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Optimizing methods a logistics plan

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Abstract:- The class of transport problems includes tasks that allow you to determine the best option for a cargo transportation plan. For example, we are faced with the task of delivering the required amount of supplies to consumers. In addition, there are a certain number of warehouses where supplies are stored. Each warehouse has a different supply quantity. The costs of transporting supplies from warehouses to consumers are called tariffs. It is necessary to find the optimal plan for transporting the supply, at which the costs will be the lowest. These initial data determined the name "Transport Tasks". This article proposes an easy and accessible way for any enterprise to find the best transportation plan using a transport problem. The transport problem is one of the most important special cases of the general linear programming problem, due to the specifics of its construction and scope of application. The transport model is initially designed to select the most economical planning of cargo flows and the operation of various modes of transport. However, the scope of application of the transport model is not limited to this. Examples of using a transport model include problems of production scheduling, rational use of natural and human resources, etc. In today's market conditions, many large companies contain logistics departments in their structure, however, there are often businessmen for whom the question of whether the company needs a logistician is controversial. Logistics allows you to solve such issues as reducing transportation costs, choosing the shortest transportation route, reducing time spent, simplifying a complex product delivery scheme, and reducing all kinds of costs.

Keywords: transport problem, optimal plan, economical planning of cargo.

1. Introduction

The classical transport problem is a problem about the optimal plan for transporting a homogeneous product from homogeneous points of availability to homogeneous points of consumption on homogeneous vehicles with static data and a linear approach, this is the main conditions of the problem.

For the classical transport problem, two types of problems are distinguished: the cost criterion (achieving a minimum of transportation costs) or distances and the time criterion (a minimum of time is spent on transportation). Under the name transport problem, a wide range of problems are defined with a unified mathematical model; these problems belong to linear programming problems and can be solved by an optimal method. However, a special method for solving the transport problem can significantly simplify its solution, since the transport problem was developed to minimize the cost of transportation.

2. Statement of the transport problem

Let's consider one of the most important linear programming problems, —the so-called transport problem. Let us formulate the problem in general form.

At m departure stations $A_1, A_2, \dots, A_m$, $a_1, a_2, \dots a_m$ units of homogeneous cargo are concentrated, respectively . The cargo should be transported to n destinations $B_1, B_2, \dots, B_n$, and $b1, b2, \dots, bn$ should be added to each of the points, respectively units of cargo. The cost of transporting a unit of cargo from point A_i to point B_j is given and equal to c_{ij}. It is believed that the total stock of cargo at all departure stations is equal to the total demand for cargo at all destination stations:

$$\sum_{i=1}^{nm} a_i = \sum_{j=1}^{n} b_j \quad . \qquad (1)$$

It is required to draw up a transportation plan in which the total cost of all transportation will be minimal.

We summarize all the data in Table 1.

Table 1

Items destination . Items departures.	B_1	B_2		Bj	...	Bn	Reserves
A_1	c_{11}	c_{12}	...	c_{1j}	...	c_{1n}	a_1
...	...	...	...	...	...	...	...
A_i	c_{i1}	c_{i2}	...	c_{ij}	...	c_{in}	ai
...	...	...	...	...	...	...	...
Am	c_{m1}	c_{m2}	...	c_{mj}	...	C_{mn}	am
Needs	$b1$	$b2$	...	bj	...	Bn	$\sum b_j = \sum a_i$

Let us denote by x_{ij} the number of units of cargo intended for shipment from point A_i to point B_j. Then the amount of cargo that is planned for delivery to point B_j from all points of departure is

$$x_{1j} + x_{2j} + \ldots + x_{mj} = \sum_{i=1}^{m} x_{ij}, \quad 1 \le j \le n \ .$$

Since the demand for cargo at destination B_j is b_j units, the equality must be satisfied

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad 1 \le j \le n \ .$$

In expanded form, the last equalities represent a system of n linear equations with m unknowns:

$$\begin{cases} x_{11} + x_{21} + \cdots + x_{m1} = b_1, \\ x_{12} + x_{22} + \cdots + x_{m2} = b_2, \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ x_{1n} + x_{2n} + \cdots + x_{mn} = b_n. \end{cases} \qquad (2)$$

On the other hand, the total amount of cargo sent from point A_i to all points B_j is the sum

$$x_{i1} + x_{i2} + \cdots x_{in} = \sum_{j=1}^{n} x_{ij} = a_i, \quad 1 \le i \le m \ .$$

Thus, similarly to the system of equations (2), we obtain a system of m linear equations with n unknowns:

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$$\begin{cases} x_{11} + x_{12} + \dots + x_{1n} = a_1, \\ x_{21} + x_{22} + \dots + x_{2n} = a_2. \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots \\ x_{m1} + x_{m2} + \dots + x_{mn} = a_m. \end{cases} \tag{3}$$

We combine the resulting systems of equations (2) and (3) and write them in the form of a system of $n + m$ linear equations with nm unknowns:

$$\begin{cases} \sum_{i=1}^{m} x_{ij} = b_j, \quad 1 \le j \le n, \\ \sum_{j=1}^{n} x_{ij} = a_i, \quad 1 \le i \le m. \end{cases} \tag{4}$$

If the values x_{ij} are tabulated (Table 2), we obtain a transportation matrix by which relations (4) are easily verified: the sum of the elements x_{ij} located in the *ith* row is equal to the stock a_i at point A_i ; the sum of elements x_{ij} from column j is equal to the need b_j of item B_j .

Table 2

Items destination .	B_1	B_2	...	B_j	...	B_n	Reserves
Items departures.							
A_1	c_{11} x_{11}	c_{12} x_{12}	...	c_{1j} x_{1j}	...	c_{1n} x_{1n}	a_1
A 2	c_{21} x_{21}	c_{22} x_{22}	...	c_{2j} x_{2j}	...	c_{2n} x_{2n}	a_2
...	...	...	...	...	...	...	...
A_i	c_{i1} x_{i1}`	c_{i2} x_{i2}	...	c_{ij} x_{ij}	...	c_{in} x_{in}	a_i
...	...	...	...	...	...	...	...
Am	c_{m1} x_{m1}	c_{m2} x_{m2}	...	c_{mj} x_{mj}	...	c_{mn} x_{mn}	a_m
Needs	b_1	b_2	...	b_j	...	b_n	$\sum b_j = \sum a_i$

From the conditions of the problem it follows that the cost F of all transportation is equal to the sum

$$F = c_{11}x_{11} + c_{12}x_{12} + \dots + c_{ij}x_{ij} + \dots + c_{mn}x_{mn} = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \quad . \tag{5}$$

Since the cost F must be minimal, we naturally arrive at the following linear programming problem:

among all non-negative solutions of the system of equations (4), *find one for which the form F reaches its minimum value* .

The formulated problem is a transport problem to determine the criterion for the cost of transportation.

The stated problem, like any linear programming problem, is solved using the simplex method discussed in [1-7]. However, due to the simple structure of the system of equations (4), the simplex method in this case is significantly simplified and is reduced to the so-called distribution method, discussed below.

r of the system of equations (4) required for further analysis . The number of equations in this system is equal to $m + n$, and the number of unknowns in it— mn . If we add up all the equations of system (2), we obtain on the left side the sum of all unknowns x_{ij} , and on the right side —the sum of all demands b_j of cargo at stations B_j , i.e.

$$\sum_{j=1}^{n}\sum_{i=1}^{m} x_{ij} = \sum_{j=1}^{n} b_j \ .$$

If we further add up all equations (3), we obtain the equality

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} = \sum_{i=1}^{m} a_i \ .$$

The last relation is identical to the previous one, since condition (1) is satisfied. This means that there is a linear relationship between the equations of the system of equations (4). Therefore, the rank r of system of equations (4) does not exceed $m + n - 1$.

Let us show that the rank of system of equations (4) is exactly equal to $m + n - 1$ ($r = m + n - 1$). From linear algebra it is known that for this it is enough to indicate such $m + n - 1$ unknowns, which using the equations of system (4) are expressed through the remaining $mn - (m + n - 1)$ unknown. As such $m + n - 1$ unknowns, expressed through the rest, let's take the unknowns

$$x_{11}, x_{12}, x_{13}, \ldots, x_{1n}, x_{21}, x_{31}, \ldots, x_{m1},$$

that is, choose $m + n - 1$ unknowns located in the first row (n unknowns) and in the first column ($m - 1$ unknown) of the transportation matrix (Table 2). There are just $m + n$ *such unknowns* -1. Next, from the second equation of system (2) we express the unknown quantity x_{12} :

$$x_{12} = b_2 - x_{22} - x_{32} - \ldots - x_{m2} \ .$$

Using the remaining equations of system (2), we find expressions for the unknowns x_{1j} ($j = 2, ,\ldots, n$):

$$x_{1j} = b_j - x_{2j} - x_{3j} - \ldots - x_{mj} \ (j = 2, 3, \ldots, n). \quad (6)$$

Similarly, using all equations of system (3), except the first, we find expressions for x_{i1} ($i = 2, 3, \ldots, m$):

$$x_{i1} = a_i - x_{i2} - x_{i3} - \ldots - x_{in} \ (i = 2, 3, \ldots, m). \quad (7)$$

To express x_{11} , we use the first equation of system (2), substituting into it the found expressions for x_{i1} ($i = 2, 3, \ldots, m$) from formulas (7). Then

$$x_{11} = b_1 - x_{21} - x_{31} - \ldots - x_{m1} = b_1 - (a_2 - x_{22} - x_{23} - \ldots - x_{2n}) -$$

$$-(a_3 - x_{32} - x_{33} - \ldots - x_{2n}) - \ldots - (a_m - x_{m2} - x_{m3} - \ldots - x_{mn}) \ .$$

Thus, the selected $m + n$ *are actually expressed* -1 unknown through the rest $mn - (m + n - 1)$ unknown. This means that the rank of the system of equations (4) $r = m + n - 1$, as stated.

It is known [8-9] that if the rank of a system of equations is less than the number of unknowns, then the system of equations has an infinite number of solutions. It is in this set that one should look for the minimum value of the cost F of all transportation.

To simplify the above reasoning, x_{i1} ($i = 2, 3, \ldots, m$) and x_{1j} ($j = 2, 3, \ldots, n$) were chosen as the basic unknowns (total $m + n - 1$ unknown). Others $mn - (m + n - 1)$ unknowns are called free.

Solving a transport problem comes down to finding an admissible base solution (reference plan for the transport problem) and approximating the base plan to the optimal solution by constructing successive iterations.

3. Diagonal method (northwest corner method)

First, let's explain the essence of the diagonal method using the example of a transport problem given by table - 3. Let's try to satisfy the needs of the first destination $B\ 1$ with the reserves of the first departure point $A\ 1$ In this case, this can be done, since stocks $a\ 1 = 30$ exceed needs $b1 = 20$.

Let's fill in the cell $x\ 11 = 20$. The needs of point $B1$ are fully satisfied, and therefore the column corresponding to point $B1$ will be temporarily excluded from consideration. In this case, the remaining reserves of the point of departure $A1$ are now equal to the difference $a1 = 30 - 20 = 10$.

Next, we satisfy the needs of point $B\ 2$ with supplies $a\ 1 = 10$ of the point of departure $A\ 1$, that is, $x12 = 10$. Since $b2 = 30 > a\ 1 = 20$, the needs of point $B2$ cannot be satisfied entirely from the point of departure $A\ 1$. Therefore, we will satisfy the lack of demand at destination $B2$ at the expense of the reserves at departure point $A\ 2$. This is possible, since the stocks of departure point $A2$ equal to $a2 = 40 > b2 - a1 = 20$. Therefore, $x22 = 20$, and the balance of supplies at point of departure $A\ 2$ is $a2 = 40 - 20 = 20$. The needs of destination $B\ 2$ are fully satisfied, and the column corresponding to point $B2$ will be temporarily excluded from consideration. Also, the reserves of the point of departure $A1$ are completely exhausted, and column $A1$ will also be temporarily excluded.

Thus, the stocks of two points of departure —point $A2$ (stocks $a\ 2 = 20$) and point of departure A3 (stocks $a\ 3 = 20$). Note that the sum of all needs is still equal to the sum of all supplies.

In a similar manner, we continue to satisfy the needs of destinations $B3$ and $B4$ at the expense of the reserves of destinations $A2$ and $A3$. In this case, all supplies at the points of departure will be exhausted and the needs of destinations $B3$ and $B\ 4$ will be satisfied . The quantities of cargo units $x\ 23 = 20$, $x\ 33 = 10$, $x\ 34 = 10$ will also be determined and the corresponding cells of the table will be filled in. Having entered all the quantities of cargo units in table 3, we get table 4, i.e. transportation matrix of the transport problem under consideration. Including all the data obtained in Table 3, we obtain Table 4.

The set of unknown values in Table 4 is a feasible solution to the transport problem, since these solutions are non-negative. There are exactly as many filled cells in Table 4 as there should be basic unknowns. Namely, rank $r = m + n - 1 = 3 + 4 - 1 = 6$.

Table 3

Items appointments. / Items departures.	*B* 1	*B* 2	*B* 3	*B* 4	Reserves
A 1	**2** / **20**	**3** / **10**	2	4	30
A 2	3	**2** / **20**	**5** / **20**	1	40
A 3	4	3	**2** / **10**	**6** / **10**	20
Needs	20	30	30	10	90

According to the diagonal method of constructing a reference plan, at each step the first of the remaining departure points and the first of the remaining destinations are considered. Therefore, the diagonal method can be described by the following block diagram, in which n —number of departure points, m —number of destinations.

4. Results

The transportation problem is essentially a linear programming problem that can be solved by the simplex method. However, the specific structure of the problem conditions allows for the development of more efficient computational methods. For example with help diagonal method (northwest corner method). The basic assumption used in constructing the model is that the amount of transportation costs on each route is directly proportional to the volume of products being transported. The arc connecting the origin to the destination represents the route along which the products are transported.

Any plan of a transport problem that, based on the conditions, contains more than m + n - 1 occupied cells will never be a reference plan under any circumstances, since it corresponds to a linearly dependent system of vectors. In this case, a closed cycle is constructed in the table, with the help of which the number of occupied cells is reduced to m + n - 1. The methods for obtaining a reference plan assume balanced transport problems. If the transport model is open, then it should be balanced before using the method.

In this method(northwest corner method), the stocks of the next supplier are used to meet the demands of the next consumers until they are completely exhausted, after which the stocks of the next supplier by number are used. "Filling in the table of the transport problem begins from the upper left corner and consists of a number of similar steps. At each step, based on the stocks of the next supplier and the demands of the next consumer, only one cell is filled in and, accordingly, one supplier or consumer is excluded from consideration

Discussion

When conducting business, a person always experiences a lack of funds. In this case, there is a need to solve the problem to determine the maximum effect under given resource restrictions. Solving this problem allows us to develop the most rational ways and methods of transporting goods, eliminating excessively long, counter, and repeated transportation. All this reduces the time it takes to promote goods, reduces the costs of enterprises and firms associated with the implementation of supply processes with raw materials, materials, fuel, equipment, etc.

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