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Supra Regular B – Closed Sets in Supra Topological Spaces

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Abstract:- The objective of this paper is to establish the ideas of Supra regular b – closed sets and Supra regular b – open sets and Supra regular b – interior in supra topological spaces. The corresponding supra topological space formed by the family of these sets is also studied. We discussed the some theorems of supra regular b - closed sets and supra regular b –open sets and supra regular b- interior examples are given. Also we consider some of their properties and look into the relations between the associated supra topology.

Keywords : Srb – closed sets , Srb – open sets , Srb – interior.

1. Introduction

In 1983 , A. S Mashour [8] introduced the notion of supra topological spaces and studied S-continuous functions and S*- Continuous functions. In 1996 , D.Andrijevic [1] introduced the concept of On b-open sets . In 2008, Devi [5] introduced the concept of supra α- open set, Sα- continuous functions respectively. In 2010 O. R. Sayed and Takashi Noiri [10] introduced Supra b- open sets and Supra b-continunity an topological spaces. In 2011 , I.Arockiarani and M.Trinita Pricilla [2] introduced the concept on Supra generalized b-closed sets.In 2013 [6] K. Krishnaveni & M. Vigneshwaran introduced the concept on bT – closed sets in supra Topological space. In 2015 , L.Chinnapparaj, P.Sathishmohan , V.Rajendran and K.Indirani [4] introduced supra regular generalized star b – closed sets. In 2016 ,K.LudiJancy and K.Indirani [7] introduced Supra regular generalized star star b-closed sets in supra topological spaces .In this paper a new class of supra closed set called supra regular b- closed sets is introduced and study their basic resources and look into several resources of the new opinion.

2. Preliminaries

Definition : 2.1

A subfamily of μ of X is said to be a supra topology on X , if

(a) $X, \varphi \ \epsilon \ \mu$

(b) if $A_i \ \epsilon \ \mu$ for all $i \in J$ then $\cup \ A_i \ \in \ \mu$.

The pair (X, μ) is called supra topological space . The elements of μ are called supra open set in (X, μ) and complement of a supra open set is called a supra closed set .

Definition : 2.2

The Supra closure of a set A is defined as

$$Scl(A) = \cap \left\{B : B \text{ is supra closed set and } A \subseteq B\right\}.$$

The Supra interior of a set A is denoted by $Sint(A)$ is defined as $Sint(A) = \cup \left\{B : B \text{ is a supra open set and } A \supseteq B\right\}$

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Definition : 2.3

Let (X,μ) be a supra topological space. A set A is called a supra b – open set if $A \subseteq Scl(S\,\text{int}(A)) \cup S\,\text{int}(Scl(A))$. The complement of a supra b-open set is called a supra b - closed set.

Definition : 2.4

Let (X,μ) be a supra topological space . A set A of X is called supra generalized b – closed set ($Sgb-closed$) if $Sbcl(A) \subseteq U$ whenever A$\subseteq U$ and U is supra open . The complement of supra generalized b – closed set is supra generalized b – open set.

Definition : 2.5

A subset of (X,μ) is called a supra regular open if $A = Sint(Scl(A))$, If supra regular closed set is $A = Scl(Sint(A))$.

Definition : 2.6

Let (X, μ) be a supra topological space.A Subset A of X is called Supra α generalised-closed set (Sαg – closed) if $S\alpha cl(A) \subseteq$ U ,whenever $A \subseteq U$ and U is supra open set of X.

Definition : 2.7

Let (X, μ) be a supra topological space.A Subset A of X is called Srg- closed set if $Scl(A) \subseteq$ U ,whenever $A \subseteq U$ and U is supra regular open in X.

Definition : 2.8

Let (X, μ) be a supra topological space.A Subset A of X is called Sgr- closed set if

$Srcl(A) \subseteq$ U ,whenever $A \subseteq U$ and U is supra open in X.

Definition : 2.9

Let (X, μ) be a supra topological space.A Subset A of X is called Sgb- closed set if

$Sbcl(A) \subseteq$ U ,whenever $A \subseteq U$ and U is supra open in X.

Definition : 2.10

Let (X, μ) be a supra topological space.A Subset A of X is called $Srgb$- closed set if

$Sbcl(A) \subseteq$ U ,whenever $A \subseteq U$ and U is supra regular open in X.

3. Supra Regular b – Closed Sets ($Srb - Closed\ set$)

In this section we introduce supra regular b-closed set and look into various properties.

Definition :3.1

A subset A of a supra topological space (X,μ) is called Supra regular b – closed ($Srb-closed$) if $Srcl(A) \subseteq U$ whenever $A \subseteq U$ and U is supra b - open in X.

Theorem : 3.2 : Every supra regular closed set is $Srb - closed$ set.

Proof : Let A be a supra regular closed set in X such that $A \subseteq U$ and U is supra b – open set in X since A is supra regular closed set, $Scl(S\,\text{int}(A)) = A \subseteq U$ but every supra regular closed set is $Srb-closed$ set . Therefore $Srcl(A) \subseteq Scl(S\,\text{int}(A)) \subseteq U$ implies $Srcl(A) \subseteq U$. Hence A is $Srb-closed$ set.

The converse of the above theorem need not be true as seen from the following examples.

Example : 3.3 . Let X = { a, b, c, d } with $\mu = \{ X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, b, c\}\}$.

The subsets {a} ,{b} are Srb – closed sets but not supra regular closed .

Theorem : 3.4 : Every $Srb - closed$ set is a Srg – closed set .

Proof : Let A be a supra regular b – closed sets in X such that $A \subseteq U$ and U is supra regular open in X since A is supra regular open , $Srcl(A) = A \subseteq U$ but every regular b-closed set is Srg - closed set . Therefore $Scl(A) \subseteq Srcl(A) \subseteq U$, implies $Scl(A) = A \subseteq U$ .Hence A is Srg -closed set .

The converse of the above theorem need not be true from the following examples.

Example : 3.5 . Let X = { a, b, c } with $\mu = \{ X, \emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\} \}$ .The subsets { a , c } are Srg – closed set but not $Srb - closed$.

Theorem : 3.6. Every $Srb - closed$ set is a Sgr – closed set .

Proof : Let A be a supra regular b – closed sets in X such that $A \subseteq U$ and U is supra open in X since A is supra open , $Srcl(A) = A \subseteq U$ but every supra regular b – closed set is Sgr - closed set . Therefore $Srcl(A) \subseteq U$, implies $Srcl(A) = A \subseteq U$. Hence A is Srg - closed set .

The converse of the above need not be true as seen from the following examples.

Example :3.7. Let X = { a, b, c } with $\mu = \{ X, \emptyset, \{a\}, \{b, c\} \}$ .The subsets { a , c } are Sgr - closed set but not $Srb - closed$.

Theorem : 3.8 . Every $Srb - closed$ set is a Sgb – closed set .

Proof : Let A be a supra regular b – closed sets in X such that $A \subseteq U$ and U is supra open in X since A is supra open , $Sbcl(A) = A \subseteq U$ but every supra regular b – closed set is Sgb-closed set. Therefore $Sbcl(A) \subseteq Srcl(A) \subseteq U$, implies $Sbcl(A) \subseteq U$. Hence A is Sgb – closed set .

The converse of the above theorem need not be true from the following examples.

Example :3.9. Let X = { a, b, c, d } with $\mu = \{ X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. The subsets { a , c } and { b , d } are Sgb – closed set but not $Srb - closed$.

Theorem : 3.10 . Every $Srb - closed$ set is a $Srgb$ – closed set .

Proof : Let A be a supra regular b – closed sets in X such that $A \subseteq U$ and U is supra open in X since A is supra open , $Sbcl(A) = A \subseteq U$ but every supra regular b – closed set is $Srgb$-closed set. Therefore $Sbcl(A) \subseteq Srcl(A) \subseteq U$, implies $Sbcl(A) \subseteq U$. Hence A is $Srgb$ – closed set .

The converse of the above theorem need not be true as seen from the following examples.

Example :3.11. Let X = { a, b, c } with $\mu = \{ X, \emptyset, \{a\}, \{b\}, \{b, c\} \}$ .The subsets { a , b } are $Srgb$ – closed set but not $Srb - closed$.

Theorem : 3.12. The Union of two $Srb - closed$ sets is $Srb - closed$.

Proof : Let A and B be two $Srb - closed\ set$ .Let $A \cup B \subseteq G$ where G is Sb – open. Since A and B are $Srb - closed$ sets . Therefore $Srcl(A) \cup Srcl(B) \subseteq G$ and thus $Srcl(A \cup B) \subseteq G$ hence $A \cup B$ is $Srb - closed$.

Theorem : 3.13. A Set A is $Srb - closed$ set iff $Srcl(A) - A$ does not contain no non empty $Sb - closed\ set.$

Proof : Necessity : Let A be an $Srb - closed$ set (X, μ). Let F be a $Sb - closed\ set$ in X such that $F \subseteq Srcl(A) - A$. Since F^c is supra b – open and $A \subseteq F^c$. Since A is $Srb - closed$.we have $Srcl(A) \subseteq F^c$.consequently $F \subseteq (Srcl(A))^c$ .This implies that $F \subseteq Srcl(A) \cap [Srcl(A)]^C = \emptyset$.

Sufficiency : Suppose $A \subseteq U$ and U is Sb – open . If $Srcl(A)$ is not contained in U .Then $Srcl(A) \cap U^c \neq \emptyset$. Since supra regular b- closed set of $Srcl(A) - A$ which is a contradiction . Therefore $Srcl(A) \subseteq U$. Hence A is $Srb - closed$.

Theorem : 3.14. A set A is $Srb - closed$ and $A \subseteq B \subseteq Srcl(A)$ then B is $Srb - closed$ set .

Proof : Let U be supra b –open set in (X, μ) such that $B \subseteq U$ .since $A \subseteq U \Rightarrow A \subseteq U$ and since A is $Srb - closed$ set in (X, μ) $Srcl(A) \subseteq U$,since $B \subseteq Srcl(A)$.Then $Srcl(B) \subseteq U$. Therefore B is also $Srb - closed$ set in (X, μ).

Theorem : 3.15. If $A \subseteq Y \subseteq X$ and suppose that A is $Srb - closed$ set in X is $Srb - closed$ set in X, then A is $Srb - closed$ set relative to Y.

Proof : Given that $A \subseteq Y \subseteq X$ and A is $Srb - closed$ set in X .To prove that A is a $Srb - closed$ set relative to Y. Let us assume that $A \subseteq Y \cap C$, where U is supra b – open in X . Since A is a $Srb - closed$ set , $A \subseteq U$ implies $Srcl(A) \subseteq U$. $Y \cap Srcl(A) \subseteq Y \cap C$, (i.e)

A is a $Srb - closed$ set relative to Y .

4. Supra Regular b- open sets ($Srb - open\ set$)

Definition : 4.1 : A set A of a topological spaces (X, μ) is called supra regular b – open ($Srb - open$) if and only if A^c is $Srb - closed$ in X .

Theorem : 4.2 .A subset A of a topological space (X, μ) is $Srb - open$ if and only if

$F \subseteq Sb\text{int}(A)$ whenever $F \subseteq A$ and F is supra b – closed in X .

Suppose that is $Srb - open$. Let $F \subseteq A$ and F be supra b – closed . Then $A^c \subseteq F^c$ and F^c is supra b – open . Since A is Srb -open , A^c is $Srb - closed$. Hence $Srcl \subseteq F^c$. Since $Srcl(A^c) = [Sr\text{int}(A)]^c$. Hence $F \subseteq Sr\text{int}(A)$.

Conversely , suppose that $F \subseteq Sb\text{int}(A)$ whenever $F \subseteq A$ and F is supra b-closed in X . Let U be supra b-open in X and $A^c \subseteq U$ .Then U^c is supra b – closed and $U^c \subseteq A$. Hence by assumption $U^c \subseteq Sr\text{int}(A)$ therefore $[Sr\text{int}(A)]^c \subseteq U$ (i.e) $Srcl(A) \subseteq U$. Therefore A^c is $Srb - closed$. Hence A is $Srb - open$.

Theorem : 4.3. Let (X, μ) be supra topological space . A set A is supra regular b – open in X if and only if G = X whenever G is supra b –open and $Sr\text{int}(A) \cup A^c \subseteq G$.

Proof : Let A be supra regular b – open , G be supra b-open and $Sr\text{int}(A) \cup A^c \subseteq G$. This given $G^c \subseteq (Sr\text{int}(A))^c \cap (A^c)^c = Srcl(A^c) \setminus A^c$. Since A^c is supra regular b-closed and G^c is supra b-closed by theorem 4.2., it follows that $G^c = \phi$ therefore X = G .

Conversely , suppose that F is supra b –closed and $F \subseteq A$. Then $Sr\,\mathrm{int}(A) \cup A^c \subseteq Sr\,\mathrm{int}(A) \cup F^c$. It follows that $Sr\,\mathrm{int}(A) \cup F = X$ and hence $F \subseteq Sr\,\mathrm{int}(A)$. Therefore A is supra regular b-open .

Proposition 4.4. Let (X,μ) be supra topological space if $Sr\,\mathrm{int}(A) \subseteq B \subseteq A$ and A is supra regular b-open in X , then B is supra regular b- open .

Proof : Suppose $Sr\,\mathrm{int}(A) \subseteq B \subseteq A$ and supra regular b-open in X . Then $A^c \subseteq B^c \subseteq Srcl(A^c)$ and since A^c is supra regular b – closed by theorem 3.10, B is supra regular b – open in X .

Theorem: 4.5.

Let (X,μ) be supra topological space . A set A is supra regular b – closed set. If and only if $Srcl(A) - A$ is supra regular b – open in X.

Proof: Necessity: Suppose that A is supra regular b-closed in X . Let $F \subseteq Srcl(A) - A$ Where F is supra b-closed. By theorem 4.2., $F \neq \phi$. Therefore $F \subseteq Sr\,\mathrm{int}(Srcl(A) - A)$ and by Theorem 4.2 $Srcl(A) - A$ is supra regular b-open.

Sufficiency: Let $A \subseteq U$ and U be supra b – open set then $Srcl(A) \cap U^c \subseteq Srcl(A) \cap A^c = Srcl(A) - A$.

Since $Srcl(A) \cap U^c$ is supra b- closed set and $Srcl(A) - A$ is supra regular b- open ,

by theorem 4.2. We have $Srcl(A) \cap U^c \subseteq Sr\,\mathrm{int}(rcl^{\mu}(A) - A = \phi$,This show that $Srcl(A) \subseteq U$.

Hence A is supra regular b- closed set.

5. Supra Regular b- Interior (Srb – Interior)

Definition 5.1. Let A be a subset of X. A point $x \in A$ is said to be *Srb* – interior point of A if A is *a Srb*-nbhd of x . The set of all *Srb* – interior points of A is called the *Srb* – interior of A and is denoted by $Srb - \mathrm{int}(A)$.

Theorem 5.2. If A be a subset of X. Then $Srb - \mathrm{int}(A) = \cup\{G : G \text{ is } Srb - open,\ G \subset A\}$.

Proof. Let A be a subset of X.

$x \in Srb - \mathrm{int}(A) \Leftrightarrow x$ is a *Srb* – interior point of A.

$\Leftrightarrow A$ is a *Srb*-nbhd of point x.

$\Leftrightarrow$ there exists *Srb*-open set G such that $x \in G \subset A$.

$\Leftrightarrow x \in \cup\{G :\ G$ is *Srb*-open, $G \subset A\}$.

Hence $Srb - \mathrm{int}(A) = \cup\{G : G$ is *Srb*-open, $G \subset A\}$. ■

Theorem:5.3. Let A and B be subsets of X. Then

(i) $Srb - \mathrm{int}(X) = X$ *and Srb-int(φ) = φ.*

(ii) $Srb - \mathrm{int}(A) \subset A$.

(iii) If B is any Srb-open set contained in A, then B $\subset$ $Srb-\text{int}(A)$.

(iv) If A $\subset$ B, then $Srb-\text{int}(A) \subset Srb-\text{int}(B)$

(v) $Srb-\text{int}(Srb-\text{int}(A)) = Srb-\text{int}(A)$.

Proof. (i) Since X and φ are *Srb*-open sets, by Theorem 5.1. $Srb-\text{int}(X) =$ $\cup\{G : G \text{ is Srb-open}, G \subset X\} = X \cup \{A \text{ is a Srb-open}\} = X$. That is $Srb-\text{int}(X) = X$. Since φ is the only *Srb*-open set contained in φ, *Srb-int(φ) = φ.*

(ii) Let x ∈ $Srb-\text{int}(A)$ ⇒ x is a *Srb* -interior point of A.⇒ A is a *Srb* -nbhd of x.

⇒ x ∈ A . Thus x ∈ $Srb-\text{int}(A)$ ⇒ x ∈ A. Hence $Srb-\text{int}(A) \subset A$.

(iii) Let B be any *Srb* -open sets such that B ⊂ A. Let x ∈ B, then since B is a *Srb* -open set contained in A. x is a *Srb* -interior point of A. That is x ∈ $Srb-\text{int}(A)$. Hence B ⊂ $Srb-\text{int}(A)$.

(iv) Let A and B be subsets of X such that A ⊂ B. Let x ∈ $Srb-\text{int}(A)$. Then x is a *Srb* -interior point of A and so A is rgα-nbhd of x. Since B ⊃ A, B is also a *Srb* -nbhd of x. This implies that x ∈ $Srb-\text{int}(B)$. Thus we have shown that

x ∈ $Srb-\text{int}(A)$ ⇒ x ∈ $Srb-\text{int}(B)$. Hence $Srb-\text{int}(A) \subset Srb-\text{int}(B)$.

(v) .Let A any subset of X .By definition of *Srb* –interior, $Srb-\text{int}(A)$ is *Srb* –open and hence $Srb-\text{int}(Srb-\text{int}(A)) = Srb-\text{int}(A)$.

Theorem: 5 .4. If A and B are subsets of X, then $Srb-\text{int}(A) \cup Srb-\text{int}(B) \subset$ $Srb-\text{int}(A \cup B)$

Proof. We know that A ⊂ A ∪ B and B ⊂ A ∪ B. We have, by Theorem 5.2. (iv), $Srb-\text{int}(A) \subset Srb-\text{int}(A \cup B)$ and $Srb-\text{int}(B) \subset Srb-\text{int}(A \cup B)$. This implies that $Srb-\text{int}(A) \cup Srb-\text{int}(B) \subset Srb-\text{int}(A \cup B)$.

Theorem: 5.5. If A is a subset of X, then int (A) ⊂ Srb-int (A).

Proof. Let A be a subset of a space X.

Let x ∈ int(A) ⇒ x ∈ ∪{G : G is open, G ⊂ A}.

⇒ there exists an open set G such that x ∈ G ⊂ A.

⇒ there exist a Srb-open set G such that x ∈ G ⊂ A, asevery open set is a Srb-open set in X.

⇒ x ∈ ∪{G : G is Srb-open, G ⊂ A}.

⇒ x ∈ Srb-int (A).

Thus x ∈ int (A) ⇒ x ∈ Srb-int (A). Hence int (A) ⊂ Srb-int (A).

Example: 5.6. Let X = {a, b, c} with topology τ = {X, φ, {a}, {b}, {a, b}}.

Then $SrbO(X) = \{X, \varphi, \{a\}, \{b\}, \{c\}, \{a, b\}\}$. Let $A = \{a, c\}$. Now $Srb - \text{int}(A) = \{a, c\}$ and $\text{int}(A) = \{a\}$. It follows that $\text{int}(A) \subset Srb - \text{int}(A)$ and

$\text{int}(A) \neq Srb - \text{int}(A)$.

6. Conclusion

In this paper ,we found *Srb – Closed* and *Srb – Open* sets , a new class of Supra open and Supra closed sets in Supra Topological spaces .Some of their features are also investigated in terms of *Srb – Interior* in Supra topological spaces.

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