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Inventory Optimization Model to Increase Profit Considering Parabolic Holding Cost

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Abstract:- In the dynamic corporate landscape, effective inventory management is crucial for sustaining competitiveness and boosting profits. Excess inventory remains a critical factor, as even well-organized enterprises may experience an oversupply of certain items within defined periods. In this paper, we optimize inventory level wih parabolic holding cost. Holding costs can fluctuate non-linearly in unpredictable situations such as sudden demand changes, supply chain disruptions, or economic fluctuations. Employing a parabolic holding cost enables a more accurate representation of these variations. We utilize the Fermatean fuzzy number to address the complexities of uncertain demand, Selling price and overhaul cost in inventory optimization. Furthermore, this paper offers empirical validation through a numerical illustration.

***Keywords**:* Fermatean fuzzy number, Economic Order Quantity, Parabolic Holding Cost, Profit Optimization.

1. Introduction

In recent decades, mathematical concepts have increasingly found applications in various real-world scenarios, particularly in inventory management. The degradation of items is a prevalent issue and holds significant importance in managing inventory effectively. Fuzzy inventory modeling emerges as a method closely aligned with real-world conditions. Many products, such as perishable goods like fruits and vegetables, pharmaceuticals, and electronics, experience deterioration over time, resulting in loss of value. This deterioration poses challenges for inventory systems, including shortages and potential loss of profit or goodwill. Additionally, factors such as market competition, changes in consumer preferences, or advancements in technology can further impact product sales. A study by [1] introduces a model for an inventory system characterized by exponential decay. Additionally, the holding cost plays a crucial role in determining the overall cost of inventory planning, consequently impacting total profitability. The holding cost can exhibit either linear or nonlinear behavior depending on time parameters. Several researchers have explored extensions of inventory planning models to account for time-dependent variable holding costs. An inventory planning problem for time-varying linear demand and parabolic holding cost with salvage value [4] Establishment of EOQ Model with Quadratic Time-Sensitive Demand and Parabolic-Time Linked Holding Cost with Salvage Value [16] Mathematical Modelling Of Inventory System With Parabolic Holding Cost, Weibull Distributed Deterioration Backlogging Under Pentagonal Fuzzy Parameters[3] For instance, [9] introduced a model incorporating variable holding cost rates within the Economic Order Quantity (EOQ) framework. Furthermore, [18] investigated economic order quantity models considering non-linear holding costs. In practical scenarios, a customer's daily requirements can fluctuate unpredictably. Assessing demand distribution can be challenging due to either insufficient historical data or an overwhelming amount of information. Real-life problems often exhibit high levels of complexity, characterized by uncertainty in various forms such as ambiguity, chance events, or limited knowledge. Many parameters defining these problems are expressed through linguistic statements. Therefore, considering the decision-maker's knowledge as fuzzy data can lead to improved outcomes. Fuzzy modeling provides a mathematical framework for capturing ambiguity and fuzziness in human-centric systems. Zadeh [5] introduced the concept of fuzzy sets (FS) to address the uncertainty in real-world problems. While FS theory proved useful, it struggled to adequately represent satisfaction and dissatisfaction in human judgments. To address this limitation, Atanassov [2] proposed intuitionistic fuzzy sets (IFS) as an extension of FS theory, offering enhanced capabilities for handling imprecise information. Yager [11,12] introduced Pythagorean fuzzy sets (PFS) in 2013, imposing a constraint that the sum of squared membership and non-membership degrees should not exceed 1. However, this constraint limited the applicability of PFS. To overcome this limitation, Senapati and Yager [13–15] introduced Fermatean fuzzy sets (FFS) as a more versatile model, where the sum of the cubes of membership and non-membership degrees should be less than or equal to 1. Further discussions and diverse applications of Fermatean fuzzy sets are explored in [7, 8]. This paper includes a numerical example to validate the effectiveness of the proposed model.

2. Objectives

***** To enhance profit optimization amidst unpredictable circumstances such as limited transportation services, adverse weather conditions, natural disasters, and lockdowns during the COVID-19 pandemic. This is achieved by incorporating time-dependent holding costs and using Fermatean fuzzy numbers for uncertain demand.

3. Methods

3.1. Assumptions

(i) Supply chain disruptions such as transportation delays, supplier shortages, or production interruptions, can occur unexpectedly and impact inventory availability.

(ii) Holding costs associated with inventory management vary over time in a manner resembling a parabolic curve, as indicated by the following expression (with x and y representing positive constants) $H_m = x + yt^2$.

(iii) Deteriorating products.

(iv) Demand fluctuates based on variations in the selling price of the product. Mathematically, this relationship is represented by the function $D_s = u - vS_m$, where u is a positive constant that varies with the selling price and $0 < u < vs$. Moreover, both u and v are considered to be triangular Fermatean fuzzy numbers.

(v) Indefinite planning time frame.

(vi) Shortages are not allowed.

(vii) Since all replenishment cycles are the same, only a standard planning cycle of duration L_T is considered. Consequently, the planning horizon spans the interval [0, L_T].

3.2 Notations

$Q_o -$ Quantity of orders placed.

$S_p -$ Percentage of excess occurrences that are discarded.

$f(d) -$ Density function.

$L_T -$ Length of the cycle.

$P_Q -$ Probability that a quantity Q_o will result in excess in the next cycle.

$C_D -$ Cost of disposing of goods from landfill.

$f(pd) -$ Probability density function.

$H_m -$ Maximum stock level per unit of time.

$H_c -$ Holding cost per unit of time for maintaining inventory.

$C_P -$ Cost of product distribution.

$S_m -$ Selling price per unit for items sold at maximum supply.

$O_f -$ Fixed cost associated with placing an order.

$S_s -$ Selling price per unit for excess products, where $S_s < S_m$.

T_n − Total number of new customers.

C_r − Overhaul cost.

C_a − Cost of acquiring a new customer.

3.3 Preliminaries

Fermatean Fuzzy Number

In reference [12], consider a universal set denoted by χ. A Fermatean fuzzy set (FFS), denoted as on χ is characterized by: $A = \{\langle z, \mu_A(z), \nu_A(z)\rangle : z\epsilon\chi\}$ where $\mu_A(z)\colon \chi \to [0,1]$represent the membership and non-membership degrees of each element z in the set A, respectively. Additionally, the constraints $0 \leq \left(\mu_A(z)\right)^3 + (\nu_A(z))^3 \leq 1$ hold for all z in χ. Furthermore, for each z in χ, the degree of hesitation within A is given by: $\pi_A(z) = \sqrt[3]{1 - \left(\left(\mu_A(z)\right)^3 + (\nu_A(z))^3\right)}$

Triangular Fermatean Fuzzy Number

A triangular Fermatean Fuzzy Number, denotes as $A = \{(a_l, a_m, a_r); q, s\}$, is a Fermatean Fuzzy Set characterized by its membership function μ_A, and non-membership function ν_A, as described below:

$$\mu_A(z) = \begin{cases} \dfrac{(z - a_l)q}{a_m - a_l}, & a_l \leq z < a_m \\ q, & z = a_m \\ \dfrac{(a_r - z)q}{a_r - a_m}, & a_m < z \leq a_r \\ 0, & z < a_l \text{ or } z > a_r \end{cases}$$

$$\mu_A(z) = \begin{cases} \dfrac{a_m - z + s(z - a_l)}{a_m - a_l}, & a_l \leq z < a_m \\ s, & z = a_m \\ \dfrac{[z - a_m + s(a_r - z)]}{a_r - a_m}, & a_m < z \leq a_r \\ 1, & z < a_l \text{ or } z > a_r \end{cases}$$

Here, the maximum μ_A and minimum ν_A value are represented as q and s respectively, ∋

$q \in [0,1], s \in [0,1]$, and $0 \leq q^3 + s^3 \leq 1$. Taking the value $q = 1$ and $s = 0$, the Triangular Fermatean Fuzzy Number A in the form $A = \{(a_l, a_m, a_r); (a_l', a_m, a_r')\}$ whose membership and nonmembership function is defined as

$$\mu_A(z) = \begin{cases} \dfrac{z - a_l}{a_m - a_l}, & a_l \leq z < a_m \\ 1, & z = a_m \\ \dfrac{a_r - z}{a_r - a_m}, & a_m < z \leq a_r \\ 0, & z < a_l \text{ or } z > a_r \end{cases}$$

$$\mu_A(z) = \begin{cases} \dfrac{a_m - z}{a_m - a_l'}, & a_l' \leq z < a_m \\ 0, & z = a_m \\ \dfrac{z - a_m}{a_r' - a_m}, & a_m < z \leq a_r' \\ 1, & z < a_l' \text{ or } z > a_r' \end{cases}$$

In this case, $a_l' \leq a_l \leq a_m \leq a_r \leq a_r'$.

A Triangular Fermatean Fuzzy number is denoted as $A = \{(a_l, a_m, a_r); (a_l', a_m, a_r')\}$ and its ranking function is defined as $\mathcal{R}(A) = \frac{(a_l +4 a_m + a_r)+(a_l' + 4a_m + a_r ')}{12}$.

3.4 *Model Formulation*

To accommodate more realistic scenarios, it's feasible to consider that several parameters, may vary within certain limits or bounds. By taking D_s, u, v, S_m, C_r *and* C_d as a Fermatean Fuzzy numbers. Then,

$u = \langle(u_1, u_2, u_3); (u_1', u_2, u_3')\rangle$, $v = \langle(v_1, v_2, v_3); (v_1', v_2, v_3')\rangle$,

$D_s = \langle(D_{s_1}, D_{s_2}, D_{s_3}); (D_{s_1}', D_{s_2}, D_{s_3}')\rangle$, $S_m = \langle(S_{m_1}, S_{m_2}, S_{m_3}); (S_{m_1}', S_{m_2}, S_{m_3}')\rangle$

$C_r = \langle(C_{r_1}, C_{r_2}, C_{r_3}); (C_{r_1}', C_{r_2}, C_{r_3}')\rangle$, $C_d = \langle(C_{d_1}, C_{d_2}, C_{d_3}); (C_{d_1}', C_{d_2}, C_{d_3}')\rangle$

By using deffuzzification method

The above parameters are defined as $D_s' = \frac{(D_{s_1}+4 D_{s_2}+ D_{s_3})+(D_{s_1}'+4 D_{s_2}+ D_{s_3}')}{12}$,

$v' = \frac{(v_1+4v_2+ v_3)+(v_1'+4v_2+ v_3')}{12}$, $u' = \frac{(u_1+4u_2+ u_3)+(u_1'+4u_2+ u_3')}{12}$,

$S_m' = \frac{(S_{m_1}+4S_{m_2}+ S_{m_3})+(S_{m_1}'+4S_{m_2}+ S_{m_3}')}{12}$, $C_d' = \frac{(C_{d_1}+4C_{d_2}+ C_{d_3})+(C_{d_1}'+4C_{d_2}+ C_{d_3}')}{12}$,

$C_r' = \frac{(C_{r_1}+4C_{r_2}+ C_{r_3})+(C_{r_1}'+4C_{r_2}+ C_{r_3}')}{12}$

The total principle of the inventory is presented as;

$$Tp(Q_o) = S_m'\left(1 - P_Q\right) D_s' + S_s P_Q D_s'$$

Inventory total cost is formulated as;

$$Tc(Q_o) = \frac{O_f D_s'}{Q_o} + \frac{T_n C_a D_s'}{Q_o} + C_r' P_Q D_s' + \frac{H_m Q_o}{2} + \frac{Q_o {P_Q}^2}{2}(H_c - H_m) + \frac{C_d' P_Q S_p D_s'}{Q_o} + \frac{D_s' C_p}{Q_o}$$

The total profit for the inventory= $Tp(Q_o) - Tc(Q_o)$

$$TP(Q_o) = \left(S_m'(1 - P_Q)D_s' + S_s P_Q D_s'\right) - \left(\frac{O_f D_s'}{Q_o} + \frac{T_n C_a D_s'}{Q_o} + C_r' P_Q D_s' + \frac{H_m Q_o}{2} + \frac{Q_o {P_Q}^2}{2}(H_c - H_m) + \frac{C_d' P_Q S_p D_s'}{Q_o} + \frac{D_s' C_p}{Q_o}\right)$$

As S_p is a stochastic variable, the expected total profit $ETP(Q_o)$ is expressed as,

$$ETP(Q_o) = \left(S_m'(1 - E(P_Q))D_s' + S_s E(P_Q)D_s'\right) - \left(\frac{O_f D_s'}{Q_o} + \frac{T_n C_a D_s'}{Q_o} + C_r' E(P_Q)D_s' + \frac{H_m Q_o}{2} + \frac{Q_o E({P_Q}^2)}{2}(H_c - H_m) + \frac{C_d' E(P_Q)E(S_p)D_s'}{Q_o} + \frac{D_s' C_p}{Q_o}\right)$$

The optimum quantity to order

$$Q_o = \sqrt{\frac{2D_s'(O_f + T_n C_a + C_d' E(P_Q)E(S_p) + C_p)}{H_m + E({P_Q}^2)(H_c - H_m)}}$$

3.5 Numerical Illustration

We have utilized Fermatean Fuzzy numbers for certain inputs to accommodate uncertain scenarios. The specific parameters are sourced from the references [10, 19].

$u = 50002.25, v = 49.75, O_f = 200, H_m = x + yt^2, x = 2, y = 3, H_c = 6, S_m = 49.75, S_s = 35, C_r = 4.75, C_a = 30, T_n = 2, C_d = 2.75, C_p = 300$

We assume that both S_p and P_Q follow a uniform distribution, characterized by the following density function:

$f(P_Q) = \begin{cases} 4 & 0 \le P_Q \le 0.25 \\ 0 & otherwise \end{cases}$ and $f(S_p) = \begin{cases} 20 & 0 \le P_Q \le 0.25 \\ 0 & otherwise \end{cases}$

$E(P_Q) = 0.1250, E({P_Q}^2) = 0.021 \; and \; E(S_p) = 0.1$

By using Python software for solving numerical problem we obtain the optimum order quantity $Q_o = 3239$ and the Expected total profit$ETP(Q_o) = 2253016$.

4. Results

In order to take into consideration parabolic holding costs and uncertainty in inventory management, this project effectively developed and implemented a hybrid inventory optimization model using fuzzy Fermatean numbers. The methodology worked well for maximising profits while minimising expenses, as shown by the numerical example. Businesses may make well-informed judgements and optimise their inventory levels by utilising this research's more accurate and useful method to inventory optimisation, which incorporates fuzzy logic. Enhancing profitability and reducing expenses across many different types of businesses, the proposed approach makes a substantial contribution to the context. To enhance the model's capabilities, future studies can expand by multi-product inventory optimization, Fuzzy stochastic inventory models and Dynamic pricing.

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