Tuijin Jishu/Journal of Propulsion Technology
ISSN: 1001-4055
Vol. 45 No. 3 (2024)

Assessment of Reliability Allocation for a Component in Standby Redundancy Propulsion System

[1]Chetan Kumar Sharma, [2]Nitin Kasana*, [3]Neeraj Singh#, [4]Rajeev Kaushik, [5]Ashok Kumar

[1]*Associate Professor, Department of Mathematics, Noida International University, India*

[2,3]*Research Scholar, Department of Mathematics, Noida International University, India*

[4]*Assistant Professor, Department of Physics, Shri K K Jain (P/G) College, Khatauli, Muzaffar Nagar, India*
[5]*Professor, Department of Mathematics, Meerut College, Meerut, India*

**Corresponding Author, #Principal Author*

***Abstract*:-** In this manuscript, we analyses that the critical factors to take into account are constructing a reliable system and reliability allocation for components. Standby redundancy provides higher dependability for any system, but active redundancy is easier to model. The standby arrangement's reliability analysis is somewhat more reliable than that of the parallel running units. On the other hand, their typical interval between failures is significantly longer, and one or more units are operating; in standby arrangements, failure detection is typically needed to activate the next unit when the first fails. One or more units are operational when the sensing device's reliability falls below 100%, as it more frequently does in this scenario. So, one effective way to extend the life of a system is to assign redundant components to it and also analyze the reliability, failure rate, and failure density functions at different points in time of a component with related hazard rate and inverted hazard rate orders established, and also discusses the components and redundancies follow a general lifetime distribution.

Keywords: *Reliability, Failure Density Function, Failure rate Function, Mean Time between Failures, Standby Redundancy*

1. Introduction

The swift advancement of science and technology has presented a challenge to engineers, designers, and manufacturers about the dependability of their systems, especially in the fields of space exploration, defense, biology, and industry Gupta, et al., [1] discusses the failure analysis is a process that involves assembling malfunctioning parts, using a range of methods to investigate the reasons further, and doing cost analysis. Analysis of Three States' Two-Unit Electronic Parallel Redundant System Kumar et al., [2] and dependability Measurement for Chance failures in the exponential scenario Talk about data analysis According to Goel, et. al., [3] and, Kumar et al., [4] a system is considered to have parallel redundancy when duplicate components function simultaneously. A parallel redundancy provides robustness against single component failure and error detection. An electronic machine circuit can determine which replication is erroneous when a unit vote is observed and Aksu, et al., [5] discusses on a vessel with two fixed and two rotating pod units as part of a combined four-pod propulsion system, a reliability evaluation approach was applied and the evaluation method made use of Markov analysis, FTA, and FMEA. In redundancy allocation, Ikegwuru, O. I. [6] analyze systems with several redundancy levels. Over the past few decades, the field of reliability analysis has focused on analyzing and discussing systems with associated failures. In order to determine probable failure modes in a system and their causes, Taboada, et al., [7] and, Sharma, et al., [8] addressed the use of failure rate models in conjunction with a statistical failure mode ratio database. For benchmark situations requiring multi-level redundancies, Yeh, et al., [9] and, Ebeling, et. al., [10] an integrated multi-level redundancy allocation can produce better results than sequential redundancy allocation. Zhao, et al., [11] define the traits, applications, and constraints of single component maintenance strategies corrective, predictive, and other approaches that are covered in detail from the standpoint of maintenance time.

In the concept of mathematically models, Jahromi, et al., [12] and, Zaslavskyi, V. [13] serves as the basis for mathematical models and algorithms that tackle optimization problems in the creation of extremely reliable safety and security systems. It is believed that when components in cold-standby mode are kept under severe standby conditions for an extended period of time, their performance would deteriorate. The other cold-standby components, excluding the online running components, are routinely inspected and maintained preventively to maintain or restore them to a proper state. It is considered that the failure rates of the cold-standby components are increasing even when they are restored to like-new state. Qiu, et al., [14] and, Tavana, et al., [15] analysis the goal of the effort is to find nearly ideal allocation schemes that maximize system reliability while remaining economical. The proposed formulation by Gholinezhad, et al., [16] and, Nath, et al., [17] is integrated into two different algorithms are used in the evolving algorithmic framework. Wang, et al., [18] investigates a redundancy allocation problem for cold-standby systems that have components that are deteriorating. It is believed that when components in cold-standby mode are kept under severe standby conditions for an extended period of time, their performance would deteriorate.

It investigates what happens to a general coherent structure when cold standby redundancy is added at the system and component levels according to Eryilmar, [19] that The effects of adding cold standby redundancy to a system at the system and component levels on dependability can be of interest to dependability designers. Adding cold standby redundancy at the component (system) level extends the life of a series (parallel) system. Specifically, expressions based on signatures are generated for the system's survival function following standby redundancy at both the system and component levels and Jia, et al., [20] discusses the Power generation systems have made extensive use of warm standby, which ensures safe and dependable operation with a shorter leading time and less energy consumption than hot and cold standby, respectively. Multi-state features appear in power generation systems because there exist a number of intermediate states for producing units between flawless functioning and total failure and also Zhao, et al., [21] substituting the standby components for a multi-state balanced system, a new rebalancing technique is suggested. The system is composed of m subsystems, each of which has n-1 standby components and one operating component. When all of the system's functional parts remain intact and its degree of balance stays within a certain range, the system is said to be balanced.

In addition to common cause failure, Gao, et al., [22] discusses a single repairman and redundant series system with a backup unit for every primary unit. the system may malfunction when all of the units in a subsystem are unavailable or when a failing primary unit is attempted to be moved to the standby without success. The repairman takes a postponed vacation if the system has no failed units also, Gao, [23] consider a K-mixed redundancy approach is taken into consideration for The bi-objective optimal problem and reliability analysis for a redundant system with two dependent failures load-sharing failure and common cause failure are included. The system in this case has N active parts, W heated standbys, and C cold standbys. Malhotra et al., [24] assesses the symmetrical availability, dependability, and other system efficacy metrics for two stochastic models with different demand, which has a two-unit hot standby system and a two-unit cold standby system; the standby unit must be turned on in order for it to start working. Zhu, et al., [25] focused on the counting procedure is used to examine the dependability of the series-parallel system with active, cold standby, and warm standby redundancy. The optimization model, which maximizes system dependability while adhering to weight and cost restrictions, is developed based on the reliability model.

A new reliability redundancy allocation problem is proposed, where each subsystem's component allocation may differ from one another, by Gholinezhad, [26] define reliability redundancy allocation problem can improve the reliability of the system in different cases in terms of the maximum possible improvement index, and Guo, et al., [27] describe the dependency structures between the components and hot/cold standbys in series systems, this article employs hierarchical copula. In order to improve the performance of the system, the optimal allocations for hot and cold standbys in series systems with dependent components and redundancies are presented. Benabid, et al., [28] discusses, the reliability assessment of redundant electrical power supply systems is discussed and Examined and enhanced with consideration for various redundancy scenarios is the electrical power supply system's reliability. Darmstadt, et al., [29] analysis the concept vehicles' failure modes and hazards and conduct functional hazard analyses and failure modes and effects criticality analyses for each. Yoon, et al., [30] to analyze the redundancy plan of the electric power management system is designed to facilitate the development of digital twin technology for electric propulsion systems, while accounting for failure types and effects.

2. Objective

Our objective of current study is to the effective operation of standby redundancy systems through a comprehensive failure mode analysis and mathematical relationships between one or more units or components working in which another component is ready to take over in the event that the first component fails.

3. Methodology of MTBF for a Component in Standby Redundancy System

Important considerations for system reliability design include reliability allocation for components, redundancy allocation, and reliability redundancy allocation. Although active redundancy has simpler modelling than standby redundancy, standby redundancy generally provides superior reliability for any system. Cold standby method, in which backups are carried out to ensure that a backup portion can effectively assume charge of the endeavour when the currently operating fizzles out, is one of the most often used procedures to attain high-reliability necessity. Such a collection of standby parts can be thought of as a single unit or system that can fail multiple times before it ceases to function.

Notations and Abbreviations:

R_s : System Reliability

Q_S : System Unreliability

n : Number of Components

λ : Failure rate

M_s: Mean time to between failures (system)

MTBF: Mean Time to Between Failures.

CFD: Cumulative Failure Distribution

FDF: Failure Density Function

We have n components in the system if $(n+1)$ components are ready to support one running component. A failure of any of the $(n+1)^{th}$ components would be the only thing that would cause the system to fail.

Since, $e^{(-\lambda t)}\, e^{(\lambda t)} = 1\ or,$

$$e^{(-\lambda t)}\left[1+\lambda t+\frac{(\lambda t)^2}{2!}+\frac{(\lambda t)^3}{3!}+\cdots\right]=1 \quad (1)$$

From (1), the chance that there won't be a failure is represented by the phrase $e^{(-\lambda t)}\,{}^{*}1$, the probability that there will be precisely one failure is represented by the term $e^{(-\lambda t)}\,{}^{*}(\lambda t)$, the probability that there will be exactly two failures is represented by the term $e^{(-\lambda t)^{*}}(\frac{(\lambda t)^2}{2!})$, etc. Therefore, The expectation that there will be two, one, or no failure, or the probability that there won't be more than two failures equal:

$$e^{(-\lambda t)}\,{}^{*}1+e^{(-\lambda t)}\,{}^{*}(\lambda t)+\ e^{(-\lambda t)^{*}}(\frac{(\lambda t)^2}{2!}) \quad (2)$$

From (2), we have R_s and Q_S represent the reliability and the unreliability of the system**,**

$$\text{Then,} R_s+Q_S=\ e^{(-\lambda t)}\left[1+\lambda t+\frac{(\lambda t)^2}{2!}+\frac{(\lambda t)^3}{3!}+\cdots\right]\cong\ e^{(-\lambda t)}\,{}^{*}1+e^{(-\lambda t)}\,{}^{*}(\lambda t)+\ e^{(-\lambda t)^{*}}(\frac{(\lambda t)^2}{2!}) \quad (3)$$

From (3), this enlarged form allows for one failure, after which the dependability of a standby system made up of one working component and another ready to step in and take over is provided by

$R_s = e^{(-\lambda t)}\,[1 + \lambda t]$ (4)

Using (4), the MTBF for a components system $i.e.,\ M_s = \int_0^{\infty} R_s\, dt \cong \frac{2}{\lambda}$

From (2), for a standby system consisting of three units with an identical failure rate, where one unit is in use and the other two are ready to take over in turn, we have

$R_s = e^{(-\lambda t)}\left[1 + \lambda t + \frac{(\lambda t)^2}{2!}\right]$ (5)

Using (5), the MTBF for a components system $i.e.,\ M_s = \int_0^{\infty} R_s\, dt \cong \frac{3}{\lambda}$

In General, when n identical parts or units are ready to assist one that is in operation, we have

$R_s = e^{(-\lambda t)} \sum_{i=0}^{n} \frac{(\lambda t)^i}{i\,!}$ (6)

Using (6), the MTBF for a components system $i.e.,\ M_s = \int_0^{\infty} R_s\, dt = \frac{(n+1)}{\lambda}$

4. Analysis of a Component in Standby Redundancy under MTBF

Due to uncertainties in production and stress, the actual failure rate can only be determined using statistical procedures. Standby arrangements are often used, with one unit working while another is ready to take over in case of failure. Failure detection and switchover devices are typically needed for standby arrangements, but there is no difference in consistent failure rates between standby and operational components. Consider the data on failure rates to determine component reliability. One unit is placed in *standby* redundancy to the main unit the failure rate of each unit is estimated to be $\lambda = 0.10/hrs$ but when sensing and switching device is 100% reliable then,

$R_s = \exp(-\lambda t)\ [1 + \lambda t] \cong 0.736$, and MTBF= $20hrs$ when, sensing and switching device is 90% reliable then,

$R_s = \exp(-\lambda t)\ [1 + \lambda t] \cong 0.699$, and MTBF= $19hrs$ when, sensing and switching device is 80% reliable then,

$R_s = \exp(-\lambda t)\ [1 + \lambda t] \cong 0.662$, and MTBF= $18hrs$ when, sensing and switching device is 70% reliable then,

$R_s = \exp(-\lambda t)\ [1 + \lambda t] \cong 0.625$, and MTBF= $17hrs$ when, sensing and switching device is 60% reliable then,

$R_s = \exp(-\lambda t)\ [1 + \lambda t] \cong 0.589$, and MTBF= $16hrs$ when, sensing and switching device is 50% reliable then,

$R_s = \exp(-\lambda t)\ [1 + \lambda t] \cong 0.552$, and MTBF= $15hrs$ when, sensing and switching device is 40% reliable then,

$R_s = \exp(-\lambda t)\ [1 + \lambda t] \cong 0.515$, and MTBF= $14hrs$ when, sensing and switching device is 30% reliable then,

$R_s = \exp(-\lambda t)\ [1 + \lambda t] \cong 0.478$, and MTBF= $13hrs$ when, sensing and switching device is 20% reliable then,

$R_s = \exp(-\lambda t)\ [1 + \lambda t] \cong 0.441$, and MTBF= $12hrs$ when, sensing and switching device is 10% reliable then,

$R_s = \exp(-\lambda t)\ [1 + \lambda t] \cong 0.405$, and MTBF= $11hrs$ when, sensing and switching device is 0% reliable then,

$R_s = \exp(-\lambda t)\ [1 + \lambda t] \cong 0.368$, and MTBF = $10hrs$

Construct a Table 3 in the relationship between MTBF and System Reliability as below:

Table 3: MTBF and System Reliability of the components

R_D (Device Reliability in %)	**0**	**10**	**20**	**30**	**40**	**50**	**60**	60	70	80	90	100
MTBF (hrs.)	10	11	12	13	14	15	16	16	17	18	19	20
R_S	0.368	0.405	0.441	0.478	0.515	0.552	0.589	0.589	0.625	0.662	0.699	0.736

Construct a Figure 5, between MTBF and System Reliability as below:

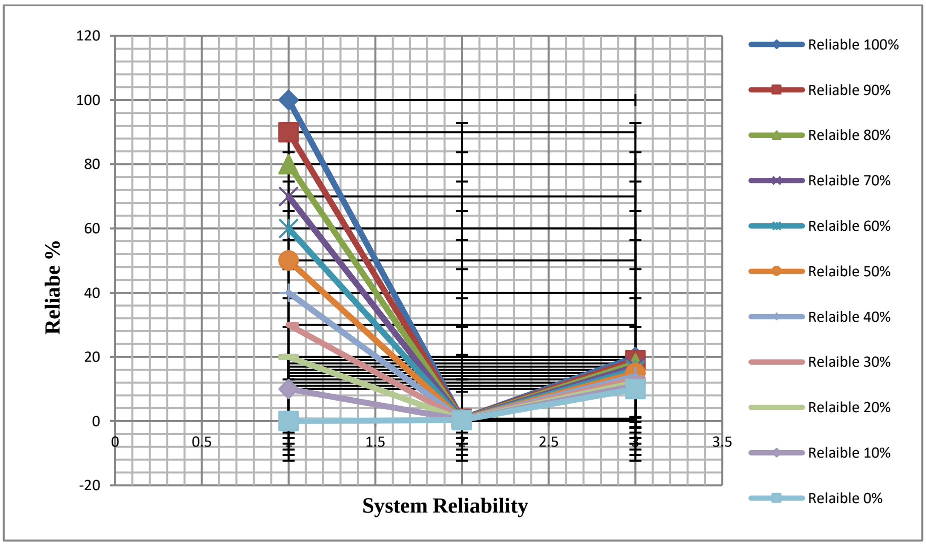

Figure 1: System Reliability under MTBF

5. Analysis of the Number of Failure Components in Operating Time

A system's equipment and component failures can have a variety of specific causes, including poor system or component design, incorrect manufacturing processes, a lack of comprehensive knowledge and experience, complex equipment, subpar maintenance facilities, complex and rigid organizational structures, and human error. In Table 1, the failure data of 1000 electronic components with operating hours as given below:

Table 1: Failure components data in operating hours

Operating Time (hrs.)	0-10	10-20	20-30	30-40	40-50
Number of Failures	190	139	121	99	88
Operating Time (hrs.)	50-60	60-70	70-80	80-90	90-100
Number of Failures	79	67	59	71	87

Establish Table 2, for failure data analysis of the components in operating hours.

Table 2: Failure data analysis of the components

Time (t)	Failure (f)	Cumulative Failures $N_f(t)$	Number of Survivors $N_s(t)$	Reliability Function $R(t)$	CFD $F(t)$	FDF $f(t)$	Failure Rate $\lambda(t)$
I	**II**	**III**	**IV**	**V**	**VI**	**VII**	**VIII**
0	0	0	1000	1	0	0.0190	0.0190
10	190	190	810	0.810	0.190	0.0139	0.0171
20	139	329	671	0.671	0.329	0.0121	0.0180
30	121	450	550	0.550	0.450	0.0099	0.0180
40	99	549	451	0.451	0.549	0.0088	0.0195
50	88	637	363	0.363	0.637	0.0079	0.0217
60	79	716	284	0.284	0.716	0.0067	0.0235
70	67	783	217	0.217	0.783	0.0059	0.0271
80	59	842	158	0.158	0.842	0.0071	0.0449
90	71	913	87	0.087	0.913	0.0087	0.1000
100	87	1000	0	0	1	-	-

6. Discussion

6.1 For Column I & II: The number of failures (frequencies) indicated against each class interval of time is the sum of the numbers of all components that fail during that time. We simply enter time in hours, starting from the lower bound of the first class interval up to the higher bound of the last class interval. These frequencies are entered in relation to the upper bound of the observed class interval.

6.2 For Column III: In this column, we represent the $\boldsymbol{N_f(t)}$**,** number of components that they have failed by time $\boldsymbol{t}$**,** followed by the cumulative frequencies of the failures written in column II.

6.3 For Column IV: In this column, we represent the $\boldsymbol{N_s(t)}$**,** number of components that are performing their intended function adequately at time $\boldsymbol{t}$**.**

6.4 For Column V: In this column, we represent the $\boldsymbol{R(t)}$**,** reliability at different points in time $\boldsymbol{t}$**,** then $R(t) = \frac{N_s(t)}{N_0}$ where, N_0 denotes the total number of component in the sample so, construct the graph as shown in Fig. 1, for reliability function of the components:

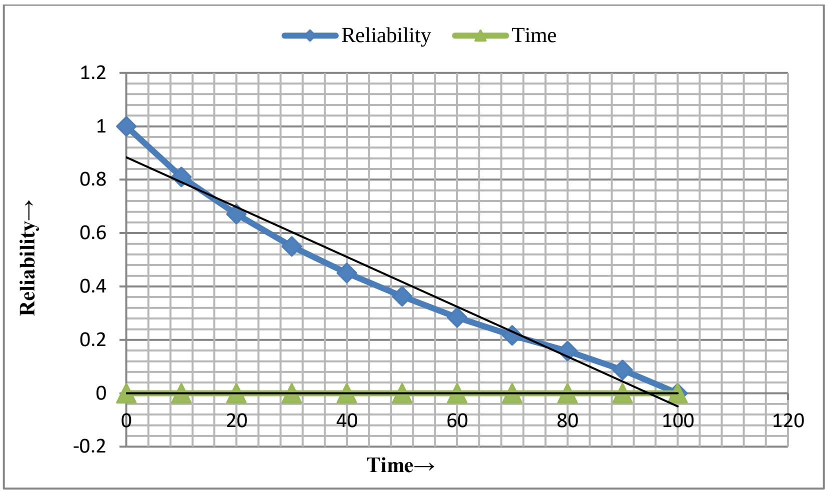

Figure 2: Reliability Function of Components

6.5 For Column VI: In this column, we represent the $\boldsymbol{F(t)}$**,** cumulative failure distribution at different points in time $\boldsymbol{t}$**,** then $F(t) = \frac{N_f(t)}{N_0}$ where, N_f denotes the number of components that have failed at time $\boldsymbol{t}$ so, construct the graph as shown in Fig. 2, for CFDF (unreliability) of components.

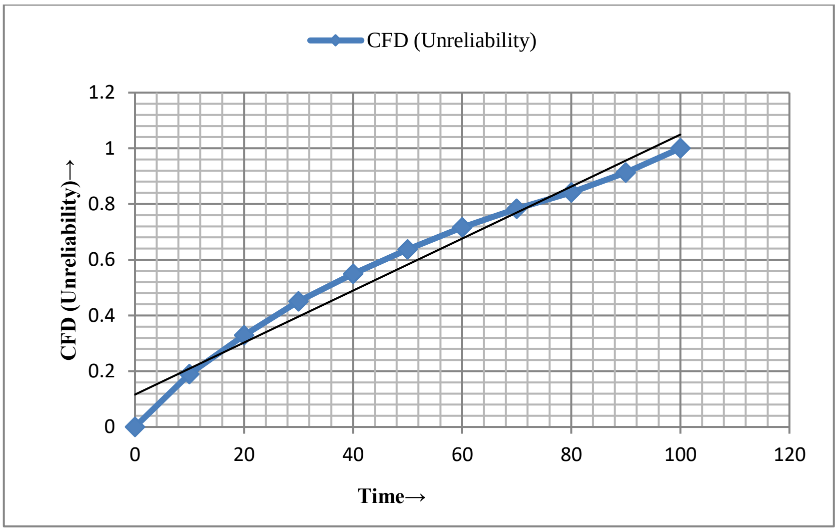

Figure 3: Cumulative Failure Distribution Function of Components

6.6 For Column VII: In this column, we represent the $\boldsymbol{f(t)}$**,** values of failure density function at different points in time $\boldsymbol{t}$**,** then

$$f(t) = \frac{\frac{d}{dt} N_f(t)}{N_0}$$

$$= \lim_{\Delta t \to \infty} \frac{N_f(t+\Delta t) - N_f(t)}{N_0 \, \Delta t} \text{ then,}$$

The approximate value of $f(t) = \frac{N_f(t+\Delta t) - N_f(t)}{N_0 \, \Delta t}$ where,$\Delta t = 10$ & $t = 0, 10, 20, \dots$ and, $N_0 = 1000$, denotes the total number of component in the sample so, construct the graph as shown in Fig. 3, for failure density function of components.

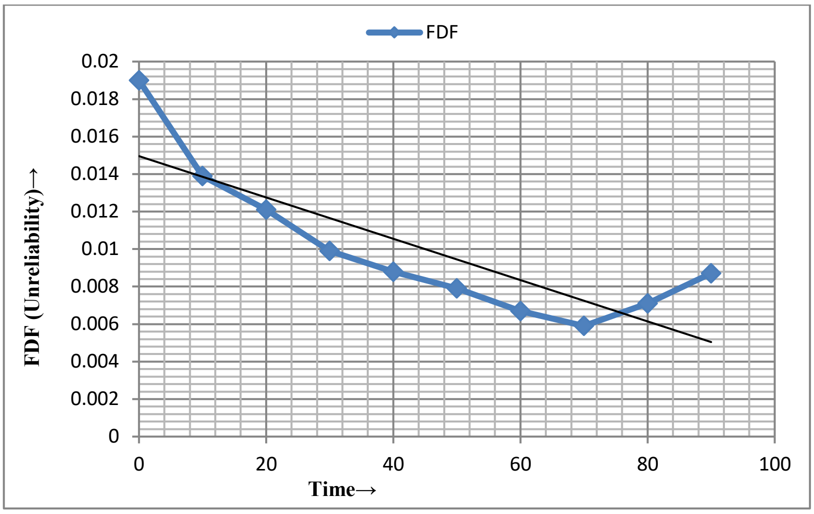

Figure 4: Failure Density Function of Components

6.7 For Column VIII: In this column, we represent the $\boldsymbol{\lambda(t)}$**,** failure rate at different points in time $\boldsymbol{t}$**,** then

$$\lambda(t) = \frac{\frac{d}{dt}N_f(t)}{N_s(t)}$$

$$= \lim_{\Delta t \to 0} \frac{N_f(t+\Delta t) - N_f(t)}{N_s(t)\,\Delta t} \text{ then,}$$

The approximate value of $\lambda(t) = \frac{N_f(t+\Delta t) - N_f(t)}{N_s(t)\,\Delta t}$ where, $\Delta t = 10$ & $t = 0, 10, 20, \ldots$ so, construct the graph as shown in Fig. 4, for failure rate of components.

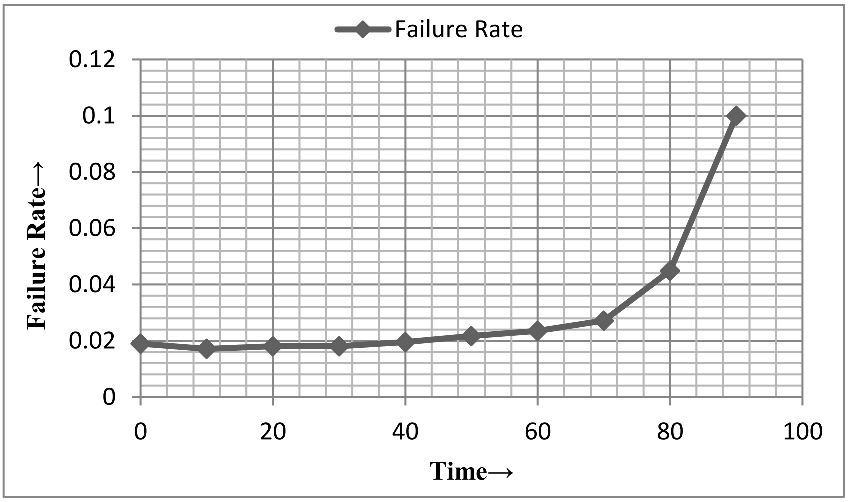

Figure 5: Failure Rate of Components

7. Conclusion and Future Scope

The current study focuses on issues in standby for multiple stages, including transient intervals, switching errors, false alarms, and missing failures. It emphasizes the importance of parameter selection and commutation in achieving optimal system performance for reliability criteria. Redundancy, reliability redundancy, and component allocation are crucial for building a reliable system. Active redundancy offers easier modeling, while standby redundancy generally delivers superior reliability. The cold standby approach is widely used for high-reliability needs. Standby systems are marginally more reliable than parallel working units due to the imprecise nature of sensing and switching over devices. This study also demonstrates how the unit's dependable state may fluctuate over time and under different conditions. In future we will also analyze by implementing multiple an effective structures, and will give important consideration to allocate the reliability of components to improve the functional system.

Acknowledgment

I would like to sincerely appreciate and express my gratitude to Prof. D K Ghosh, and all faculty members of the Department of Mathematics for their insightful advice, which helped us to conduct present work.

Author Contribution

Chetan Kumar Sharma and Rajeev Kaushik organized the methodology and managed the work; Neeraj and Nitin wrote the original draft of the simulations and literature review; and Prof. Ashok Kumar helped with editing and supervision.

Conflicts Of Interest

On the behalf of all authors, the corresponding author states that there is no conflict of interest.

References

[1] Gupta P P , Sing S B and Goel C K (2001) "*Analytic study of complex standby redundant system involving the concept of a multi failure human error under head of the line repair policy* " Bulletin of Pure and Applied Sciences, V-20, pp345-355

[2] Ashok Kumar, Madan Mohan Gupta and Chetan Kumar Sharma (2012) **"***Cost Analysis of 2-Unit Electronic Parallel Redundant System for Three States***"** International Research Journal of Management Science and Technology, Volume-02, Issue-03, pp290-296

[3] Goel C.K, Singh S. B and Jai Kishore (2005) "*Reliability and MTBF of a standby redundant system with common cause failure* "Volume 31 (02), pp365-370

[4] Kumar, A., Gupta, M. M., & Sharma, C. K. (2011). Reliability measurement for the exponential case of chance failures discuss with data analysis. *International Research Journal of Management Science and Technology*, *2*(3), 212-218.

[5] Aksu, S., Aksu, S., & Turan, O. (2006). Reliability and availability of pod propulsion systems. *Quality and Reliability Engineering International*, *22*(1), 41-58.

[6] Ikegwuru, O. I. (2016). *Integrated performance-reliability optimisation of systems with multi-level redundancies*. University of Northumbria at Newcastle (United Kingdom).

[7] Taboada, H. A., Baheranwala, F., Coit, D. W., & Wattanapongsakorn, N. (2007). Practical solutions for multi-objective optimization: An application to system reliability design problems. *Reliability Engineering & System Safety*, *92*(3), 314-322.

[8] Sharma, C. K., Gupta, M. M., & Kumar, A. (2010). Reliability analysis with analytical discussion of the system with correlated failures. *International Research Journal of Management Science and Technology*, *1*(3), 104-115.

[9] Yeh, W. C., Su, Y. Z., Gao, X. Z., Hu, C. F., Wang, J., & Huang, C. L. (2021). Simplified swarm optimization for bi-objection active reliability redundancy allocation problems. *Applied Soft Computing*, *106*, 107321.

[10] Ebeling E. Charles (2008) "*A reliability and maintainability for common failure*" Microelectronic and Reliability, V-43, pp 205-231

[11] Zhao, J., Gao, C., & Tang, T. (2022). A review of sustainable maintenance strategies for single component and multicomponent equipment. *Sustainability*, *14*(5), 2992.

[12] Jahromi, A. E., & Feizabadi, M. (2017). Optimization of multi-objective redundancy allocation problem with non-homogeneous components. *Computers & Industrial Engineering*, *108*, 111-123.

[13] Zaslavskyi, V. (2017). System principles, mathematical models and methods to ensure high reliability of safety systems. In *XI Conference on Reconnaissance and Electronic Warfare Systems* (Vol. 10418, pp. 8-21). SPIE.

[14] Qiu, X., Ali, S., Yue, T., & Zhang, L. (2017). Reliability-redundancy-location allocation with maximum reliability and minimum cost using search techniques. *Information and Software Technology*, *82*, 36-54.

[15] Tavana, M., Khalili-Damghani, K., Di Caprio, D., & Oveisi, Z. (2018). An evolutionary computation approach to solving repairable multi-state multi-objective redundancy allocation problems. *Neural Computing and Applications*, *30*, 127-139.

[16] Gholinezhad, H. (2024). A new model for reliability redundancy allocation problem with component mixing. *Reliability Engineering & System Safety*, *242*, 109815.

[17] Nath, R., & Muhuri, P. K. (2024). A novel evolutionary solution approach for many-objective reliability-redundancy allocation problem based on objective prioritization and constraint optimization. *Reliability Engineering & System Safety*, *244*, 109835.

[18] Wang, W., Wu, Z., Xiong, J., & Xu, Y. (2018) "Redundancy optimization of cold-standby systems under periodic inspection and maintenance" *Reliability engineering & system safety*, *180*, 394-402.

[19] Eryilmaz, S. (2017). The effectiveness of adding cold standby redundancy to a coherent system at system and component levels. *Reliability Engineering & System Safety*, *165*, 331-335.

[20] Jia, H., Ding, Y., Peng, R., Liu, H., & Song, Y. (2020). Reliability assessment and activation sequence optimization of non-repairable multi-state generation systems considering warm standby. *Reliability Engineering & System Safety*, *195*, 106736.

[21] Zhao, X., Wang, C., & Wang, S. (2024). Reliability analysis of multi-state balanced systems with standby components switching mechanism. *Reliability Engineering & System Safety*, *242*, 109774.

[22] Gao, S., Wang, J., & Zhang, J. (2023). Reliability analysis of a redundant series system with common cause failures and delayed vacation. *Reliability Engineering & System Safety*, *239*, 109467.

[23] Gao, S. (2023). Reliability analysis and optimization for a redundant system with dependent failures and variable repair rates. *Mathematics and Computers in Simulation*, *208*, 637-659.

[24] Malhotra, R., Alamri, F. S., & Khalifa, H. A. E. W. (2023). Novel Analysis between Two-Unit Hot and Cold Standby Redundant Systems with Varied Demand. *Symmetry*, *15*(6), 1220.

[25] Zhu, L., & Jia, X. (2023, October). Reliability-Redundancy Allocation for System Under Mixed Active, Cold and Warm Standby Redundancy With Weibull Distribution. In *2023 Global Reliability and Prognostics and Health Management Conference (PHM-Hangzhou)* (pp. 1-8). IEEE.

[26] Gholinezhad, H. (2024). A new model for reliability redundancy allocation problem with component mixing. *Reliability Engineering & System Safety*, *242*, 109815.

[27] Guo, M. Y., Zhang, J., Zhang, Y., & Zhao, P. (2024). Optimal redundancy allocations for series systems under hierarchical dependence structures. *Quality and Reliability Engineering International*, *40*(4), 1540-1565.

[28] Benabid, R., Merrouche, D., Bourenane, A., & Alzbutas, R. (2018, October). Reliability assessment of redundant electrical power supply systems using fault tree analysis, reliability block diagram, and Monte Carlo simulation methods. In *2018 International Conference on Electrical Sciences and Technologies in Maghreb (CISTEM)* (pp. 1-7). IEEE.

[29] Darmstadt, P. R., Catanese, R., Beiderman, A., Dones, F., Chen, E., Mistry, M. P., ... & Preator, R. (2019). *Hazards analysis and failure modes and effects criticality analysis (fmeca) of four concept vehicle propulsion systems* (No. NASA/CR-2019-220217).

[30] Yoon, K., & Kim, J. (2021). FMEA of Electric Power Management System for Digital Twin Technology Development of Electric Propulsion Vessels. *Journal of the Korean Society of Marine Environment & Safety*, *27*(7), 1098-1105.

Appendix I: *Description of Keywords*

Reliability:

In the simplest of terms, reliability is the probability that a failure won't happen within a specified time frame. "*The **probability** that a unit (or product) performs its **intended function** adequately for a given period of **time** under the stated **operating condition** or environment*" is a more meticulous definition of reliability. We describe to an element, system, component of a system, or whatever is similar as a unit. Four components are highlighted in the reliability definition, namely:

- Probability
- Intended function
- Time, and
- Operating condition

A failure occurs when a device (or system) loses or changes some or all of its properties to the point where it can no longer function properly or at all. While some components fail with clear indications, others do not. For instance, there are clearly defined failures with switches and electric lamps under all circumstances. In contrast, a variety of operating conditions apply to devices like as resistors and voltage stabilizers. It is expected that there will be a significant number of initial failures when we put a large collection of units into operation. Initial failures or infant mortality resulting from manufacturing faults are the terms used to describe these early failures.

Reliability function for devices/item with continuous lifetime distribution is as follows:

$R(t) = 1 - F(t) = \Pr(T > t)\,; t > 0$

or,

$$R(t) = 1 - \int_0^t f(x)dx \equiv \int_t^\infty f(x)dx$$

Hence, $R(t)$ define, probability that the item/device operate properly in the time interval $(0, t]$, and $f(x)$ is the probability density function of time to failure, also $F(t)$ is the cumulative density function of lifetime T.

Failure Density Function:

The probability density function (pdf) of a random variable is the derivative of its cumulative distribution function; however, in the context of reliability, this probability density function is referred to as the failure density function. While $\lambda(t)$ measures the immediate speed of failure, the failure density function $f(t)$ measures the overall speed at which failures are occurring and defined as

$$f(t) = \frac{d\,F(t)}{dt}$$

Where, $F(t)\left(i.e., \int_0^t f(t)dt \;\forall\, t \geq 0\right)$ is the item of unreliability at time t, and $f(t)$ is the failure (or, probability) density function.

Failure Rate Function:

The probability that an item will fail in the interval $(t, t + \Delta t]$ and the item is functioning at time t is

$$\Pr(t < T \leq t + \Delta t | T > t) = \frac{\Pr(t < T \leq t + \Delta t)}{\Pr(T > t)} = \frac{F(t + \Delta t) - F(t)}{R(t)}$$

$$\lambda(t) = \lim_{\Delta t \to 0} \frac{\Pr(t < T \leq t + \Delta t | T > t)}{\Delta t} \cong \lim_{\Delta t \to 0} \frac{F(t + \Delta t) - F(t)}{\Delta t\, R(t)}$$

$$\lambda(t) = \frac{f(t)}{R(t)}$$

Here, $\lambda(t)$ is defining the item failure rate or time-dependent failure rate. This implies that, Δt is very small,

$$\Pr(t < T \leq t + \Delta t | T > t) \approx \lambda(t).\Delta t$$

Mean Time Between Failures:

The average amount of time that a system or product lasts between two consecutive failures is called its maximum test-break frequency, or MTBF. A measure of reliability called MTBF is frequently discussed in relation to product development, warranties, and maintenance schedules. Mean Time between Failure (MTBF) is a reliability metric that analyzes the average time between failures, which help inform an asset's reliability. It quantifies the likelihood of an equipment or component failure within a time frame.

The Mean Time between Failures (MTBF) is computed by dividing the total operational hours in a particular period by the total number of failures that happened during that same time. The hours of runtime before technology or equipment malfunctions are revealed by its solution.

$i.e.,\ \text{MTBF} = \frac{\text{Number of Operation Hours}}{\text{Number of Failures}}$

Standby Redundancy:

In order for a system to function successfully, redundancy indicates that there is a backup plan in place. Component or unit operations cannot always be carried out in parallel, necessitating the application of so-called stand-by arrangements, which involve having one or more components or units working in parallel and ready to take over in the event that the primary unit fails. Standby redundancy is the redundancy that supplements active redundancy. When we talk about active redundancy, we mean that every single parallel component is turned on when the system boots up and keeps working until anything breaks.

Every component is concurrently in the working mode when there is active redundancy. Conversely, with standby redundancy, the parts are linked in parallel but do not begin to function simultaneously as soon as the system is operational. The component(s) in operating mode in standby redundancy are referred to as typically operating components. Standby component(s) are the component(s) that are maintained in reserve or standby mode. There's a changeover device in addition to this. The switch device's job is to detect whether a regularly functioning component fails and, if so, to switch a standby component back to normal operation. We make the assumption that the standby component or components won't malfunction when in standby mode. Let us now assume that component I successfully carries out its planned function, where $i = 1, 2, 3, \ldots, n.$ Let q_i denote the unreliability of the component i, given that components have failed. Further, if R_s and Q_s denote the reliability and unreliability of the standby system, then

$Q_s = P(\overline{e_1} \cap \overline{e_2} \cap \overline{e_3} \cap \overline{e_4} \cap \ldots \cap \overline{e_n})$, where $\overline{e_i}$ denotes complement of event e_i

$$= P(\overline{e_1})P(\overline{e_2}/\overline{e_1})P(\overline{e_3}/\overline{e_1}\ \overline{e_2}) \ldots P(\overline{e_n}/\overline{e_1}\overline{e_2} \ldots \overline{e_{n-1}}) \equiv q_1 q_2 q_3 \ldots q_n$$

When there is one component that operates normally and one that is on standby, the system's unreliability is described as $Q_s = q_1 q_2$ and reliability given by $R_s = 1 - Q_s$