For the optimum values of T_1 and T the first order partial derivatives, $\frac{\partial TC(T_1,T)}{\partial T_1}$ and $\frac{\partial TC(T_1,T)}{\partial T}$ must be equal to zero i.e. $\frac{\partial TC(T_1,T)}{\partial T_1} = 0$ and $\frac{\partial TC(T_1,T)}{\partial T} = 0$.

At the optimum values of T_1 and T, the second-order partial derivatives of $TC(T_1,T)$ satisfies the conditions $\frac{\partial^2 TC(T_1,T)}{\partial T_1^2} > 0$ and

$$\left(\frac{\partial^2 TC(T_1,T)}{\partial T_1^2}\right)\left(\frac{\partial^2 TC(T_1,T)}{\partial T^2}\right) - \left(\frac{\partial^2 TC(T_1,T)}{\partial T_1 \partial T}\right)^2 > 0.$$

From equation (17), we obtain the first-order partial derivatives,

$$\frac{\partial TC(T_1,T)}{\partial T_1} = \left(\frac{e^{-as}}{T}\right)\Big[\{-m^2 p_c + (s_p + c_e c_t)(2-m^2)\} + \{(1-m^2)c_2 - c_3 + (1+m-\delta)(p_c + s_p)$$

$$+l_c\delta + (1+m-m^2+\delta)c_e c_t\}T_1 + \{c_3 + \delta(p_c - l_c - s_p - c_e c_t)\}T + \frac{1}{2}\{-4\delta c_3$$

$$+(3m+1)(c_2+c_ec_t)\}T_1^2-2\delta c_3T^2+4\delta c_3TT_1+\frac{1}{2}(c_2+c_ec_t)T_1^3+\frac{1}{2}(c_2+c_ec_t)T_1T^2\Big] \quad (18)$$

$$\frac{\partial TC(T_1,T)}{\partial T_1}=\left(\frac{e^{-as}}{T}\right)\Big[(p_c-s_p-c_ec_t)+\{-c_3+\delta(-p_c+l_c+s_p+c_ec_t)\}T+\{c_3+\delta(p_c-l_c-s_p-c_ec_t)\}T_1+2\delta c_3T^2-4\delta c_3TT_1+2\delta c_3T_1^2+\frac{1}{2}(c_2+c_ec_t)TT_1^2\Big]$$
$$+\left(\frac{e^{-as}}{T^2}\right)\Big[c_1e^{as}+\{-m^2p_c+(s_p+c_ec_t)(2-m^2)\}T_1+(p_c-s_p-c_ec_t)T+\left(\frac{1}{2}\right)\{c_2(1-m^2)-c_3+p_c(1+m-\delta)+l_c\delta+s_p(1+m-\delta)+c_ec_t(1+m-m^2+\delta)\}T_1^2+\frac{1}{2}\{-c_3+\delta(-p_c+l_c+s_p+c_ec_t)\}T^2+\{c_3+\delta(p_c-l_c-s_p-c_ec_t)\}TT_1+\frac{1}{6}\{(3m+1)(c_2+c_ec_t)-4\delta c_3\}T_1^3+\frac{2\delta c_3}{3}T^3-2\delta c_3T_1T^2+2\delta c_3TT_1^2+\frac{1}{8}(c_2+c_ec_t)T_1^4+\frac{1}{4}(c_2+c_ec_t)T^2T_1^2\Big] \quad (19)$$

The second-order partial derivatives are as follows,

$$\frac{\partial^2TC(T_1,T)}{\partial T_1^2}=\left(\frac{e^{-as}}{T}\right)\Big[\{(1-m^2)c_2-c_3+(1+m-\delta)(s_p+p_c)+l_c\delta+(1+m-m^2+\delta)c_ec_t\}+\{(3m+1)(c_2+c_ec_t)-4\delta c_3\}T_1+4\delta c_3T+\frac{3}{2}(c_2+c_ec_t)T_1^2+\frac{1}{2}(c_2+c_ec_t)T^2\Big] \quad (20)$$

$$\frac{\partial^2TC(T_1,T)}{\partial T_1\partial T}=\left(\frac{e^{-as}}{T}\right)\Big[-\{m^2p_c+(2-m^2)(s_p+c_ec_t)\}-\{(1-m^2)c_2-c_3+(1+m-\delta)(s_p+p_c)+l_c\delta+(1+m-m^2+\delta)c_ec_t\}T_1-\frac{1}{2}\{(3m+1)(c_2+c_ec_t)-4\delta c_3\}T_1^2-2\delta c_3T^2-\frac{1}{2}(c_2+c_ec_t)T_1^3+\frac{1}{2}(c_2+c_ec_t)T_1T^2\Big] \quad (21)$$

$$\frac{\partial^2TC(T_1,T)}{\partial T^2}=\left(\frac{e^{-as}}{T}\right)\Big[\{-c_3+\delta(-p_c+l_c+s_p+c_ec_t)\}+4\delta c_3T-4\delta c_3T_1+\frac{1}{2}(c_2+c_ec_t)T_1^2\Big]$$
$$-\left(\frac{2e^{-as}}{T^2}\right)\Big[(p_c-s_p-c_ec_t)+\{-c_3+\delta(-p_c+l_c+s_p+c_ec_t)\}T+\{c_3+\delta(p_c-l_c-s_p-c_ec_t)\}T_1+2\delta c_3T^2-4\delta c_3TT_1+2\delta c_3T_1^2+\frac{1}{2}(c_2+c_ec_t)TT_1^2\Big]$$
$$+\left(\frac{2e^{-as}}{T^3}\right)\Big[c_1e^{as}+\{-m^2p_c+(s_p+c_ec_t)(2-m^2)\}T_1+(p_c-s_p-c_ec_t)T+\left(\frac{1}{2}\right)\{c_2(1-m^2)-c_3+p_c(1+m-\delta)+l_c\delta+s_p(1+m-\delta)+c_ec_t(1+m-m^2+\delta)\}T_1^2+\frac{1}{2}\{-c_3+\delta(-p_c+l_c+s_p+c_ec_t)\}T^2+\{c_3+\delta(p_c-l_c-s_p-c_ec_t)\}TT_1+\frac{1}{6}\{(3m+1)(c_2+c_ec_t)-4\delta c_3\}T_1^3+\frac{2\delta c_3}{3}T^3-2\delta c_3T_1T^2+2\delta c_3TT_1^2+\frac{1}{8}(c_2+c_ec_t)T_1^4+\frac{1}{4}(c_2+c_ec_t)T^2T_1^2\Big] \quad (22)$$

Numerical Example: Let us consider the data in appropriate units for the parameters of the proposed model as follows,

$a = 0.05,\ \delta = 0.1,\ c_1 = \$500,\ c_2 = \$8,\ c_3 = \$5,\ p_c = \$15,\ l_c = \$4.5,\ c_e = \$1.5,\ c_t = \$3.5,$

$s_p = \$2.5,\ s = \$100,\ m = 2\ Months$

Table 1, Sensitivity analysis of parameters

Decision Variables/ Parameters		T_1	T	$TC(T_1, T)$
δ	0.1	0.13846	49.49570	14.75210
	0.3	0.42356	33.95780	21.73601
	0.5	0.69262	28.64109	25.77710
	0.7	0.94339	25.65940	28.72760
	0.9	1.17583	23.67382	31.07110
m	2	0.13846	49.49652	14.75210
	4	0.15618	49.48697	14.74641
	6	0.18664	49.46324	14.73540
	8	0.23108	49.41222	14.71631
	10	0.29159	49.31223	14.68511
a	0.01	0.14579	14.15859	48.44410
	0.02	0.13694	19.14557	36.63361
	0.03	0.13489	26.12225	27.31620
	0.04	0.13607	35.87430	20.15381
	0.05	0.13846	49.49652	14.75210
	0.06	0.14102	68.51696	10.73511
	0.08	0.14521	132.12929	5.61878
s	20	0.14579	14.15865	48.44410
	30	0.14022	16.44310	42.21151
	40	0.13694	19.14557	36.63360
	50	0.13534	22.34213	31.68181
	60	0.13489	26.12225	27.31620
	100	0.13846	49.49652	14.75210

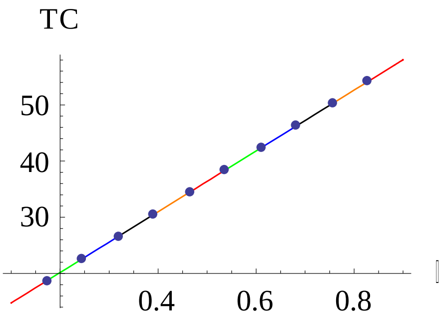

Figure 2, Variation w.r.to δ

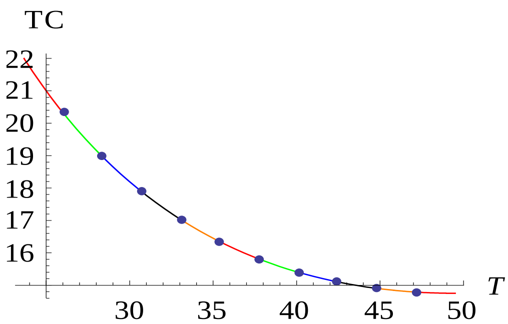

Figure 3, Variation w.r.to T

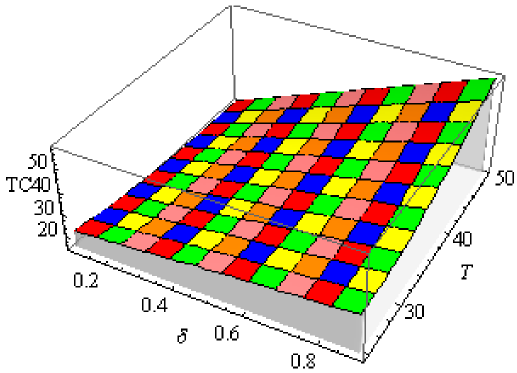

Figure 4, Variation in total cost

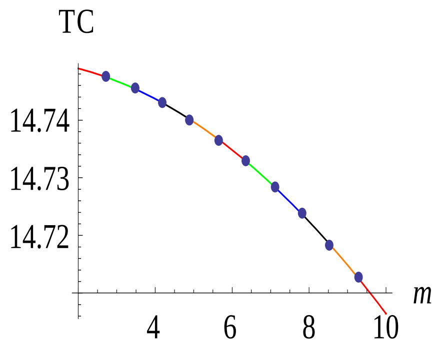

Figure 5, Variation w.r.to m

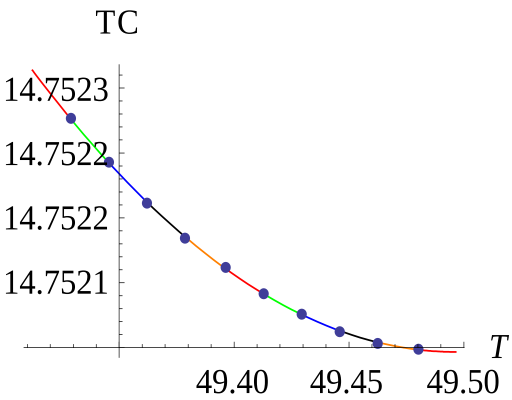

Figure 6, Variation w.r.to T

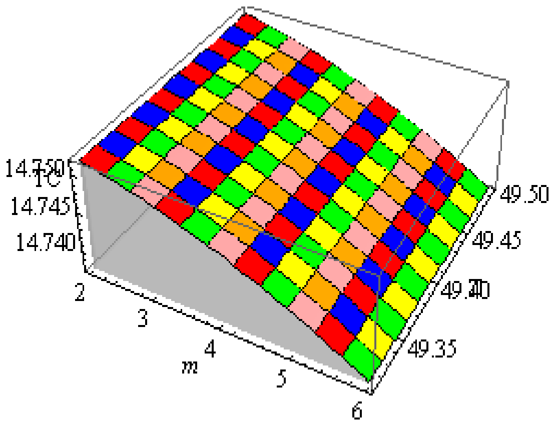

Figure 7, Variation in total cost

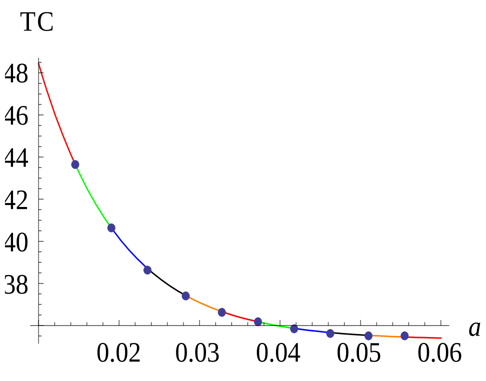

Figure 8, Variation w.r.to a

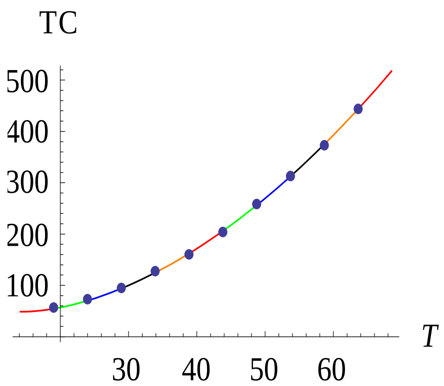

Figure 9, Variation w.r.to T

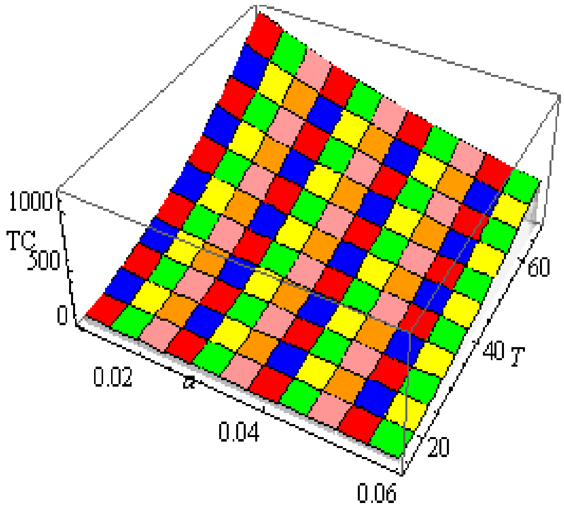

Figure 10, Variation in total cost

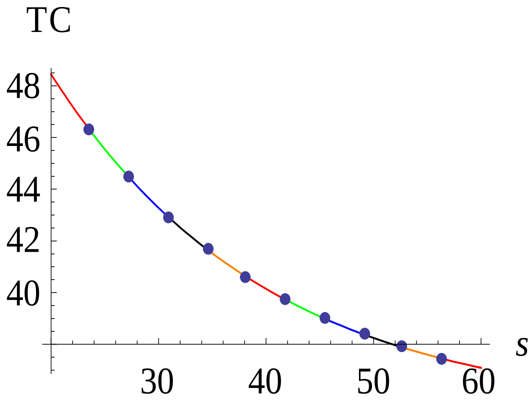

Figure 11, Variation w.r.to s

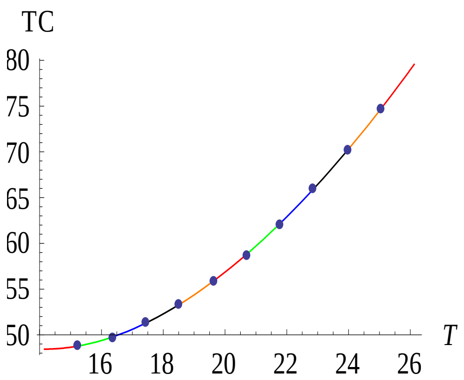

Figure 12, Variation w.r.to T

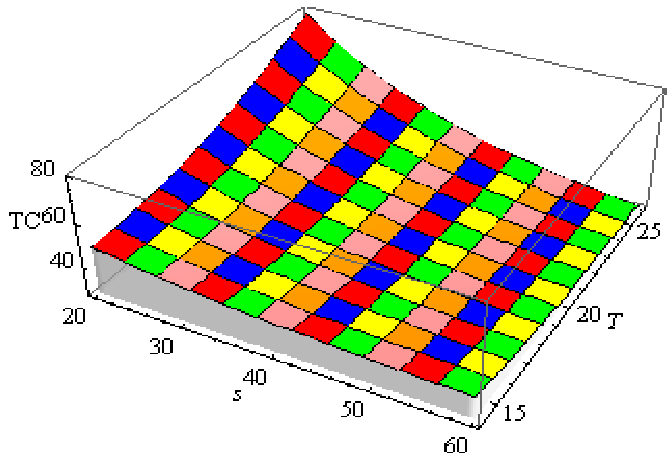

Figure 13, Variation in total cost

4. Sensitivity Analysis: In this section, the sensitivity analysis has been performed to show the response of total cost concerning the change values of various parameters such as backlogging parameter, expiration date, demand parameter, and selling price.

The observations based on the findings are as follows:

a. As the backlogging parameter increases the total cost increases. With regards to the other parameters an increase in backlogging parameter also increases an on-hand inventory period and decreases the optimal cycle length. The cause of this is the increase in holding costs and carbon tax. Moreover, an increase in order size leads to an increase in carbon emission and holding costs.

b. As the expiration date of products increases then the total cost decreases. With regards to the other parameters an increase in expiration date also increases an on-hand inventory period and decreases the optimal cycle. It occurs due to the decrease in deterioration rate, carbon tax, and shipping price.

c. As the demand parameter increases then the total cost decreases. With regards to the other parameters an increase in demand parameter also decreases an on-hand inventory period and increases the optimal cycle length. This leads to an increase in the selling rate of products and a decrease in the deterioration and carbon emission rates.

d. As the selling price of products increases then the total cost decreases. With regards to the other parameters an increase in selling price also decreases an on-hand inventory period and increases an optimal cycle length. This occurs due to the decrease in holding cost, shipping cost, and carbon tax.
e. The products having long expiration dates are preferred by the customers. As such types of products have a low deterioration and carbon emission rates.
f. The sustainability and optimality of products depend on the selling rate, expiration date, shipping cost, and carbon emission rate.

5. Conclusion: This study considers a sustainable inventory system together with carbon emissions. Globally, carbon emission affects the environment and it becomes a major issue for every developing country. So a proper balance is required between the maximum supply at low cost, carbon emission, and sustainable development. The products having long expiration dates and low deterioration rates are preferred by everyone. The present paper develops a sustainable inventory system for decaying products. Price-sensitive demand and partial backlogging are considered. The carbon tax policy is introduced to mitigate the carbon emissions. The control of deterioration is based on the lifespan of products. The analysis based on a numerical example shows that the demand and backlogging parameters have much impact on the total cost in comparison to the associated lifespan and price parameters. In the future, several realistic assumptions can be considered for the decaying products to mitigate the carbon-emission.

Conflict of Interest:

It is confirmed that there is no conflict of interest among authors about this publication.

References:

[1] Ghare P. M., and Schrader G. F. (1963), An inventory model for exponentially decayingproducts. Journal of Industrial Engineering, 14(2), 238-243.
[2] Fujiwara O., and Perera U. (1993), EOQ models for continuously deteriorating items with linear and exponential type penalty costs. European Journal of Operational Research, 70(1), 104-114.
[3] Abad P. L. (1996), An optimal pricing and lot-sizing inventory model for decaying products and allowing shortages. Manag Science 42(8), 1093-1104.
[4] Polatoglu H., and Sahin I. (2000), Optimal procurement policies for an inventory system of decaying products with price-dependent demand. International Journal of Production Economics, 65(2), 141-171.
[5] You P. S. (2005), An inventory system for perishable products with price and time dependent demand and having an inventory policy for decaying products. Journal of Operational Research Society 56(7), 870-873.
[6] Hsu P. H., Wee H. M., and Teng H. M. (2006), An optimal lot-sizing inventory system for perishable products with expiration date. Journal of Information and Optimum Science, 27(2), 271-286.
[7] Mo. J., Mi F., Zhou F., and Pan H (2009), A note on an EOQ model for deteriorating items with price and stock-sensitive demand. Math Computer Model, 49(9-10), 2029-2036.
[8] Metcalf G. E. (2009), Market-based policy options for an inventory system of decaying

products to control U S greenhouse gas emissions. Journal of Economics Perspectives, 23(2), 5-27.

[9] Bonney M., and Jaber M. Y. (2011), Environmentallyresponsible inventory models, non-classical models for a non-classical era. International Journal of Production Economics, 133, 43-53.

[10] Sarkar B. (2012), An EOQ model for decaying products with time-varying deterioration rate and delay in payment. Mathematical Computational Model, 55(3-4), 367-377.

[11] Cheng X., Bonjaafar S., and Elomri A. (2013), An EOQ model for deteriorating products with carbon emissions. Operations Research Letters, 41(2), 172-179.

[12] Horbon A. (2014), Dynamic pricing vs. acquiring information on consumers, heterogeneous sensitivity to product freshness. International Journal of Production Research, 52(3), 918-933.

[13] He P., Zhang W., Xu X., and Bian Y. (2015), A production lot-sizing inventory system for perishable products with carbon emission, cap, and trade under carbon taxregulations.Journal of clean production, 103, 241-248.

[14] Teng J. T., Cardenas Barron L. E., Chang H-J, Wu J., and Hu Y. (2016), Inventory lot-sizing policies for perishable products with advance payment and expiration dates. Applied Mathematical Model, 40(19-20), 8605-8616.

[15] Chen S. C., Min J., Teng J. T., and Li F.(2016), Inventory and shelf space optimization for fresh products with expiration dates under freshness and stock-dependent demand rate. Journal of Operational Research Society, 67(6), 884-896.

[16] Ahmed W., Sarkar M., Sarkar B., and Kim S. (2017), Effect of carbon tax and uncertainty in an economic policy for the second-generation biofuel supply chain. Proceedings of the Full Conference of the Korean Institute of Industrial Engineers, 1700-1707.

[17] Ghosh A., Sarmah S. P., and Jha J. K. (2018), A two-echelon supply chain inventory system for decaying products with uncertain demand under carbon tax policy. Sadhana43(9), 144.

[18] Wu J., Teng J-T., Chan Y-L(2018), Inventory policies for perishable products with an advance cash credit schemes and expiration dates. International Journal of System Science, 5(4), 310-326.

[19] Ullah M., Khan I., and Sarkar B. (2019), An inventory system for decaying products with stochastic price-dependent demand and dynamic pricing in a multi-period newsvendor. Mathematics, 7(6), 520.

[20] Dey B. K., Sarkar B., Sarkar M., and Pareek S. (2019), An integrated inventory system with selling price-dependent demand, discrete setup cost reduction, and investment. RAIRO Operations Research, 53(1), 39-57.

[21] Iqbal M. W., and Sarkar B. (2019), Recycling of lifetime dependent decaying products through different supply chains. Rairo Operations Research, 53(1), 129-156.

[22] Gautam P., Khanna A., and Jaggi C. K. (2020), Preservation technology investment for an inventory system of decaying products with variable deterioration rates, price-sensitive demand and expiration dates. Yugoslav Journal of Operations Research, 30(3), 289-305.

[23] Khanna A., and Yadav S. (2020), Effect of carbon tax and cap and trade mechanism on an inventory system with price-sensitive demand and preservation technology investment. Yugoslav Journal of Operations Research, 30(3), 361-380.

[24] Mishra U., Wu J. Z., and Sarkar B. (2020), A sustainable production inventory system for decaying products and controllable carbon emission rate under shortages. Journal of

clean Production, 256, 120268.
[25] Yadav S., and Khanna A. (2021), Sustainable inventory model for perishable products with price-reliant demand and expiration date under carbon tax policy. Process Integration and Optimization for Sustainability, 5, 475-486.
[26] Mishra R. K., and Mishra V. K. (2022) An optimum sustainable inventory model for non-instantaneous deteriorating and quality assessment under carbon emissions and complete backlogging shortages. Arab Journal of Science and Engineering, 47, 3929-3944.
[27] Sarkar B., Ullah M., and Sarkar M. (2022) Environmental and economic sustainability through innovative green products by remanufacturing. Journal of Clean Production, 332.
[28] Sindhuja S., and Arathi P. (2023), An inventory model for deteriorating products with time dependent quality demand under preservation technology. Cogent Engineering, 10, 1-25.
[29] Pervin M., Roy S. K., Prasenjit S., and Weber G. W. (2023), Sustainable inventory model for non-instantaneous deteriorating items with composite demand under an environmental impact. Rairo Operations Research, 57, 237-261.
[30] Kumar S., Sami S., Agarwal S., and Yadav D. (2023) Sustainable fuzzy inventory model for deteriorating items with partial backlogging along with social and environmental responsibility under the effect of learning. Alexandria Engineering Journal, 69, 221-241.
[31] Luis A. S. J., Joaquin S., Leopoldo E., Caredenas B., and Manuel. R. (2024), Asustainable inventory model for deteriorating items with power demand and full backlogging under a carbon emission tax. International Journal of Production Economics, 268, Id-109098.