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A Study on Binary Generalized ℘− Closed Set in Binary Topological Space

Parvathy. C. R[1],Narmatha. S[2]
[1]*Associate Professor & Head, Department of Mathematics*
[2]*Assistant Professor, Department of Mathematics*
[1,2]*PSGR Krishnammal College for Women, Coimbatore, Tamilnadu, India.*

Abstract:- In this paper, we introduced the Binary Generalized ℘−Closed set and Binary Generalized ℘− Open set in Binary Topological Space. Also we discuss some of their properties and the relations between the associated binary topological space.

Keywords: Binary topological spaces, generalized binary topological space, binary generalized ℘−Closed set

1. Introduction

The generalized closed set and its complements are introduced by Levine in 1970[6]. In 1983 Njasted [11] introduced and studied the concepts of $\propto -$Sets.$\propto -$closed sets, $\propto -$open functions in topological spaces concepts are studied by Mashhours et.al [7] Binary Topology was introduced by S.Nithyanantha Jothi and P. Thangavelu in 2011[4]. Further contribution to this research in 2014, they introduced the concepts of generalized binary closed sets and discussed some properties [8]. In this paper, we introduced the *Binary Generalized ℘ − Closed set* in Binary Topological Space and discuss some properties.

Let X and Y be any non-empty sets. A binary topology [8] from X to Y is a binary structure $\mathcal{M} \subseteq \rho(X)\times\rho(Y)$ that satisfies the following axioms

(i) $(\phi, \phi)$and (X,Y) $\epsilon\mathcal{M}$,

(ii) $(K\cap K_1, L \cap L_1)\ \epsilon\mathcal{M}$ whenever $(K,L)\epsilon\mathcal{M}$ and $(K_1\cap L_1)\ \epsilon\mathcal{M}$,

(iii) If $\{(K_\propto, L_\propto): \propto\in\triangle\}$is a family members of $\mathcal{M}$ then $(\cup_{\propto\in\triangle} K_\propto, \cup_{\propto\in\triangle} L_\propto)\epsilon\mathcal{M}$.

Proposition [8] Let (X,Y, $\mathcal{M}$) be a $\mathfrak{B}\mathfrak{J}$ (K,L) ⊆(M,N) ⊆(X,Y) then the following statements hold

- *b-int(K,L)⊆(K,L)*
- If (K,L) is binary open, then *b-int*(K,L) = (K,L)
- *b-int* (K,L) ⊆*b-int* (M,N)
- *b-int(b-int* (K,L)) = *b-int* (K,L)
- (K,L)⊆*b-cl* (K,L)
- If (K,L) is binary closed, then *b-cl*(K,L) = (K,L)
- *b-cl*(K,L)⊆*b-cl* (M,N)
- *b-cl(b-cl*(K,L))⊆*b-cl*(K,L)

2. Binary Generalized ℘ −Closed Set And Binary Generalized℘ − Open Set

Definition 2.1: Let (K,L) be a subset of a binary topological space (X,Y,$\mathcal{M}$) is called a Binary generalized ℘ −closed set [briefly $\mathcal{BG}\wp -$ closed] if *bscl(K,L)* ⊆ *(M,N)* whenever *(K,L)* ⊆ *(M,N) and (M,N) is binary* $\propto$ open.

Theorem2.2: Every binary closed set is$\mathcal{BG}\wp-$ closed set

Proof: Let (A,B) be a binary closed set of (X,Y,$\mathcal{M}$) then (A,B) ⊆(M,N), where (M,N) is binary ∝ open in (X,Y,$\mathcal{M}$). Since *bcl*(A,B) ⊆ (A,B). Therefore *bint(bcl*(A,B)) ⊆(A,B). So (A,B) is *bscl*(A,B) ⊆ (M,N). Where (M,N) is binary ∝ open in (X,Y,$\mathcal{M}$). Hence every binary closed set is a $\mathcal{BG}\wp-$ closed.

The converse of the above theorem need not be true from the following example.

Example 2.3 Let X = {a,b}, Y = {1,2,3} and $\mathcal{M}$= {((φ, φ), (φ,{1}), ({a},{1}), ({a},{1,2}), ({b},φ), ({b},{1}) , ({b},{3}), ({b},{1,3}), (X,{1}), ({X}, {1,2}), (X, {1,3}), (X,Y) }. Then the set ({a},{3}) is $\mathcal{BG}\wp-$ closedset but not a binary closed set.

Theorem 2.4: Every binary g closed set is $\mathcal{BG}\ \wp-$ closed set.

Proof: Let (A,B) be a Binary generalized $\wp$ −closed set. Let (A,B) ⊆ (M,N)⊆(X,Y), then b-cl(A, B)=(A,B), (A,B)⊆(M,N) , (M,N) is binary open then b-cl(A,B) ⊆(M,N), b-int(b-cl(A,B)) ⊆*b-int*(M,N),

Therefore b-int(b-cl(A,B)) ⊆(M,N), since bint(M,N)= (M,N). Hence scl(A,B) ⊆(M,N), Where (M,N) is binary ∝ open. Since using every binary open is binary ∝ open set then (A,B) is $\mathcal{BG}\ \wp-$ closed set.

The converse of the above theorem need not be true as seen from the following example.

Example 2.5 In the example 2.3 (X,{3}) is $\mathcal{BG}\ \wp-$ closed set but not g closed set.

Theorem 2.6: Every generalized binary regular closed set is $\mathcal{BG}\ \wp-$ closedset.

Proof: Let (A,B) be a generalized binary Regular closed set. By the definition b-cl(A,B)⊆(M,N) where (A,B) ⊆(M,N) ⊆(X,Y), then b-cl(A,B) ⊆(M,N) , and b-int(b-cl(A,B)) ⊆b-int (M,N). Since b- int (M,N) ⊆(M,N), it follows that b-scl(A,B) ⊆(M,N). Every binary regular open set is binary ∝ open. Hence (A,B) is binary generalized $\wp$ −closed set.

The converse of the above theorem need not be true as seen from the following example.

Example 2.7 Let S={0,1}, T={a,b,c}, and $\mathcal{M}$= {(φ, φ),({0}, {a}), ({1},{b}),(S,{a,b}),(S,T)}. Let ({1},{b}) is a $\mathcal{BG}\wp-$ closed set but not binary regular closed set.

Theorem 2.8: Every binary g^*closed set is $\mathcal{BG}\ \wp-$ closedset.

Proof: Let (A,B) be a binary g^*closed set of (X,Y). Let (A,B) ⊆(M,N). Assume (M,N) is binary g open in (X,Y). Since (A,B) is binary g^*closed, bcl(A,B)= (A,B).

However b s-cl(A,B) ⊆bcl(A,B), which implies that bcl(A,B) ⊆(M,N). Therefore (A,B) ⊆(M,N), is binary g open in (X,Y). Every binary generalized open set is binary-∝open set. Therefore (A,B) is binary generalized $\wp$ −closed set.

The converse of the above theorem need not be true from the following example.

Example 2.9 : In the example 2.3 (X,{3}) is $\mathcal{BG}\wp-$ closed set but not g^*closed set.

Theorem2.10: Every Binary generalized $\wp$ −closed set is Binary gs closed set.

Proof: Let (A,B) be a Binary generalized$\wp$ −closed set of (X,Y,$\mathcal{M}$) and let (M,N) is a binary ∝ open in (X,Y,$\mathcal{M}$)such that (A,B) ⊆ (M,N).Since every binary semi closure of (A,B)) ⊆ (M,N), and Some binary ∝ open setsare binary open, then (M,N) is binary open.

We know that *bscl(A,B)* ⊆ (M,N), and (M,N) is binary open . Therefore (A,B) is Binary gs closed set.

The converse of the above theorem need not be true as from the following example.

Example 2.11: In the example 2.3 (X,{2}) is Binary gs closed set but not $\mathcal{BG}\wp-$ closed set.

Theorem 2.12: Every $\mathcal{BG}\wp-$ closedset is binary ∝g closed set.

Proof: From the definition of binary generalized ℘ −closed set, consider the subset (A,B) of (X,Y). Let (M,N) be a binary open set in (X,Y), such that (A,B)⊆ (M,N). Since every binary ∝ open sets are binary open set, we have b-scl(A,B) ⊆ (M,N), and b-int(b-cl(A,B)) ⊆ (M,N).

Additionally, b-cl(b-int(bcl(A,B)))⊆bcl(M,N). We know that bcl(M,N) ⊆(M,N), therefore, b∝cl(A,B) ⊆ (M,N) and (A,B)⊆ (M,N). Since (M,N) is a binary open set in (X,Y), it follows that (A,B) is ∝g closed set. Hence it is proved.

The converse of the above theorem need not be true as can be seen in the following example.

Example 2.13: In the example 2.3 ({a},{2,3}) is binary ∝g closed set but not $\mathcal{BG}$℘ − closed set Hence the converse of the theorem 2.11 is not possible.

Theorem 2.14: Every $\mathcal{BG}$℘ − closedset is Binary sg closed set.

Proof: Let (A,B) be a Binary generalized℘ −closed set of (X,Y,$\mathcal{M}$) and let (M,N) is a binary ∝ open in (X,Y,$\mathcal{M}$), such that (A,B) be a subset of (M,N). Since every binary semi closure of (A,B) ⊆ (M,N), and binary ∝ open sets are binary open set, (M,N) is binary open.

Every binary openset is binary semi open. (ie)*bscl(A,B)* ⊆ (M,N), whenever (A,B) ⊆ (M,N), (M,N) is binary semi open. Therefore (A,B) is a Binary sg closed set.

The converse of the above theorem need not be true as can be seen in the following example.

Example 2.15 In the example 2.3 ({a},{2,3}) is binary sg closed set but not $\mathcal{BG}$℘ − closed set.

Theorem 2.16: Every $\mathcal{BG}$℘ − closedset is binary g^*s closed set.

Proof: Let (A,B) be a binary generalized ℘ −closed set of (X,Y), and let (A,B) ⊆(M,N). (M,N) is a binary ∝open set in (X,Y). From the definition of binary generalized ℘ −closed set, (A,B) is a binary semi closure of (A,B). If (A,B) is binary ∝open set, then (M,N) ⊆*b-int(b-cl(b-int(M,N)))* .

We know that *b-int(M,N)* ⊆*(M,N)* thus (M,N) ⊆*b-int(b-cl(M,N)).* Taking the binary interior on both sides, we get *bint(M,N)*⊆*bint(b-int(b-cl(M,N))),* which implies (M,N) ⊆*b*cl(M,N). Taking the complement on both sides, the result will be binary g open set . If *bscl*(A,B) ⊆(M,N) and (M,N) is binary g open in (X,Y), then the subset (M,N) is binary g^*s closed set. Hence, it is proved.

The converse of the above theorem need not be true as can be seen in the following example.

Definition:2.17 Let (K,L) be a subset of a binary topological space (X,Y,$\mathcal{M}$) is called a Binary generalized ℘ −Open sets (briefly $\mathcal{BG}$℘ − open) if $(K,L)^c$ is also binary generalized ℘-open set.

Theorem 2.18: Every binary open set is $\mathcal{BG}$℘ − open set

Proof: Proof follows from the theorem 2.2

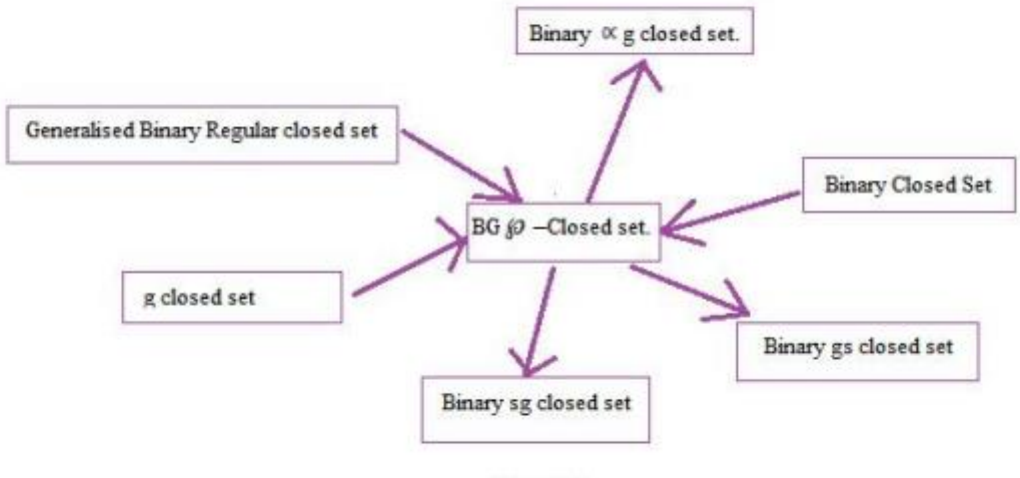
Theorem 2.19: Every generalized Binary regular open set is $\mathcal{BG}$℘ − open set

Proof: Proof follows from the theorem2.6

Theorem 2.20: If *bsint* (K,L)⊆ (P,Q) ⊆ (K,L) and if (K,L) is binary generalized ℘ open set.

Proof: Let*bsint* (K,L)⊆ (P,Q) ⊆ (K,L). Then $(K,L)^c$⊂ $(P,Q)^c$⊆ bscl $((K,L)^c)$ where $(A,B)^c$ is a binary generalized ℘ closed set. Hence, $(P,Q)^c$ is also binary generalized ℘ closed set. Therefore, (P,Q) is binary generalized ℘ open set.

From the above discussion we can figure out the relation between Binary generalized ℘ closed set with other closed sets.



3. Characteristics Of Binary Generalized ℘ −Closed Set

Theorem 3.1: Union of any two $\mathcal{BG}\wp$ −Closed set is $\mathcal{BG}\wp$ −Closed set.

Proof: Let (K,L) and (R,S) are $\mathcal{BG}\wp$ −Closed sets in (X,Y,$\mathcal{M}$), and let (M,N) be any Binary α open set containing (K,L) and(R,S). Therefore, *bscl*(K,L)⊆(M,N) and*bscl*(R,S)⊆(M,N). Since (K,L) ⊆(M,N) and (R,S) ⊆(M,N), we have (K,L) ∪ (R,S) ⊆(M,N).

As (K,L) and (R,S) are $\mathcal{BG}\wp$ −Closed sets in (X,Y,$\mathcal{M}$), *bscl(*K,L) ⊆(M,N) and *bscl*(R,S) ⊆(M,N). Now,*bscl*((K,L)∪(R,S)) = *bscl*(K,L)∪bscl(R,S)⊆(M,N). Since (M,N) is a binary α open set in (X,Y,$\mathcal{M}$), (K,L)∪ (R,S) is $\mathcal{BG}\wp$ −Closed set.

Example 3.2 In the example 2.3 (φ,{2}) and ({b}, {2}) are $\mathcal{BG}\wp$ −closed set and its union ({b}, {2}) is also the $\mathcal{BG}\wp$ −Closed set.

Theorem 3.3: Let (K,L)$\mathcal{BG}\wp$ −Closed set of (X,Y,$\mathcal{M}$). If (K,L))⊆(R,S) ⊆ $bscl(K,L)$ then (K_1,L_1) is also $\mathcal{BG}\wp$ −Closed set of (X,Y,$\mathcal{M}$).

Proof: Let (M,N) be a binary α open set in (X,Y). If (R,S) is a subset of (M,N), then (K,L) ⊆ (R,S) implies (K,L) ⊆ (M,N).Since (K,L) is a binary Generalized℘ −Closed set, *bscl*(K,L) ⊆ (M,N). Also, (R,S) ⊆ $bscl$(K,L) implies *bscl*(R,S) ⊆ *bscl*(K,L). Thus *bscl*(R,S) ⊆ (M,N) and so, (R,S) is a binary generalized℘ −Closed set.

Theorem 3.4: Let (K,L) be a $\mathcal{BG}\wp$ −Closed subset of (X,Y,$\mathcal{M}$). If (K,L) ⊆(P,Q) ⊆ bs-cl(K,L), then (P,Q) is also $\mathcal{BG}\wp$ −Closed subset of (X,Y,$\mathcal{M}$)

Proof: Let (P,Q) ⊆(M, N), where (M,N) is a binary ∝open set in (X,Y,$\mathcal{M}$). Then (K,L) ⊆(P,Q) implies that (K,L) ⊆(M,N). Since (K,L) is $\mathcal{BG}\wp$ −Closed set, *bs-cl(K,L)⊆(M,N).* Also (P,Q) ⊆bs-cl(K,L) implies bscl(P,Q)⊆ (M,N). Therefore (P,Q) is $\mathcal{BG}\wp$ −Closed.

Conclusion

In this paper, we discussed a new form of Binary generalized ℘ −closed set, $\mathcal{BG}$ ℘ −open set and its characterization are discussed.

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