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Degree Sum Adjacency Energy and Energy of Some Special Graphs

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Abstract:- A Degree sum adjacency matrix of graph G is defined as the addition of degrees of vertices if both are adjacent and 0 otherwise. The Degree sum adjacency eigenvalues of the graph are the eigenvalues of the Degree sum adjacency matrix of the graph and an absolute sum of the Degree sum adjacency eigenvalues is called Degree sum adjacency energy of the graph. Splitting graph, m-Splitting graph, Shadow graph and m-Shadow graph are obtained by graph operation on the given graph. In this paper, we found a relation between Degree sum adjacency energy of the k-regular graph and Degree sum adjacency energy of its Splitting graph and m-Splitting graph. In addition, we found Degree sum adjacency energy of the Shadow graph and m-Shadow graph of the connected graph in the form of a Degree sum adjacency energy of the graph.

Keywords: Degree sum adjacency energy, Splitting graph, Shadow graph, m-Shadow graph, and m-Splitting graph.

1. Introduction

Let $u_1, u_2, \ldots, u_k$ are vertices of simple graph G and d_i represent degree of vertex u_i. The Degree sum matrix [9] of the graph G is defined by $DS(G) = [a_{ij}]$ where $a_{ij} = \begin{cases} d_i + d_j, & \text{if } u_i \neq u_j, \\ 0, & \text{if } u_i = u_j. \end{cases}$

By adding an adjacency condition in above matrix, we get another symmetric matrix for the graph G. The Degree sum adjacency matrix of the graph G is defined by

$$DSA(G) = [a_{ij}] \quad \text{where } a_{ij} = \begin{cases} d_i + d_j, & \text{if } u_i \text{ and } u_j \text{ are adjacent}, \\ 0, & \text{otherwise}. \end{cases}$$

The eigenvalues $\alpha_1, \alpha_2, \ldots \alpha_k$ of DSA(G) are called Degree sum adjacency eigenvalues of the graph G and $DSAE(G) = \sum_{l=1}^{k} |\alpha_l|$ is called Degree sum adjacency energy of the graph G.

The fundamental properties of the energy of graph are given in the [1,4]. Vaidya et al. [11] obtained the energy of $S'(G)$ of the graph G is $\sqrt{5}E(G)$ and the energy of $D_2(G)$ of the graph G is 2E(G), where E(G) represents the energy of G. Also, they derived energy of m-splitting and m-shadow graphs in the form of the energy of original graph [10]. Jahfar et al. [3] presented the Randić energy of the m-shadow graph, m-splitting graph, and obtain infinitely many pairs of Randić equienergetic graphs. Vaidya et al. [12] derived a relation between the Randić energy of original graph and its m-shadow and m-splitting graphs. Mirajkar et al. have studied Degree product adjacency energy of the graph in great detail. In [5] the same authors have discovered bounds for the degree product adjacency eigenvalues and the degree product adjacency energy of the graph. In [7] the same authors discovered the explicit formulas for the complement graph and the line graph of k-regular graph. In [6] the same authors have found the maximum eigenvalue for the regular graphs and studied equienergetic graphs for the nonco-spectra and co-spectra. Mirajkar et al. [8] validated Smith, hyperenergetic, and hypoenergetic graphs using degree product adjacency matrices for all graph classes.

The matrix $A \otimes B = \begin{bmatrix} a_{11}B & \cdots & a_{1n}B \\ \vdots & \ddots & \vdots \\ a_{m1}B & \cdots & a_{mn}B \end{bmatrix}_{mp\times nq}$ is called Kronecker product of matrices $A=[a_{ij}]_{m\times n}$ and $B=[b_{ij}]_{p\times q}$

.

Lemma 1.1. let α and β are an eigenvalues of the matrices $A=[a_{ij}]_{m\times n}$ and $B=[b_{ij}]_{p\times q}$ respectively corresponding to eigenvectors x and y then $\alpha\beta$ is an eigenvalue of $A\otimes B$ corresponding to eigenvector $x \otimes y$ [2].

2. Splitting graph and m-Splitting graph

In this section, we have found relation between Degree sum adjacency energy of the k-regular graph and Degree sum adjacency energy of its Splitting graph and m-Splitting graph. [11]

Definition 2.1. If $u_1, u_2, \ldots, u_p$ are vertices of graph G then Splitting graph $S'(G)$ of G constructed by adding $v_1, v_2, \ldots, v_p$ new vertices in G such that $N(v_i) = N(u_i)$ for each $i = 1, 2, \ldots, p$.

Theorem 2.1. Let G be a k-regular graph then $DSAE(S'(G)) = 2\sqrt{13}kE(G)$.

Proof. Let G be a k-regular graph with $u_1, u_2, \ldots, u_p$ vertices and the Splitting graph $S'(G)$ of G constructed by adding $v_1, v_2, \ldots, v_p$ new vertices in G such that $N(v_i) = N(u_i)$ for each $i = 1, 2, \ldots, p$.

Let $A = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ b_{p1} & b_{p2} & \cdots & b_{pp} \end{bmatrix}$ be an adjacency matrix of G and $\beta_1, \beta_2, \ldots, \beta_p$ are adjacency eigenvalues of G. So, Degree Sum Adjacency matrix of $S'(G)$ with respect to the sequence of vertices $u_1, u_2, \ldots, u_p, v_1, v_2, \ldots, v_p$ is

$$DSAM(S'(G)) = \begin{bmatrix} 4kA & 3kA \\ 3kA & 0 \end{bmatrix} = \begin{bmatrix} 4k & 3k \\ 3k & 0 \end{bmatrix} \otimes A$$

where A is an adjacency matrix of G.

Hence by lemma 1.1, the Degree Sum Adjacency eigenvalues of $S'(G)$ are $\left(2k \pm \sqrt{13}k\right)\beta_j$ for $j = 1, 2, \ldots, p$. where $\left(2k \pm \sqrt{13}k\right)$ are eigenvalues of the matrix $\begin{bmatrix} 4k & 3k \\ 3k & 0 \end{bmatrix}$ and $\beta_1, \beta_2, \ldots, \beta_p$ are adjacency eigenvalues of G.

$$\begin{aligned} DSAE(S'(G)) &= \sum_{j=1}^{p} \left| \left(2k \pm \sqrt{13}k\right)\beta_j \right| \\ &= \sum_{j=1}^{p} \left(\sqrt{13} + 2 + \sqrt{13} - 2\right) k \left|\beta_j\right| \\ &= 2\sqrt{13}k \sum_{j=1}^{p} \left|\beta_j\right| \\ &= 2\sqrt{13}kE(G). \end{aligned}$$

Corollary 2.1. Let G be a k-regular graph then $DSAE(S'(G)) = \sqrt{13}kDSAE(G)$.

Proof. If A is an adjacency matrix of the k-regular graph G then $DSAM(G) = 2kA$

Hence, $DSAE(G) = 2kE(G)$

By the above theorem, $DSAE(S'(G)) = 2\sqrt{13}kE(G) = \sqrt{13}DSAE(G)$.

Example 2.1. Consider complete graph K_3. $S'(K_3)$ is a Splitting graph of the K_3.

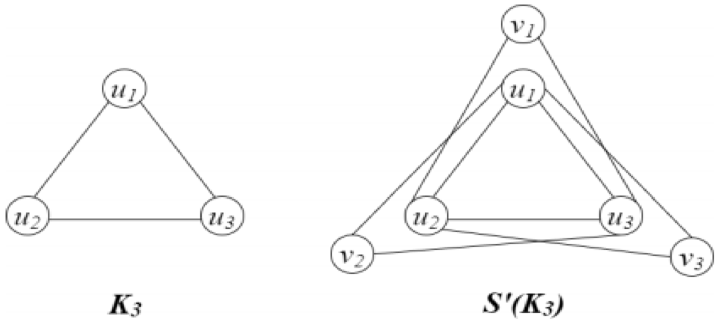

Figure 1

We know that $E(K_3) = 4$ and $k = 2$.

So, that $DSAE(K_3) = 2(2)E(K_3) = 16$.

$$DSAM(S'(K_3)) = \begin{bmatrix} 0 & 8 & 8 & 0 & 6 & 6 \\ 8 & 0 & 8 & 6 & 0 & 6 \\ 8 & 8 & 0 & 6 & 6 & 0 \\ 0 & 6 & 6 & 0 & 0 & 0 \\ 6 & 0 & 6 & 0 & 0 & 0 \\ 6 & 6 & 0 & 0 & 0 & 0 \end{bmatrix}$$

So, the spectrum of the matrix $DSAM(S'(K_3))$ is $\begin{pmatrix} 8+4\sqrt{13} & 8-4\sqrt{13} & -4+2\sqrt{13} & -4-2\sqrt{13} \\ 1 & 1 & 2 & 2 \end{pmatrix}$

Hence, $DSAE(S'(K_3)) = 16\sqrt{13} = 2\sqrt{13}kE(K_3) = \sqrt{13}DSAE(K_3)$.

Definition 2.2. The m-Splitting graph $Spl_m(G)$ obtained by adding $v_1, v_2, ..., v_m$ new m vertices for each vertex u of G such that $N(v_i) = N(u)$ for each $i = 1, 2, ..., m$. [3]

Theorem 2.2. Let G be a k-regular graph then $DSAE(Spl_m(G)) = \left(\sqrt{m^3 + 5m^2 + 6m + 1}\right)2kE(G)$.

Proof. Let G be a k-regular graph with $u_1, u_2, ..., u_p$ vertices and the m-Splitting graph $Spl_m(G)$ obtained by adding $v_j^1, v_j^2, ..., v_j^m$ new m vertices for each vertex u_j of G such that $N(v_j^i) = N(u_j)$ for each $i = 1, 2, ..., m$.

Let A be an adjacency matrix of G and $\beta_1, \beta_2, ..., \beta_p$ are adjacency eigenvalues of G.

The Degree Sum Adjacency matrix of $Spl_m(G)$ is

$$DSAM\left(Spl_m(G)\right)=\begin{bmatrix} 2(m+1)kA & (m+2)kA & \cdots & (m+2)kA \\ (m+2)kA & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ (m+2)kA & 0 & \cdots & 0 \end{bmatrix}_{p(m+1)\times p(m+1)}$$

$$=\begin{bmatrix} 2(m+1)k & (m+2)k & \cdots & (m+2)k \\ (m+2)k & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ (m+2)k & 0 & \cdots & 0 \end{bmatrix}_{(m+1)\times(m+1)} \otimes A \qquad (2.1)$$

Let $B=\begin{bmatrix} 2(m+1)k & (m+2)k & \cdots & (m+2)k \\ (m+2)k & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ (m+2)k & 0 & \cdots & 0 \end{bmatrix}_{(m+1)x(m+1)}$

Rank of matrix B is 2. So, B have eigenvalues 0 with multiplicity m-1 and two non-zero eigenvalues with multiplicity 1. Let x and y are two non-zero eigenvalues of B. Hence,

$$x + y = tr(B) = 2(m+1)k \qquad (2.2)$$

Also, we know that $x^2+y^2=tr(B^2)=4(m+1)^2k^2+2m(m+2)^2k^2$ (2.3)

where $B^2=\begin{bmatrix} 4(m+1)^2k^2+m(m+2)^2k^2 & 2(m+1)(m+2)k^2 & \cdots & 2(m+1)(m+2)k^2 \\ 2(m+1)(m+2)k^2 & (m+2)^2k^2 & \cdots & (m+2)^2k^2 \\ \vdots & \vdots & \ddots & \vdots \\ 2(m+1)(m+2)k^2 & (m+2)^2k^2 & \cdots & (m+2)^2k^2 \end{bmatrix}.$

From the equations (2.2) and (2.3), we got

$$x=\frac{2(m+1)k+\sqrt{4(m+1)^2k^2+4m(m+2)^2k^2}}{2} \quad \text{and} \quad y=\frac{2(m+1)k-\sqrt{4(m+1)^2k^2+4m(m+2)^2k^2}}{2}$$

From the equation (2.1)

$$DSAE\left(Spl_m(G)\right)=\sum_{i=1}^{p}\left(|x|+|y|\right)|\beta_i|$$

$$=\sum_{i=1}^{p}\left(\frac{2(m+1)k+\sqrt{4(m+1)^2k^2+4m(m+2)^2k^2}}{2}+\frac{\sqrt{4(m+1)^2k^2+4m(m+2)^2k^2}-2(m+1)k}{2}\right)|\beta_i|$$

$$=\sum_{i=1}^{p}\left(\sqrt{4(m+1)^2k^2+4m(m+2)^2k^2}\right)|\beta_i|$$

$$=2k\left(\sqrt{m^3+5m^2+6m+1}\right)\sum_{i=1}^{p}|\beta_i|$$

$$=2k\left(\sqrt{m^3+5m^2+6m+1}\right)E(G)$$

Corollary 2.2. Let G be a k-regular graph then $DSAE\left(Spl_m(G)\right)=\left(\sqrt{m^3+5m^2+6m+1}\right)DSAE(G)$.

Proof. As we know that $DSAE(G)=2kE(G)$ where G is k-regular graph.

By the above theorem,

$$DSAE(Spl_m(G)) = \left(\sqrt{m^3+5m^2+6m+1}\right)2kE(G) = \left(\sqrt{m^3+5m^2+6m+1}\right)DSAE(G).$$

Example 2.2. Consider complete graph K_3. $Spl_3(K_3)$ is a 3-Splitting graph of the K_3.

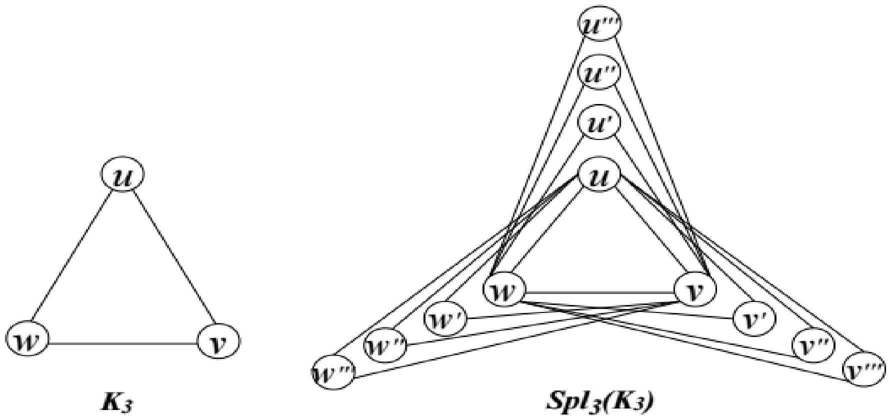

Figure 2

We know that $E(K_3) = 4$ and $k = 2$.

So, that $DSAE(K_3) = 2(2)E(K_3) = 16.$

$$DSAM(Spl_3(K_3)) = \begin{pmatrix} 0 & 16 & 16 & 0 & 10 & 10 & 0 & 10 & 10 & 0 & 10 & 10 \\ 16 & 0 & 16 & 10 & 0 & 10 & 10 & 0 & 10 & 10 & 0 & 10 \\ 16 & 16 & 0 & 10 & 10 & 0 & 10 & 10 & 0 & 10 & 10 & 0 \\ 0 & 10 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 10 & 0 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 10 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 10 & 0 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 10 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 10 & 0 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 10 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

So, the spectrum of the matrix $DSAM(Spl_3(K_3))$ is $\begin{pmatrix} 0 & 8+4\sqrt{91} & 8-4\sqrt{91} & -8+2\sqrt{91} & -8-2\sqrt{91} \\ 6 & 1 & 1 & 2 & 2 \end{pmatrix}$

Hence, $DSAE(Spl_3(K_3)) = 16\sqrt{91} = 2\sqrt{91}kE(K_3) = \sqrt{91}DSAE(K_3)$

3. Shadow graph and m-Shadow graph

We have found Degree sum adjacency energy of the Shadow graph and m-Shadow graph of connected graph in the form of a Degree sum adjacency energy of the graph in this section.

Definition 3.1. The Shadow graph $D_2(H)$ of a connected graph H is obtained by taking H_1 and H_2 two copies of H such that join each vertex v_1 in H_1 to the neighbour of the corresponding vertex v_2 in H_2. [11]

Theorem 3.1. If G is a connected graph, then $DSAE(D_2(G)) = 4DSAE(G)$.

Proof. Let $D_2(G)$ be a Shadow graph of a connected graph G.

Let $D = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1p} \\ d_{21} & d_{22} & \cdots & d_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ d_{p1} & d_{p2} & \cdots & d_{pp} \end{bmatrix}$ be a Degree Sum Adjacency matrix of G and $\beta_1, \beta_2, ..., \beta_p$ are Degree Sum Adjacency eigenvalues of G.

So, Degree Sum Adjacency matrix of $D_2(G)$ is

$$DSAM(D_2(G)) = \begin{bmatrix} 2D & 2D \\ 2D & 2D \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \otimes D$$

where D is a Degree Sum Adjacency matrix of the graph G.

Hence by lemma 1.1, the Degree Sum Adjacency eigenvalues of $D_2(G)$ are 0 and $4\beta_j$ for $j = 1, 2, ..., p$ where 0 and 4 are eigenvalues of the matrix $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ and $\beta_1, \beta_2, ..., \beta_p$ are Degree Sum Adjacency eigenvalues of G.

$$DSAE(D_2(G)) = \sum_{i=1}^{p} |4\beta_i|$$
$$= 4\sum_{i=1}^{p} |\beta_i|$$
$$= 4DSAE(G).$$

Example 3.1. Consider complete graph K_3. $D_2(K_3)$ is a Shadow graph of the K_3

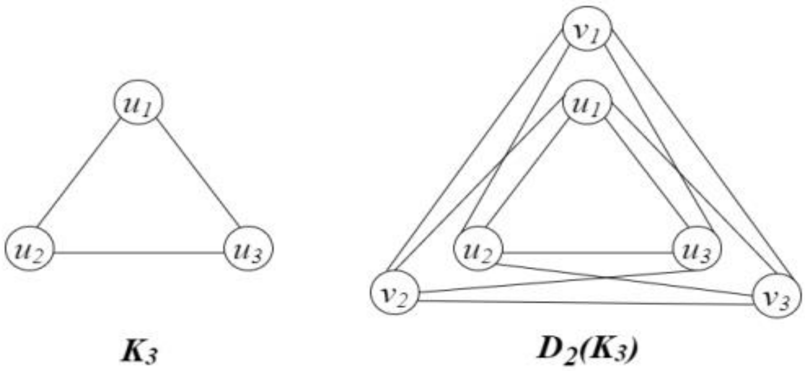

Figure 3

We know that $DSAE(K_3) = 16$.

$$DSAM(D_2(K_3)) = \begin{pmatrix} 0 & 8 & 8 & 0 & 8 & 8 \\ 8 & 0 & 8 & 8 & 0 & 8 \\ 8 & 8 & 0 & 8 & 8 & 0 \\ 0 & 8 & 8 & 0 & 8 & 8 \\ 8 & 0 & 8 & 8 & 0 & 8 \\ 8 & 8 & 0 & 8 & 8 & 0 \end{pmatrix}$$

So, the spectrum of the matrix $DSAM\left(D_2\left(K_3\right)\right)$ is $\begin{pmatrix} 0 & 32 & -16 \\ 3 & 1 & 2 \end{pmatrix}$

Hence, $DSAE\left(D_2\left(K_3\right)\right) = 64 = 4DSAE\left(K_3\right)$.

Definition 3.2. Let H be a connected graph. The m-Shadow graph $D_m(H)$ of H is obtained by taking $H_1, H_2, ..., H_m$, m copies of H such that join each vertex u_i in H_i to the neighbour of the corresponding vertex u_j in H_j, $1 \le i, j \le m$. [3]

Theorem 3.2. If G is a connected graph, then $DSAE(D_m(G)) = m^2 DSAE(G)$.

Proof. Let $D_m(G)$ be a m-Shadow graph of a connected graph G.

Let $D = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1p} \\ d_{21} & d_{22} & \cdots & d_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ d_{p1} & d_{p2} & \cdots & d_{pp} \end{bmatrix}$ be a Degree Sum Adjacency matrix of G and $\beta_1, \beta_2, ..., \beta_p$ are Degree Sum Adjacency eigenvalues of G.

So, Degree Sum Adjacency matrix of $D_m(G)$ is

$$DSAM\left(D_m(G)\right) = \begin{bmatrix} mD & mD & \cdots & mD \\ mD & mD & \cdots & mD \\ \vdots & \vdots & \ddots & \vdots \\ mD & mD & \cdots & mD \end{bmatrix} = \begin{bmatrix} m & m & \cdots & m \\ m & m & \cdots & m \\ \vdots & \vdots & \ddots & \vdots \\ m & m & \cdots & m \end{bmatrix} \otimes D$$

where D is a Degree Sum Adjacency matrix of the graph G.

Hence by lemma 1.1, the Degree Sum Adjacency eigenvalues of $D_m(G)$ are 0 with multiplicity (m-1)p and $m^2\beta_j$ for $j = 1, 2, ..., p$ where 0 with (m-1)p multiplicity and m^2 are eigenvalues of the matrix $\begin{bmatrix} m & m & \cdots & m \\ m & m & \cdots & m \\ \vdots & \vdots & \ddots & \vdots \\ m & m & \cdots & m \end{bmatrix}$ and $\beta_1, \beta_2, ..., \beta_p$ are Degree Sum Adjacency eigenvalues of G.

$$DSAE\left(D_m(G)\right) = \sum_{i=1}^{p} \left| m^2 \beta_i \right| = m^2 \sum_{i=1}^{p} \left| \beta_i \right| = m^2 DSAE(G).$$

Example 3.2. Consider complete graph K_3. $D_3(K_3)$ is a 3-Shadow graph of the K_3

We know that $DSAE\left(K_3\right) = 16$ and m = 3.

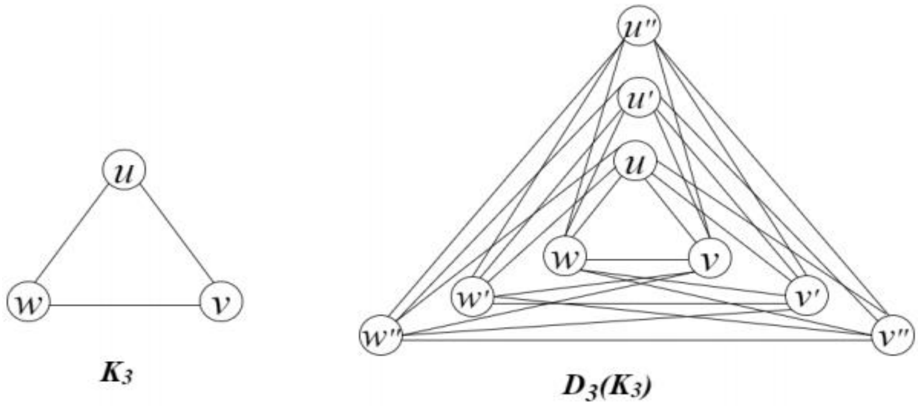

Figure 4

$$\mathrm{DSAM}\left(\mathrm{D}_3\left(\mathrm{K}_3\right)\right)=\begin{pmatrix} 0 & 12 & 12 & 0 & 12 & 12 & 0 & 12 & 12 \\ 12 & 0 & 12 & 12 & 0 & 12 & 12 & 0 & 12 \\ 12 & 12 & 0 & 12 & 12 & 0 & 12 & 12 & 0 \\ 0 & 12 & 12 & 0 & 12 & 12 & 0 & 12 & 12 \\ 12 & 0 & 12 & 12 & 0 & 12 & 12 & 0 & 12 \\ 12 & 12 & 0 & 12 & 12 & 0 & 12 & 12 & 0 \\ 0 & 12 & 12 & 0 & 12 & 12 & 0 & 12 & 12 \\ 12 & 0 & 12 & 12 & 0 & 12 & 12 & 0 & 12 \\ 12 & 12 & 0 & 12 & 12 & 0 & 12 & 12 & 0 \end{pmatrix}$$

So, the spectrum of the matrix $\mathrm{DSAM}\left(\mathrm{D}_3\left(\mathrm{K}_3\right)\right)$ is $\begin{pmatrix} 0 & 72 & -36 \\ 6 & 1 & 2 \end{pmatrix}$.

Hence, $\mathrm{DSAE}\left(\mathrm{D}_3\left(\mathrm{K}_3\right)\right)=144=(3)^2\,\mathrm{DSAE}\left(\mathrm{K}_3\right)$.

4. Conclusion and Future scope

Many results about the relationship between the various types of energy of the original graph and energy of its Splitting, m-Splitting, shadow, and m-Shadow graphs are derived. This paper aims to found the relationship between the Degree Sum Adjacency energy of the graph and the Degree Sum Adjacency energy of its Splitting, m-Splitting, shadow, and m-Shadow graphs. Using distinct graph operations, one might obtain alternative outcomes for the Degree Sum Adjacency energy.

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