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Analyzing Performance Measure of Fuzzy Queuing in the Postal Vote System

M.Shahul Hameed[1], V. Kamal Nasir [2], S.M.A Shahul Hameed [3,]
[1] *Assistant Professor, Department of Mathematics,Government Arts college , Nandanam, Chennai-35*
[2] *Associate Professor,Department of Mathematics,The New College, Chennai-14*
[3] *Assistant Professor, Department of Mathematics, Ramakrishna Mission Vivekananda College, Mylapore, Chennai-04*

Abstract: In this article we have emphasized the L.R method to analyse the queuing model for the evaluation of system performance measures using this technique we derived a novel approach for FM/FM/1, a simple fuzzy queue with a Trapezoidal fuzzy number, and compared the result with α cut method with suitable numerical examples.

Keywords: Trapezoidal fuzzy number, L-R type Trapezoidal fuzzy number, α cut, Postal vote.

1. Introduction:

In India, school and college teachers and many more Government officials have worked as polling officers on the day of the election, they are not able to cast their vote in their respective booths. For this, an alternative method was introduced by the election commission of India a postal ballot through which officials involved in election duty exercise their rights of casting their votes.98% of the eligible Indian citizens cast their vote in the booth, while the 2% of the government employees who are involved in the election duty cast their vote through postal ballot. People who employ themselves in election duties can vote through postal ballot. The process of casting their votes through the postal ballot is of filling out two forms on one form enclosed with the list of candidates who stand for MP/MLA for a particular constituency named as Ballot Paper where he/she selects his preferred candidate with the mark seal/tick with a pen. Later one consists of a Declaration form. But with this process, there are more disadvantages than advantages as it has been listed as its

Demerits of postal vote:

1. Data missing: Preparation of the list for postal votes, the data of polling officers may be excluded or deleted from the list.

2. Postal delay: It may not reach the polling officer's address and counting station at the correct time.

3. Damage of Postal votes: (i) Voters sign or fold the ballot paper incorrectly.

(ii)Voters put the forms in the wrong envelope.

(iii) Tick mark impression falls on the other candidate.

(iv) Polling officer’s signature or date of birth not matching.

(v) Addresses not filling accurately.

(vi)Marking away from the space allocated for the choice of candidate will result in rejection of the vote.

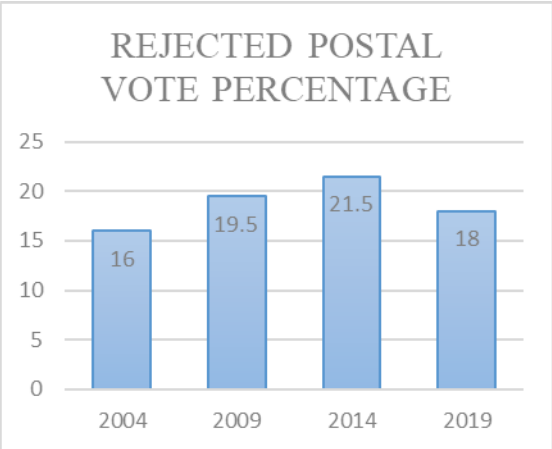
On the day of counting, many postal votes are invalid because of above said points. By this factor, a candidate may fall with a narrow margin.

The following tables give the results of candidates who lost by a narrow margin.

Table No.1: Candidates win with a narrow margin

S.I	Election	Constituency	Vote margin(Win)	Reason
1	1962 Parliament Election	-	9	Two M.Ps lost
2	2004-Karnataka Assembly	-	1	
3	2008-Rajasthan	-	1	
4	2016-Tamilnadu	Radhapuram	49	203 postal votes rejected
5	2017-Gujarat Assembly election	Dhokla	327	429 postal votes rejected
6	2021-Tamilnadu Assembly	T.Nagar	137	
7	2021-Tamilnadu Assembly	Modakkurichi	281	
8	2021-Tamilnadu Assembly	Thenkasi	281	
9	2023-Karnataka Assembly	Jayanagar	16	Rejected 177 postal ballot votes
10	2023-Karnataka Assembly	Sringeri	201	
11	2023-Karnataka Assembly	Malur	248	
12	2024-Parliament election	Mumbai North West	48	

Figure 1: Rejected postal vote for Parliament election in India



Though only 2% of the votes are cast through postal ballot, it plays a deciding factor in the outcomes of the final result in many constituencies in the Indian elections. In the earlier election, the constituency of MLA or MP won the seats by a very meager amount of votes. So, the votes of the learned scholars play a vital role in changing the future of India. So in order not to waste a single vote, we wanted to develop a paper in such a manner that if these government employees are all allowed to cast their vote through the EVM machine know spill of blood from the lioness mouth will be wasted. We develop the paper in the context of allowing government employee to cast their vote in an EVM machine. Our problem was studied, by using fuzzy Queueing theory techniques to find the service rate, waiting time, and much more.

So, the Government will waste no money on Postal Ballot like printing of papers and delivery charges.

2. Algorithm:

The following assumptions are made

(i) The voter's ID is linked with Aadhar so that his details are available through Online mode.

(ii) Polling Official's duties are not allotted in the same constituency(He or She is not eligible for EDC)

(iii) Polling Officers on duty for this purpose are allowed to cast their vote in the same EVM machine

Step 1:

Preparation of List: We list out officers from school, colleges and the government sectors who works as polling officers.

Step 2:

Verify their voter ID linked with Aadhar using Biometric.

Step 3:

A polling slip was received with his details.

Step 4:

Slips are handed to polling personnel.

Step 5:

Allowed to cast their vote in allocated booth.

3. Literature review

Fuzzy queues are introduced by R.J.Li and E.S Lee,[1] in the year 1989 . After that, D.S.Negi, E.S Lee[3] used this technique in the simulation concept. Later, this technique was analyzed by J.P. Mukeba Kanyinda, R. Mabela Makengo [4] who used L-R Method in Fuzzy Queueing Performance. S.Sanmuga Sundaram et. al[5] discussed a study on single server Queuing Model using DSW algorithm with Heptagonal and Octagonal Fuzzy Number. M. Shanmugasundari and S. Aarthi[6] who gave a different approach to solve fuzzy queuing. Recently, S.Vijaya discussed the Performance Measure of Fuzzy Queueing Problem with Trapezoidal Fuzzy Numbers by L.R Method [11]. Many of the authors are analyzed to compute system performance measures using the alpha-cuts method. No one has discussed the postal vote system mathematically. However, we discussed the alternate solution for the postal vote through the fuzzy queue. In this paper, we used L.R fuzzy numbers and trapezoidal fuzzy numbers with alpha-cut solutions.

4.Preliminaries

A. Fuzzy set:

A fuzzy set $\tilde{A}$ on R, which satisfies the following three properties is called a fuzzy number

(a) $\tilde{A}$ is convex, which means that there exists $x, y \in R$ and $\lambda \epsilon$ [0,1] such that
$\mu_{\tilde{A}}(\lambda x + (1-\lambda)y) \geq \min\{\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y)\}$
(b) $\tilde{A}$ is normal,which means that there exists an $x \in R$ such that $\mu_{\tilde{A}}(x) = 1$.
(c) $\tilde{A}$ is piecewise continuous.

B. Trapezoidal Fuzzy Number:

A trapezoidal Fuzzy number $\tilde{A}(x)$ is represented by $\tilde{A}(a_1, a_2, a_3, a_4)$ with the membership function

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a_1 \\ \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ 1, & a_2 \leq x \leq a_3 \\ \frac{a_4-x}{a_4-a_3}, & a_3 \leq x \leq a_4 \\ 0, & x > a_4 \end{cases}$$

C. L-R Type trapezoidal fuzzy number:

A fuzzy number $\tilde{B} = (m, n, a, b)_{LR}$ is said to be L-R Type trapezoidal fuzzy numbers if its membership function is given by

$$\mu_{\tilde{B}}(x) = \begin{cases} L\left(\frac{m-x}{a}\right); if\ x \le m, a \ge 0 \\ R\left(\frac{x-n}{b}\right);\ \ if\ x \ge n, b \ge 0 \\ \quad 1, \quad otherwise \end{cases}$$

D. Arithmetic operations on Trapezoidal Fuzzy Numbers:

Let the two triangular fuzzy numbers be $\tilde{X}$ =(a_1, a_2, a_3, a_4) and $\tilde{Y}$ =(b_1, b_2, b_3, b_4) and then the arithmetic operations on trapezoidal fuzzy numbers be given as follows:

(i) $\tilde{X}$ +$\tilde{Y}$ $= (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$

(ii) $\tilde{X}$ -$\tilde{Y}$ $= (a_1 - b_2, a_2 - b_1, a_3 + b_4, a_4 + b_3)$

(iii) $\tilde{X}\ \tilde{Y} = (a_1b_1, a_2b_2, a_1b_3 + a_3b_1, a_2b_4 + a_4b_2)$

(iv) $\frac{\tilde{X}}{\tilde{Y}} \quad = (\frac{a_1}{b_2}, \frac{a_2}{b_1}, \frac{a_3}{b_4}, \frac{a_4}{b_3})$

Expected "Number of Customers" and their "Waiting Time" in a Queue using Fuzzy Environment: FM/FM/1: $\infty/FCFS$ Model

If FM/FM/1 is a simple fuzzy queue, whose arrival rate λ and service rate μ respectively.

The system performance measures namely average number of customers and waiting time in

the system given by the following

1. $L_s = \frac{\lambda}{\mu-\lambda}$
2. $W_s = \frac{1}{\mu-\lambda}$
3. $L_q = \frac{\lambda^2}{\mu(\mu-\lambda)}$
4. $W_q = \frac{\lambda}{\mu(\mu-\lambda)}$

5.L-R Method

Theorem:

If FM/FM/1 is a simple fuzzy queue, whose arrival rate and service rate are taken as positive

Trapezoidal fuzzy numbers $\tilde{\lambda}$=(a_1, a_2, a_3, a_4) and $\tilde{\mu}$==(b_1, b_2, b_3, b_4) respectively with $a_4 < b_1$, then the expected number of customers

$$\tilde{N} = (\frac{a_2}{b_3-a_2}, \frac{a_3}{b_2-a_3}, \frac{x}{v+x}, \frac{y}{u+y})$$

Waiting time of customer in the system

$$\tilde{T_s} = \frac{1}{(b_2-a_3, b_3-a_2, u+x, v+y)_{LR}}$$

Proof:

Let us assume that $\tilde{\lambda}$, $\tilde{\mu} > 0$.

We assume these number are to be vague as it is in fuzzy environment

We also choose these number to be vague are represented in the fuzzy environment.

Let $\tilde{N}$ be the expected number of customers of this queue at the steady state.

L-R fuzzy values can be represented in the form of $\tilde{A}(a, b, c, d) = =< b, c, b - a, d - c >$

L-R representation of arrival and service rate represented as

$$\tilde{\lambda}(a_1, a_2, a_3, a_4) = (a_2, a_3, a_2 - a_1, a_4 - a_3)$$

a_2, a_3 are left and right spreads.

$$\tilde{\mu}(b_1, b_2, b_3, b_4) = (b_2, b_3, b_2 - b_1, b_4 - b_3)$$

b_2, b_3 are left and right spreads.

Average number of customer in the system

$$\tilde{N} = \frac{\tilde{\lambda}}{\tilde{\mu}-\tilde{\lambda}} \quad (1)$$

Let $x = a_2 - a_1, y = a_4 - a_3$ then $\tilde{\lambda}=(a_2, a_3, x, y)$

$u = b_2 - b_1, v = b_4 - b_3$ then $\tilde{\mu}=(b_2, b_3, u, v)$

$$\tilde{N} = \frac{(a_2,a_3,x,y)_{LR}}{(b_2,b_3,u,v)_{LR}-(a_2,a_3,x,y)_{LR}}$$

$$\tilde{N} = \frac{(a_2, a_3, x, y)_{LR}}{(b_2 - a_3, b_3 - a_2, u + y, v + x)_{LR}}$$

$$\tilde{N} = \left(\frac{a_2}{b_3-a_2}, \frac{a_3}{b_2-a_3}, \frac{x}{v+x}, \frac{y}{u+y}\right) \quad (2)$$

Waiting time of customer in the system

$$T_s = \frac{1}{\tilde{\mu}-\tilde{\lambda}} \quad (3)$$

$$= \frac{1}{(b_2,b_3,u,v)-(a_2,a_3,x,y)}$$

$$T_s = \frac{1}{(b_2-a_3,b_3-a_2,u+x,v+y)_{LR}} \quad (4)$$

L-R method solution

The data for the Chennai North region for the postal vote was collected from the website of ECI and amounted to 3161 postal votes. For our methodology, these votes cannot be polled in a single booth, we formulate by subdividing from 5 to 7 booths, with a maximum of 11 hours on the day of the election. Further, the value of arrival and service rate are taken in a Trapezoidal fuzzy number, the data are in approximated nature which is not of clear form with the presence of vagueness carried through the fuzzy environment. This is shown in the table below.

Table No.3: Chennai -North parliament constituency in 2019 election-postal vote and its performance measure by L.R method

Parliament	Postal Vote	No. of Booths	Single Booth	$\tilde{\lambda}=(a_1, a_2, a_3, a_4)$	$\tilde{\mu}=(b_1, b_2, b_3, b_4)$	$\tilde{N}$	$\tilde{T_s}$
CN	3161	7	452	(40,41,42,43)	(44,45,46,47)	(8.2,14)	(0.2,0.333)
		6	527	(47,48,49,50)	(51,52,53,54)	(9.6,16.3)	(0.2,0.333)
		5	632	(53,54,55,56)	(57,58,59,60)	(10.8, 18.33)	(0.2,0.333)

We determine the L-R representations of fuzzy numbers $\tilde{\lambda}$ *and* $\tilde{\mu}$ for seven polling booths

$\tilde{\lambda}$=(41,42,1,1), $\tilde{\mu}$=(45,46,1,1)

From (2) $\widetilde{N} = \frac{(41,42,1,1)}{(45,46,1,1)-(41,42,1,1)}$

$\widetilde{N} = \frac{(41,42,1,1)}{(3,5,2,2)}$

$\widetilde{N} = (8.2,14,0.5,0.5)$

From (4)

$\widetilde{T_s} = \frac{1}{(45,46,1,1)-(41,42,1,1)}$

$\widetilde{T_s} = (0.2,0.333,0.5,0.50\ )$

6. $\propto$ ***cut method***

Let us suppose that the arrival and service rate of trapezoidal fuzzy numbers are given by

$\tilde{\lambda}=(a_1, a_2, a_3, a_4)$ and $\tilde{\mu}==(b_1, b_2, b_3, b_4)$

$\widetilde{N} = \frac{\tilde{\lambda}}{\tilde{\lambda}-\tilde{\mu}}$ $\qquad$ $\tilde{T} = \frac{\widetilde{N}}{\tilde{\lambda}}$

We define the following $\propto cuts$ of the fuzzy numbers $\tilde{\lambda}$ and $\tilde{\mu}$

Let $\lambda_\alpha = [\propto (a_2 - a_1) + a_1, a_4 - \propto (a_4 - a_3)]$

$\mu_\alpha = [\propto (b_2 - b_1) + b_1, b_4 - \propto (b_4 - b_3)]$

We calculate the $\propto cuts$ of $\widetilde{N}$ and $\tilde{T}$ as follows

$$\widetilde{N}_\alpha = \frac{\lambda_\alpha}{\mu_\alpha - \lambda_\alpha} = \frac{[\propto(a_2-a_1)+a_1, a_4-\propto(a_4-a_3)]}{[\propto(b_2-b_1)+b_1, b_4-\propto(b_4-b_3)]-[\propto(a_2-a_1)+a_1, a_4-\propto(a_4-a_3)]}$$

$$=\frac{[\propto(a_2-a_1)+a_1, a_4-\propto(a_4-a_3)]}{[\propto(b_2-b_1+a_4-a_3)+b_1-a_4, b_4-a_1-\propto(b_4-b_3-a_2+a_1)]}$$

= [min Z,max Z]

Where Z= $[\frac{\propto(a_2-a_1)+a_1}{[\propto(b_2-b_1+a_4-a_3)+b_1-a_4}, \frac{\propto(a_2-a_1)+a_1}{b_4-a_1-\propto(b_4-b_3-a_2+a_1)}, \frac{a_4-\propto(a_4-a_3)}{\propto(b_2-b_1+a_4-a_3)+b_1-a_4}, \frac{a_4-\propto(a_4-a_3)}{b_4-a_1-\propto(b_4-b_3-a_2+a_1)}]$

Finally we find $\widetilde{N}_\alpha = [\frac{\propto(a_2-a_1)+a_1}{b_4-a_1-\propto(b_4-b_3-a_2+a_1)}, \frac{a_4-\propto(a_4-a_3)}{\propto(b_2-b_1+a_4-a_3)+b_1-a_4}]$

For $\qquad \widetilde{T}_\alpha = \frac{\widetilde{N}_\alpha}{\lambda_\alpha}$

$$=\frac{[\frac{\propto(a_2-a_1)+a_1}{b_4-a_1-\propto(b_4-b_3-a_2+a_1)}, \frac{a_4-\propto(a_4-a_3)}{\propto(b_2-b_1+a_4-a_3)+b_1-a_4}]}{[\propto(a_2-a_1)+a_1, a_4-\propto(a_4-a_3)]}$$

$= [\min Z^*, maxZ^*]$

Where $Z^* = \{\frac{1}{b_4-a_1-\propto(b_4-b_3-a_2+a_1)}, \frac{\propto(a_2-a_1)+a_1}{(b_4-a_1-\propto(b_4-b_3-a_2+a_1))(a_4-\propto(a_4-a_3))}, \frac{a_4-\propto(a_4-a_3)}{(\propto(b_2-b_1+a_4-a_3)+b_1-a_4)(\propto(a_2-a_1)+a_1)}, \frac{1}{\propto(b_2-b_1+a_4-a_3)+b_1-a_4}\}$

$$\widetilde{T}_\alpha = [\frac{\propto(a_2-a_1)+a_1}{(b_4-a_1-\propto(b_4-b_3-a_2+a_1))(a_4-\propto(a_4-a_3))}, \frac{a_4-\propto(a_4-a_3)}{(\propto(b_2-b_1+a_4-a_3)+b_1-a_4)(\propto(a_2-a_1)+a_1)}]$$

Calculation using by $\propto$ ***cut method***

Table No. 4: The Performance measures using the alpha cut.

∝	No of booths=7		No of booths=6		No of booths=5	
	L_s	W_s	L_s	W_s	L_s	W_s
0	[5.71,43]	[0.13,1.08]	[6.71,50]	[0.13,1.06]	[7.57,56]	[0.14,1.06]
0.1	[5.9,35.75]	[0.14,.0.89]	[6.93,41.58]	[0.14,.0.88]	[7.81,46.58]	[0.14,.0.88]
0.2	[6.09,30.57]	[0.14,0.76]	[7.15,35.57]	[0.14,0.75]	[8.06,39.86]	[0.14,0.75]
0.3	[6.3, 26.69]	[0.15,0.66]	[7.39,31.06]	[0.15,0.66]	[8.33,34.81]	[0.15,0.65]
0.4	[6.52,23.67]	[0.15,0.59]	[7.65,27.56]	[0.15,0.58]	[8.61,30.89]	[0.15,0.58]
0.5	[6.75,21.25]	[0.16,0.52]	[7.92,24.75]	[0.16,0.52]	[8.92,27.75]	[0.16,0.52]
0.6	[7, 19.27]	[0.17,0.47]	[8.21,22.45]	[0.17,0.47]	[9.24,25.18]	[0.17,0.47]
0.7	[7.27,17.63]	[0.17,0.43]	[8.52,20.54]	[0.17,0.43]	[9.59,23.04]	[0.17,0.43]
0.8	[7.56,16.23]	[0.18,0.4]	[8.85,18.92]	[0.18,0.4]	[9.96,21.23]	[0.18,0.39]
0.9	[7.87,15.04]	[0.19,0.37]	[9.21,17.54]	[0.19,0.37]	[10.37,19.68]	[0.19,0.37]
1	[8.2, 14]	[0.2,0.34]	[9.6,16.33]	[0.2,0.34]	[10.8,18.33]	[0.2,0.34]

7. Discussion

For our study, we could conclude that the method FM/FM/1, Gives the best and most accurate of reducing its waiting time with this effect expected number of customers in the system gets reduced. Also, the crisp value that we have calculated through the M/M/1 model lies inside the interval of FM/FM/1 for different values of alpha cut as shown in the following table.

Table No.5: Comparison performance measures between Queuing with L.R method and alpha cut method

Model	No of booths=7		No of booths=6		No of booths=5	
	L_s	W_s	L_s	W_s	L_s	W_s
M/M/1	10.25	0.25	12	0.25	13.5	0.25
FM/FM/1 L.R Method	(8.2,14)	(0.2,0.333)	(9.6,16.3)	(0.2,0.333)	(10.8,18.33)	(0.2,0.333)

Fig 2: Expected number of customers for seven polling booth

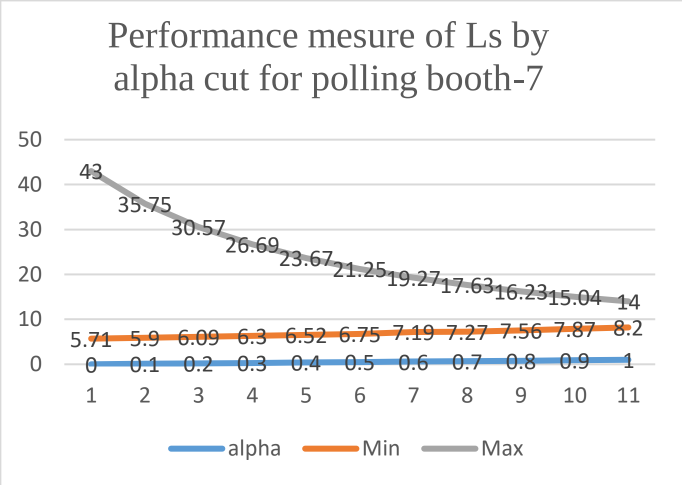
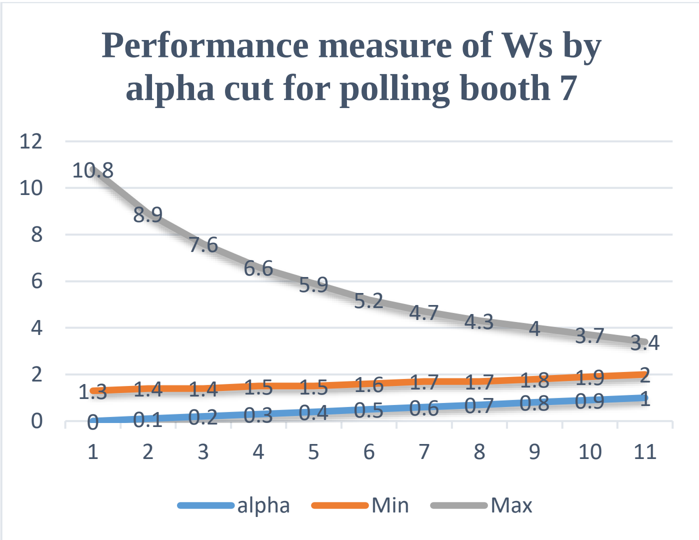


Fig 3: Waiting time of customers for seven polling booths



8.Conclusion

In this study, we suggested a different approach to solve simple fuzzy queuing problem with Trapezoidal fuzzy number. The performance measure L_s *and* W_s in queuing system for both L.R method and $\propto$ cut method were enumerated and compared. The findings using fuzzy gave a lot more details.

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