
Homomorphism on Fuzzy Translation and Fuzzy Multiplication of BV- Algebras

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Abstract: In this paper, we introduced the concept of Homomorphism on fuzzy translations and fuzzy multiplications of BV-Algebras and discussed some of their properties in detail by using the concepts of fuzzy BV- Ideal and fuzzy BV-subalgebra.
***Keywords*:** Fuzzy α −translation of BV-Algebras, Fuzzy α −multiplication of BV-Algebras, BV-Algebra, Fuzzy BV–Ideal, Fuzzy BV-Sub algebra, Homomorphism and Cartesian Product.

1. Introduction:

The concept of fuzzy set was introduced by L.A.Zadeh in 1965 [3]. Several researchers explored at the generalization of the perception of fuzzy subset. The principle of fuzzy subsets has developed in several guidelines and determined packages in a extensive variety of domain names considering the fact that its introduction. The take a look at of fuzzy subsets and its packages to diverse mathematical contexts has given upward thrust to what's now generally known as fuzzy mathematics. Fuzzy algebra is an essential department of fuzzy mathematics. Other algebraic structures, inclusive of groups, rings, modules, vector spaces, and topologies, had been investigated the usage of fuzzy ideas. K.Iseki and S.Tanaka [1] introduced the concept of BCK algebras in 1978. K.Iseki [2] newly developed the concept of BCI-algebras in 1980. Kim C B and Kim H S [8], developed the new concept of BG-algebras in 2008. In 2010, Muthuraj R, Sridharan M and Sithar Selvam P M [7], introduced the concept of fuzzy BG-ideals in BG-algebra. T.Priya and T.Ramachandran [4][5] developed the class of PS-algebras , which is a generalization of BCI / BCK/Q / KU / d algebras. A.Prasanna, M.Premkumar and S.Ismail Mohideen [10], introduced the concept of fuzzy translation and multiplication on B-algebras in 2018. In Ho Hwang, Yong Lin Liu and Hee Sik Kim [6] introduced the concept of BV-Algebras. T.Priya and T.Ramachandran [11] developed Homomorphism and Cartesian product on PS-Algebras. In this paper, we introduced the concept of Homomorphism on fuzzy translations and fuzzy multiplications of BV-Algebras and established some of its properties in detail.

2. Preliminaries:

In this section the basic definition of a BV-algebra, fuzzy BV-subalgebra, BV-ideal are recalled. We start with,

Definition 2.1:[4]

A BCK- algebra is an algebra (A,*,0) of type(2,0) satisfying the following conditions:

(i) $(x * y) * (x * z) \leq (z * y)$
(ii) $x * (x * y) \leq y$
(iii) $x \leq x$
(iv) $x \leq y \text{ and } y \leq x \Rightarrow x = y$
(v) $0 \leq x \Rightarrow x = 0, \text{ where } x \leq y \text{ is defined by } x * y = 0, \text{for all } x, y, z \in A$

Definition 2.2:[4]

A BCI- algebra is an algebra (A,*, 0) of type(2,0) satisfying the following conditions:

(i) $(x * y) * (x * z) \leq (z * y)$
(ii) $x * (x * y) \leq y$
(iii) $x \leq x$
(iv) $x \leq y$ and $y \leq x \Rightarrow x = y$
(v) $x \leq 0 \Rightarrow x = 0$, where $x \leq y$ is defined by $x * y = 0$, for all $x, y, z \in A$

Definition 2.3:[6]

An algebra (A, *, 0) is said to be a BV-algebra if it satisfies
(i) $x * x = 0$,
(ii) $x * 0 = x$,
(iii) $(x * y) * z = (0 * y) * (z * x)$ for all $x, y, z \in A$.

Example: 2.3.1[6]

Let A = {0, 1, 2, 3, 4} be a set with the following table:

*	0	1	2	3	4
0	0	2	1	4	3
1	1	0	3	2	4
2	2	4	0	3	1
3	3	1	4	0	2
4	4	3	2	1	0

Then it is easy to see that (A,*,0) is a BV-algebra.

Definition: 2.4[10]

A binary relation " $\leq$ " on a BV-algebra A defined as $x \leq y$ if and only if $x * y = 0$.

Definition: 2.5 [10]

Let S be non-empty subset of a BV-Algebra A, then S is called a BV–sub-algebra of A, if $x * y \in S \ \forall x, y \in S$

Definition: 2.6[4]

Let A be a BV-Algebra and I be a subset of A, then I is called a BV-ideal of A if it satisfies following conditions:

(i) $0 \in I$
(ii) $x * y \in I$ and $y \in I \Longrightarrow x \in I$

Definition: 2.7

Let A be a BV-Algebra. A fuzzy set δ in A is called a fuzzy BV-ideal of A if it satisfies following conditions:
(i) $\delta(0) \geq \delta(x)$
(ii) $\delta(x) \geq \min\{\delta(y * x), \delta(y)\} \ \forall x, y \in A$

Definition: 2.8 [11]

A fuzzy set δ in a BV-algebra A is called a fuzzy BV-subalgebra of A if

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$\delta(x * y) \geq \min\{\delta(x), \delta(y)\}\ \forall x, y \in A$

Definition: 2.9 [11]

A denotes a BV-algebra, and for any fuzzy set δ of A, we denote $T = 1 - sup\{\delta(x) : x \in A\}$ unless otherwise specified.

Definition: 2.10 [11]

Let δ be a subset of A and $\alpha \in [0, T]$. A mapping $\delta_{\alpha}^{t}: X \to [0,1]$ is said to be a fuzzy-α-translation of δ if it satisfies $\delta_{\alpha}^{t}(x) = \delta(x) + \alpha\ \forall x \in A$.

Definition: 2.11[11]

Let δ be a subset of A and $\alpha \in [0,1]$. A mapping $\delta_{\alpha}^{M}: X \to [0,1]$ is said to be a fuzzy-α-multiplication of δ if it satisfies $\delta_{\alpha}^{M}(x) = \alpha\delta(x)\ \forall x \in A$.

Example: 2.11.1

Let A = {0, 1, 2, 3, 4} be a set with the following table:

*	0	1	2	3	4
0	0	2	1	4	3
1	1	0	3	2	4
2	2	4	0	3	1
3	3	1	4	0	2
4	4	3	2	1	0

Then (A, *, 0) is a BV-algebra.

Define fuzzy set δ of A by $\delta(x) = \begin{cases} 0.4 \text{ if } x \neq 1 \\ 0.1 \text{ if } x = 1 \end{cases}$

Thus δ is a fuzzy BV-subalgebra of A.

Hence $T = 1 - \sup\{\delta(x)/x \in A\} = 1 - 0.4 = 0.6$, Choose $\alpha = 0.3 \in [0, T]$ and $\beta = 0.4 \in [0,1]$.

Then the mapping $\delta_{0.3}^{t}: A \to [0,1]$ is defined by $\delta_{0.3}^{t} = \begin{cases} 0.4 + 0.3 = 0.7 \text{ if } x \neq 1 \\ 0.1 + 0.3 = 0.4 \text{ if } x = 1 \end{cases}$

which satisfies $\delta_{0.3}^{t} = \delta(x) + 0.3\ \forall x \in A$, is a fuzzy-0.3-translation.

And the mapping $\delta_{0.4}^{M}: A \to [0,1]$ is define d by $\delta_{0.4}^{M} = \begin{cases} (0.4)(0.4) = 0.16 \text{ if } x \neq 1 \\ (0.4)(0.1) = 0.04 \text{ if } x = 1 \end{cases}$ which satisfies $\delta_{0.4}^{M} = (0.4)\delta(x)\ \forall x \in A$, is a fuzzy-0.4-multiplication.

3. Homomorphism on Fuzzy Translation and Fuzzy Multiplication on BV- Algebras [6]

Definition: 3.1[11, 12]

Let $F: A \longrightarrow$ A be an endomorphism and δ_{α}^{t} be a fuzzy –α-translation of δ in A. We define a new fuzzy set in A by $(\delta_{\alpha}^{t})_{\mathcal{F}}$ in A as $(\delta_{\alpha}^{t})_{\mathcal{F}}(x) = \delta_{\alpha}^{t}(\mathcal{F}(x)) = \delta[\mathcal{F}(x)] + \alpha,\ \forall x \in$ A.

Theorem: 3.2 [11, 12]

For any fuzzy BV –subalgebra δ of A, the fuzzy -α – translation $(\delta_\alpha^t)_\mathcal{F}$ of δ is a fuzzy BV – subalgebra of A.

Proof: Let $x, y \in A$, and $\alpha \in [0, T]$

Then $\delta\,(\,x * y\,) \geq \min\,\{\delta(x), \delta(y)\}$

Now $(\delta_\alpha^t)_\mathcal{F}(x * y) = \delta_\alpha^t\left(\mathcal{F}(x * y)\right)$

$$= \delta\left(\mathcal{F}(x * y)\right) + \alpha$$
$$= \delta\left(\mathcal{F}(x) * \mathcal{F}(y)\right) + \alpha$$
$$\geq \min\{\delta(\mathcal{F}(x)), \delta(\mathcal{F}(y))\} + \alpha$$
$$= \min\,\{\delta(\mathcal{F}(x)) + \alpha, \delta(\mathcal{F}(y)) + \alpha\}$$
$$(\delta_\alpha^t)_\mathcal{F}(x * y) \geq \min\{(\delta_\alpha^t)_\mathcal{F}(x),\ (\delta_\alpha^t)_\mathcal{F}(y)\}$$

This completes the proof.

Theorem: 3.3

Let $F: A \longrightarrow$ B be a homomorphism of a fuzzy BV-algebra A into a BV-algebra B and δ_α^t be a fuzzy α-translation of δ, then the pre-image of δ_α^t denoted by $F^{-1}(\delta_\alpha^t)$ is defined as $\{F^{-1}(\delta_\alpha^t)\}(x) = \delta_\alpha^t(F(x)), \forall x \in A$. If δ is a fuzzy BV-sub algebra of B, then $F^{-1}(\delta_\alpha^t)$ is a fuzzy BV-sub algebra of A.

Proof:

$$\{F^{-1}(\delta_\alpha^t)\}(x * y) = \delta_\alpha^t(F(x * y))$$
$$= \delta[F(x * y)] + \alpha$$
$$= \delta[F(x) * F(y)] + \alpha$$
$$\geq \min\{\delta F(x), \delta F(y)\} + \alpha$$
$$= \min\{\delta F(x) + \alpha, \delta F(y) + \alpha\}$$
$$= \min\{\delta_\alpha^t(F(x)), \delta_\alpha^t(F(y))\}$$
$$= \min\{\{F^{-1}(\delta_\alpha^t)\}(x), \{F^{-1}(\delta_\alpha^t)\}(y)\}$$
$$\{F^{-1}(\delta_\alpha^t)\}(x * y) \geq \min\{\{F^{-1}(\delta_\alpha^t)\}(x), \{F^{-1}(\delta_\alpha^t)\}(y)\}$$

Hence $F^{-1}(\delta_\alpha^t)$ is a fuzzy BV-sub algebra of A.

Theorem: 3.4 [11] Let $\mathcal{F}$ be an endomorphism of BV-algebra A. If δ is a fuzzy BV-ideal of A, then $(\delta_\alpha^t)_\mathcal{F}$ is BV-ideal of A.

Proof:

Let δ be a fuzzy BV-ideal of A.

Now, $(\delta_\alpha^t)_\mathcal{F}(0) = \delta_\alpha^t(F(0))$

$$= \delta[F(0)] + \alpha$$
$$\geq \delta[F(x)] + \alpha$$
$$\geq \delta_\alpha^t(F(x))$$
$$\geq (\delta_\alpha^t)_\mathcal{F}(x)$$
$$(\delta_\alpha^t)_\mathcal{F}(0) \geq (\delta_\alpha^t)_\mathcal{F}(x)$$

Let $u, v \in \Gamma$. Then $(\delta_\alpha^t)_\mathcal{F}(x) = \delta_\alpha^t(F(x))$

$$= \delta[F(x)] + \alpha$$
$$\geq \min\,\{(\delta_\alpha^t)_\mathcal{F}(y * x), (\delta_\alpha^t)_\mathcal{F}(y)\}$$
$$= \min\{\delta[F(y) * F(x)], \delta[F(y)\} + \alpha$$
$$= \min\{\delta[F(y * x)], \delta[F(y)\} + \alpha$$
$$= \min\{\delta[F(y * x)] + \alpha, \delta[F(y) + \alpha\}$$
$$= \min\{\delta_\alpha^t(F(y * x)), \delta_\alpha^t(F(y))\}$$
$$= \min\{(\delta_\alpha^t)_\mathcal{F}(y * x), (\delta_\alpha^t)_\mathcal{F}(y)\}$$
$$(\delta_\alpha^t)_\mathcal{F}(x) \geq \min\{(\delta_\alpha^t)_\mathcal{F}(y * x), (\delta_\alpha^t)_\mathcal{F}(y)\}$$

Hence $(\delta_\alpha^t)_\mathcal{F}$ is a fuzzy BV-ideal of A.

Theorem: 3.5

Let $F: A \longrightarrow$ B be an epimorphism of fuzzy BV-ideal. If $(\delta_\alpha^t)_\mathcal{F}$ is a fuzzy BV-ideal of A, thenA is a fuzzy B-ideal of B.

Proof: Let $(\delta_\alpha^t)_\mathcal{F}$ be a fuzzy BV-ideal of A and let $y \in$ B. Then there exits $x \in$ A such that $F(x) = y$.

Now, $\delta(0) + \alpha = \delta_\alpha^t(0)$

$$(\delta_\alpha^t)_\mathcal{F}(0) = \delta_\alpha^t(F(0))$$
$$= \delta[F(0)] + \alpha$$
$$\geq \delta[F(x)] + \alpha$$
$$\geq \delta_\alpha^t(F(x))$$
$$\geq (\delta_\alpha^t)_\mathcal{F}(x)$$
$$(\delta_\alpha^t)_\mathcal{F}(0) \geq (\delta_\alpha^t)_\mathcal{F}(x)$$
$$\delta(0) \geq \delta(u).$$

Let$y_1, y_2 \in$ B.

$$\delta(y_1) + \alpha = \delta_\alpha^t(y_1)$$
$$= \delta_\alpha^t[F(x_1)]$$
$$=(\delta_\alpha^t)_\mathcal{F}(x_1)$$
$$\geq \min\{(\delta_\alpha^t)_\mathcal{F}(x_2 * x_1), (\delta_\alpha^t)_\mathcal{F}(x_2)\}$$
$$= \min\{\delta_\alpha^t[F(x_2 * x_1)], \delta_\alpha^t[F(x_2)]\}$$
$$= \min\{\delta_\alpha^t[F(x_2) * F(x_1)], \delta_\alpha^t[F(x_2)]\}$$
$$= \min\{\delta_\alpha^t[y_2 * y_1], \delta_\alpha^t[y_2]\}$$
$$= \min\{\delta[y_2 * y_1] + \alpha, \delta[y_2] + \alpha\}$$
$$\delta(y_1) + \alpha \geq \min\{\delta[y_2 * y_1], \delta[y_2]\} + \alpha$$
$$\delta(y_1) \geq \min\{\delta[y_2 * y_1], \delta[y_2]\} \Longrightarrow \delta \text{ is a fuzzy BV-ideal of B.}$$

Theorem: 3.6

Let F be a homomorphism from A to B of fuzzy BV-subalgebra. If δ is a fuzzy BV-ideal of B then $(\delta_\alpha^t)_\mathcal{F}$ is a fuzzy BV-ideal of A.

Proof:

Let δ be a a fuzzy BV-ideal of B and let $x, y \in$ A.

Then, $(\delta_\alpha^t)_\mathcal{F}(0) = \delta_\alpha^t(F(0))$

$$= \delta[F(0)] + \alpha$$
$$\geq \delta[F(x)] + \alpha$$
$$\geq \delta_\alpha^t(F(x))$$
$$\geq (\delta_\alpha^t)_\mathcal{F}(x)$$
$$(\delta_\alpha^t)_\mathcal{F}(0) \geq (\delta_\alpha^t)_\mathcal{F}(x)$$

Also $(\delta_\alpha^t)_\mathcal{F}(x) = \delta_\alpha^t(F(x))$

$$= \delta[F(x)] + \alpha$$
$$\geq \min\{\delta[F(y) * F(x)], \delta[F(y)\} + \alpha$$
$$= \min\{\delta[F(y * x)], \delta[F(y)\} + \alpha$$
$$= \min\{\delta[F(y * x)] + \alpha, \delta[F(y) + \alpha\}$$
$$= \min\{\delta_\alpha^t(F(y * x)), \delta_\alpha^t(F(y))$$
$$= \min\{(\delta_\alpha^t)_\mathcal{F}(y * x), (\delta_\alpha^t)_\mathcal{F}(y)\}$$
$$(\delta_\alpha^t)_\mathcal{F}(x) \geq \min\{(\delta_\alpha^t)_\mathcal{F}(y * x), (\delta_\alpha^t)_\mathcal{F}(y)\}$$

Hence $(\delta_\alpha^t)_\mathcal{F}$ is a fuzzy BV-ideal of A.

Definition: 3.7

Let F be an endomorphism from A to B and δ_α^M be a fuzzy α-multiplication of δ in A. We define a new fuzzy set in A by $(\delta_\alpha^M)_F$ in A as $(\delta_\alpha^M)_F(x) = (\delta_\alpha^M)F(x) = \alpha\delta(F(x)), x \in A$.

Theorem: 3.8

Let F be an endomorphism of BV- algebra A. If δ is a fuzzy BV −ideal of A, then $(\delta_\alpha^M)_\mathcal{F}$ is also fuzzy BV −ideal of A.

Proof:

Let δ be s fuzzy BG-ideal of A.

Now, $(\delta_\alpha^M)_\mathcal{F}(0) = \delta_\alpha^M[F(0)]$
$= \alpha\delta[F(0)]$
$\geq \alpha\delta[F(x)]$
$= (\delta_\alpha^M)[F(x)]$
$= (\delta_\alpha^M)_\mathcal{F}(x)$

$(\delta_\alpha^M)_\mathcal{F}(0) \geq (\delta_\alpha^M)_\mathcal{F}(x)$

Let $x, y \in$ A. Then $(\delta_\alpha^M)_\mathcal{F}(x) = (\delta_\alpha^M)[F(x)]$
$= \alpha\delta[F(x)]$
$\geq \alpha \min\{\delta[F(y) * F(x)], \delta(F(y))\}$
$\geq \alpha \min\{\delta[F(y * x)], \delta(F(y))\}$
$= \min\{\alpha\delta[F(y * x)], \alpha\delta(F(y))\}$
$= \min\{\delta_\alpha^M [F(y * x)], \delta_\alpha^M(F(y))\}$
$= \min\{(\delta_\alpha^M)_\mathcal{F}(y * x), (\delta_\alpha^M)_\mathcal{F}(y)\}$

$\therefore (\delta_\alpha^M)_\mathcal{F}(x) \geq \min\{(\delta_\alpha^M)_\mathcal{F}(y * x), (\delta_\alpha^M)_\mathcal{F}(y)\}$

Hence $(\delta_\alpha^M)_\mathcal{F}$ is a fuzzy BV-ideal of A.

Theorem: 3.9

Let F be an epimorphism from BV-algebras A to B. If $(\delta_\alpha^M)_\mathcal{F}$ is a fuzzy BV-ideal of A then δ is a fuzzy BV-ideal of B.

Proof: Let $(\delta_\alpha^M)_\mathcal{F}$ be a fuzzy BV-ideal of A and let $y \in$ B. Then there exists $x \in$ A such that $F(x) = y$.

Now, $\alpha\,\delta(0) = \delta_\alpha^M(0)$
$= \delta_\alpha^M[F(0)]$
$= (\delta_\alpha^M)_\mathcal{F}(0)$
$\geq (\delta_\alpha^M)_\mathcal{F}(x)$
$= \delta_\alpha^M[F(x)]$
$= \alpha\,\delta[F(x)]$

$\Longrightarrow \alpha\,\delta(0) \geq \alpha\,\delta[F(x)] = \alpha\,\delta(y)$ $\quad\therefore\ \delta(0) \geq \delta(y)$

Let $y_1, y_2 \in$ B.

$\alpha\,\delta(y_1) = \delta_\alpha^M(y_1)$
$= \delta_\alpha^M[F(x_1)]$
$= (\delta_\alpha^M)_\mathcal{F}(x_1)$
$\geq \min\{(\delta_\alpha^M)_\mathcal{F}(x_2 * x_1), (\delta_\alpha^M)_\mathcal{F}(x_2)\}$
$= \min\{(\delta_\alpha^M)[F(x_2 * x_1)], (\delta_\alpha^M[F(x_2)]\}$
$= \min\{(\delta_\alpha^M)[F(x_2) * F(x_1)], (\delta_\alpha^M)[F(x_2)]\}$
$= \min\{(\delta_\alpha^M)(y_2 * y_1), (\delta_\alpha^M)(y_2)\}$
$= \min\{\alpha\,\delta(y_2 * y_1), \alpha\,\delta(y_2)\}$
$= \alpha\ \min\{\delta(y_2 * y_1), \delta(y_2)\}$

$\delta(y_1) \geq \min\{\delta(y_2 * y_1), \delta(y_2)\}$

$\Longrightarrow \delta$ is a fuzzy BV-ideal of B.

Theorem: 3.10

Let $F\colon$ A $\longrightarrow$ B be a homomorphism of BV-algebra. If δ is a fuzzy BV-ideal of B then $(\delta_\alpha^M)_\mathcal{F}$ is a fuzzy BV-ideal of A.

Proof: Let δ is a fuzzy BV-ideal of B and let $x, y \in$ A.

Then $(\delta_\alpha^M)_\mathcal{F}(0) = \delta_\alpha^M[F(0)]$
$= \alpha\,\delta[F(0)]$
$\geq \alpha\,\delta[F(x)]$
$= \delta_\alpha^M[F(x)]$
$= (\delta_\alpha^M)_\mathcal{F}(x)$

$\Longrightarrow (\delta_\alpha^M)_\mathcal{F}(0) \geq (\delta_\alpha^M)_\mathcal{F}(\delta_\alpha^M)$.

Also, $(\delta_\alpha^M)_\mathcal{F}(x) = (\delta_\alpha^M)F(x)$

$= \alpha\ \delta[F(x)]$
$\geq \alpha \min\{\delta[F(y) * F(x)], \delta[F(y)]\}$
$= \min\{\alpha\ \delta[F(y * x)], \alpha\ \delta[F(y)]\}$
$= \min\{\delta_\alpha^M[F(y * x)], \delta_\alpha^M[F(y)]\}$
$= \min\{(\delta_\alpha^M)_{\mathcal{F}}(y * x), (\delta_\alpha^M)_{\mathcal{F}}(y)\}$

$\therefore (\delta_\alpha^M)_{\mathcal{F}}(x) \geq \min\{(\delta_\alpha^M)_{\mathcal{F}}(y * x), (\delta_\alpha^M)_{\mathcal{F}}(y)\}$.

Hence $(\delta_\alpha^M)_{\mathcal{F}}$ is a fuzzy BV-subalgebra of A.

Theorem: 3.11

If δ is a fuzzy BV-ideal of A, then $(\delta_\alpha^M)_{\mathcal{F}}$ is also a fuzzy BV-subalgebra of A.

Proof:

Let δ be a fuzzy BG-ideal of Γ. Let $x, y \in A$.

Now, $(\delta_\alpha^M)_{\mathcal{F}}(x * y) = (\delta_\alpha^M)[F(x * y)]$
$= \alpha\ \delta[F(x * y)]$
$= \alpha\ \delta[F(x) * F(y)]$
$\geq \alpha \min\{\delta[F(x)], \delta[F(y)\}$
$= \min\{\alpha\ \delta[F(x)], \alpha\ \delta[F(y)\}$
$= \min\{(\delta_\alpha^M)[F(x)], (\delta_\alpha^M)[F(y)\}$
$= \min\{(\delta_\alpha^M)_{\mathcal{F}}(x), (\delta_\alpha^M)_{\mathcal{F}}(y)\}$

$\Longrightarrow (\delta_\alpha^M)_{\mathcal{F}}$ is also a fuzzy BV-subalgebra of A.

Theorem: 3.12

Let F: A $\longrightarrow$ B be a homomorphism of a fuzzy BV-algebra A into a BV-algebra B and δ_α^Mbe a fuzzy-α-translation of δ, then the pre-image of δ_α^M denoted by $F^{-1}(\delta_\alpha^M)$ is defined as $\{F^{-1}(\delta_\alpha^M)\}(x) = \delta_\alpha^M(F(x)), \forall x \in A$. If δ is a fuzzy BV-sub algebra of B, then $F^{-1}(\delta_\alpha^M)$ is a fuzzy BV-sub algebra of A.

Proof:

$\{F^{-1}(\delta_\alpha^M)\}(x * y) = \delta_\alpha^M(F(x * y))$
$= \alpha\ \delta[F(x * y)]$
$= \alpha\ \delta[F(x) * F(y)]$
$\geq \alpha \min\{\delta[F(x)], \delta[F(y)]\}$
$= \min\{\alpha\ \delta[F(x)], \alpha\ \delta[F(y)]\}$
$= \min\{\delta_\alpha^M(F(x)), \delta_\alpha^M(F(y))\}$
$= \min\{\{F^{-1}(\delta_\alpha^M)\}(x), \{F^{-1}(\delta_\alpha^M)\}(y)\}$

$\{F^{-1}(\delta_\alpha^M)\}(x * y) \geq \min\{\{F^{-1}(\delta_\alpha^M)\}(x), \{F^{-1}(\delta_\alpha^M)\}(y)\}$

Hence $F^{-1}(\delta_\alpha^M)$ is a fuzzy BV-sub algebra of A.

4. Conclusion

In this paper we have introduced the notion of Homomorphism on fuzzy translation and fuzzy multiplication of BV-algebras through fuzzy BV-ideals. BV-algebras are another generalization of BCK/BCI/d/Q-Algebras. This concept can further be generalized to intuitionistic fuzzy sets, n-generated fuzzy sets for new results in our future work.

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