

Integral Equations and it's Applications

Umesh Kumar ¹

¹ Assistant Professor, Mathematical Sciences

Abstract:- The theory of Integral Equations has been an active field of research for many years and is inextricably related with other areas of mathematics. Integral equation arise naturally in applications in many areas of mathematics, science and technology. The paper introduces integral equations and it's solutions.

Keywords: integral equations, kernals, resolvent kernals, separable kernals.

1. Introduction

Integral Equations have been essential tools for various areas of Applied Mathematics and they occur naturally in many Field of Science and Engineering.

An Integral Equation is a Functional Equation in which the unknown Functions appears under one or Several Integral sign.

In more precise an Integral Equation of the Form

$$\lambda \int k(x, s) y(s) ds = f(x)$$

is called a linear voltera integral equation of First kind.

and that of the form

$$y(x) - \lambda \int k(x, s) y(s) ds = f(x)$$

Where limits of integrations are from a

to x

is called a linear voltera integral equation of the second kind.

The Function $f(x)$

is called the free term while the function $k(x, s)$

is called the kernel.

λ

is a parameter, $y(s)$

is an unknown functions while $f(x)$

and $k(x, s)$

are given by functions which are square integrable on $[a, b]$

and in the domain $a \leq x \leq b, a \leq s \leq b$

Respectively.

If the kernel $k(x, s)$

is continuous then integral equation is said to be non-singular.

If the range of integration is infinite then equation is said to be singular.

Kernal of integration function,

Types of kernal

Kernels are the symmetric kernels ,separable kernels and iterated kernels

The solution of an integral equation of any type is to find the unknown functions $y(s)$

Satisfying that equation.

A function that shows up as an integrand in an integral interpretation for a solution of an integral equation and which regularly determines the alternatives completely is called resolvent kernal.

It is known that if the kernal $k(x, s)$

is a continuous function in the domain $R \{a \leq s \leq x \leq b\}$ and $f(x)$

is continuous in the interval $[a, b]$

Then integral equation has unique solution for any λ .

Integral equations appear in two ways when solving problems related to difference by reversing different components as when trying to describe occurrences with systems that incorporate summation of the integrations.

An integral integral equation's kernal is said to be symmetric if $k(x, s) = k(s, x)$

Rsolvent kernels examples related to symmetric kernels

$$x^2 + s^2, (x - s)^2, \sin(x, s)$$

be all representation of symmetric kernels .However xs^2

is not a representation of symmetric kernels.

Separable Kernels

Separable kernels are also known as degenerate kernels. It can be expressed as the sum of all finite number of terms in relative to each of the each of the products of the functions in x and s

Resolvent Kernels

Resolvent kernels can also be understood as reciprocal kernels.

2. Conclusion

The paper focuses on integral equation, kernels and solution to integral equations with applications.

References

- [1] Fox ,L. ,Goodwin, E.T",The numerical solution of non-singular linear integral equation Phil. Trans. R. Soc. 245(902),1953,501-534.
- [2] Heins,A.E.,Maccaamy,R.C.,,"A Function theoretic solution of certain integral equation (II) Quart. J. Math. 10 (1959),280-293
- [3] Green,C.",Integral Equations Method,Nelson,London(1969).
- [4] Bart,G.Warnock, R."Linear Integral Equation of third kindly,SIAM J.Math. Anal.(1973),609-622.
- [5] Brown ,S.N.,,"On an Integral Equation,Viscous flow theory,J. Eng.Math. II(1977),219-226.
- [6] Boersma,J.,,"Notes on an Integral Equation of Viscous flow theory, J. Eng. Math. 12(3),1978,327-334.
- [7] Elliot,D,Dow,M.I,"The Numerical Solution of Singular Integral EquationSIAM,J. Number. Anal. 16,1979,34-40.
- [8] Anderson,R.S.,Dehoog,F.R.,Luces,M.A,"The Application and Numerical Solution of an Integral Equation,Netherlands,Martin Nijhoff(1980).