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Numerical Study on the Unsteady MHD Flow of Immiscible Non-Newtonian and Newtonian Fluids Through a Vertical Porous Pipe

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Abstract: - This research aims to investigate the mixed convective flow and heat transfer occurring within a vertical pipe filled with porous media, involving immiscible micropolar and Newtonian fluids. Additionally, the effects of magnetohydrodynamics are taken into account. To solve the coupled system of governing differential equations, the Crank-Nicolson finite difference numerical method is employed. Furthermore, Newton's method is utilized to address the nonlinear difference equations underlying the system. The study examines numerical results concerning fluid velocities, microrotation, and fluid temperatures across various engineering parameters. Moreover, the research delves into the variations observed in volumetric flow rate, skin friction coefficient, and Nusselt number.

Keywords*:* immiscible fluids, micropolar fluid, heat transfer, numerical solution.

1. Introduction

The pressure-driven flows through vertical pipes also known as the mixed convective flows through pipes, possess special importance in applications like cooling devices of electronic and micro-electronic equipment, blood rheology etc. Despite its relevance to real-life situations, the studies on mixed convective flows through vertical pipes are significantly unexplored in literature. Moutsoglou and Kwon [1] presented a computational study on the laminar mixed convective flow in a vertical tube. Barletta and Schio [2] studied the mixed convection flow in a vertical circular duct with periodic boundary conditions.

Within the array of non-Newtonian fluid models, a notable one known as the "micropolar fluid model" was first developed by Eringen in the early 1960s [3]. This model is capable of capturing the characteristics of intricate fluids such as lubricating oils, colloidal suspensions, animal blood, liquid crystals, slurries, and polymeric fluids, among others, whose constituent particles exhibit varying shapes and the ability to expand and contract. A detailed account of micropolar fluid theory can be found in the books of Eringen [4, 5]. Some notable works in this field can be referred from [6, 7, 8, 9, 10, 11, 12].

The principles of heat transfer find extensive application in the operation of numerous engineering devices and systems such as thermal insulators, thermocouples, thermoelectric coolers, and more. Particularly notable is its utilization in heat exchangers, which play a pivotal role in refrigeration and air conditioning systems. Refer to the works in [13, 14, 15, 16, 17, 18].

The objective of this work is to study the unsteady mixed convective flow of immiscible micropolar and Newtonian fluids through a vertical pipe under heat transfer effects. The study of immiscible fluid flows through vertical pipes is an interesting topic in itself as it involves numerous complexities. The vertical geometry of the pipe brings the buoyancy effects into the flow, which in turn makes the system of equations governing the convective flow highly coupled in flow and heat transfer variables. Further, the immiscible nature of fluids increases the degree of difficulty in dealing with the study of such flows. The problem considered in the present chapter deals with the unsteady mixed convective flow and heat transfer of two immiscible fluids through a vertical pipe. The micropolar and Newtonian fluids are assumed to be in core and peripheral regions, respectively. To obtain the solution of the governing partial differential equations, a finite difference-based approach, known as the Crank-Nicolson approach, is used followed by Newton's method for solving a system of non-linear equations. The numerical solutions obtained for fluid velocities, microrotation and fluid temperatures are displayed through graphs. The volume flow rate, skin friction coefficient and Nusselt number are also computed, numerically.

2. Mathematical Formulation of the Problem

Consider the unsteady, mixed convective, laminar and axisymmetric flow of two immiscible fluids through a porous medium in a circular pipe of radius R_0. The pipe is assumed to be vertical and at constant temperature T_w. The immiscible fluids considered in this pipe are Eringen's micropolar fluid and classical Newtonian fluid. A cylindrical polar coordinate system (r, θ, z) is used to represent the flow set-up, with z-axis taken along the axis of the pipe (see fig. 1). Fluids in both regions are assumed to be incompressible, and the gravitational force acts on both fluids in a vertically downward direction. The pipe is filled with a uniform porous medium having permeability k^*. Fluids in both regions are assumed to be electrically conductive, having electrical conductivity σ and a constant transverse magnetic field of strength H_0 is applied normally to the pipe.

Initially, both fluids are at rest with temperature ${T_0}^*$. At time $t > 0$, a constant pressure gradient is applied in z-direction. However, the pipe is held stationary throughout. Here again, as in the case of the previous chapter, the flow is due to pressure gradient and simultaneously buoyancy forces in upward z-direction

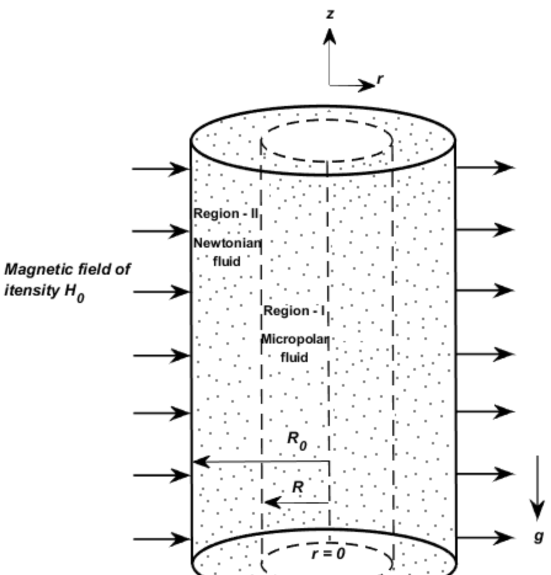

Figure 1 Geometrical configuration of problem

Under these assumptions, the present immiscible fluid flow with the fluid velocities $\bar{q} = (0,0,w_I(r,t))$, microrotation $\bar{\omega} = (0, b(r,t), 0)$ and the fluid temperatures $T_I = T_I(r,t)$ is governed by the following partial differential equations after non-dimensionalisation:

Region-I: Micropolar fluid region $(0 \le r \le 1)$

$$\frac{\partial w_1}{\partial t} = G + \frac{(1+n_1)}{Re}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w_1}{\partial r}\right) + \frac{n_1}{Re}\frac{1}{r}\frac{\partial}{\partial r}(rb) + \frac{G_R}{Re^2}T_1 - \frac{\left[M^2 + \frac{1}{Da}\right]}{Re}w_1$$

$$\frac{\partial T_1}{\partial t} = \frac{1}{ReP_R}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_1}{\partial r}\right) + \frac{B_R}{ReP_R}\left\{\left(\frac{\partial w_1}{\partial r}\right)^2 + n_1\left[\frac{\partial w_1}{\partial r} + 2b\right]^2 - 2\delta_1\frac{b}{r}\frac{db}{dr}\right.$$

Region-II: Newtonian fluid region $(1 < r \le s)$

$$\frac{\partial w_2}{\partial t}=\frac{G}{m_2}+\frac{m_1}{Rem_2}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w_2}{\partial r}\right)+\frac{G_R m_3}{Re^2}T_2-\left[\frac{M^2}{m_1}+\frac{1}{Da}\right]\frac{m_1}{Rem_2}w_2$$

$$\frac{\partial T_2}{\partial t}=\frac{K}{ReP_R m_2 C_p}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_2}{\partial r}\right)+\frac{B_R m_1}{ReP_R m_2 C_p}\left(\frac{\partial w_2}{\partial r}\right)^2$$

where, $s=\frac{R_0}{R}\geq 1, n_1=\frac{\kappa}{\mu_1}$ is micropolarity parameter, $Re=\frac{\rho_1 WR}{\mu_1}$ is the Reynolds number, $G=-\frac{\partial p}{\partial z}$ is constant pressure gradient, $M=\sqrt{\frac{\sigma {H_0}^2 R^2}{\mu_1}}$ is Hartmann number, $Da=\frac{k^*}{R^2}$ is Darcy number, $G_R=\frac{g\beta_1 \Delta T R^3 {\rho_1}^2}{{\mu_1}^2}$ is the Grashof number, W is the maximum velocity in the pipe, $P_R=\frac{\mu_1 C_{p_1}}{K_1}$ is the Prandtl number, $B_R=\frac{\mu_1 W^2}{K_1 \Delta T}$ is the Brinkmann number, $\delta_1=\frac{\gamma}{R^2\mu_1}$ and $\delta_2=\frac{\beta}{R^2\mu_1}, m_1=\frac{\mu_2}{\mu_1}$ is the ratio of viscosities, $m_2=\frac{\rho_2}{\rho_1}$ is the ratio of densities, $m_3=\frac{\beta_2}{\beta_1}$ is the ratio of thermal expansion co-efficient, $K=\frac{K_2}{K_1}$ is the ratio of thermal conductivities, $C_p=\frac{C_{p2}}{C_{p1}}$ is the ratio of specific heats, T_∞ is the ambient temperature, $T_0=\frac{{T_0}^*-T_w}{T_\infty-T_w}$.

Here again, the continuity of fluid velocities, shear stresses, fluid temperatures and heat fluxes is presumed at the fluid-fluid interface with the classical no-slip boundary condition. The non-dimensional conditions to be satisfied are listed hereunder:

Initial conditions:

$$w_1(r,0)=0 \text{ for } 0\leq r\leq 1$$
$$w_2(r,0)=0 \text{ for } 1<y\leq s$$
$$b(r,0)=0 \text{ for } 0\leq r\leq 1$$
$$T_1(r,0)=T_0 \text{ for } 0\leq r\leq 1$$
$$T_2(r,0)=T_0 \text{ for } 1<y\leq s$$

Boundary and interface conditions: For $t>0$,

$$\frac{\partial w_1}{\partial r}=0 \text{ at } r=0$$
$$w_2(r,t)=0 \text{ at } r=s$$
$$w_1(r,t)=w_2(r,t) \text{ at } r=1$$
$$b(r,t)=-\frac{1}{2}\frac{\partial w_1}{\partial r} \text{ at } r=1$$
$$\left(1+\frac{n_1}{2}\right)\frac{\partial w_1}{\partial r}=m_1\frac{\partial w_2}{\partial r} \text{ at } r=1$$
$$\frac{\partial T_1}{\partial r}=0 \text{ at } r=0$$
$$T_2(r,t)=0 \text{ at } r=s$$
$$T_1(r,t)=T_2(r,t) \text{ at } r=1$$
$$\frac{\partial T_1}{\partial r}=K\frac{\partial T_2}{\partial r} \text{ at } r=1$$

3. Numerical Solution

Flow and heat transport

It is to be noted that the above governing partial differential equations are fully coupled in terms of w_1, w_2, b, T_1, T_2; therefore, these are to be solved simultaneously. Due to the non-linear nature of the differential equations, the Crank-Nicolson finite difference approach resulted in a system of non-linear algebraic equations. The resultant non-linear system of finite difference equations is solved using Newton's method to obtain the fluid velocities, microrotation and fluid temperatures.

Discretizing the domain so that $i = 0,1,2,3, \ldots..m-1$ give spatial points of the micropolar fluid region and $i = m+1, m+2, \ldots..l-1$ represent spatial points in the Newtonian fluid region, and employing Crank-Nicolson technique for the system of PDEs, we get the following finite difference scheme, for all time levels j :

Region-I: Micropolar fluid region ($i = 0, 1, 2, 3, \ldots..m-1$)

For $i = 0$ (at origin),

$$(1 + 2M + Z_3)w_{10,j+1} - 2Mw_{11,j+1} - Z_1T_{10,j+1}$$

$$(1 + 2N)b_{0,j+1} - 2Nb_{1,j+1} = (1 - 2N)b_{0,j} + 2Nb_{1,j}$$

$$(1 + 2C_{15})T_{10,j+1} - 2C_{15}T_{11,j+1} = (1 - 2C_{15})T_{10,j} + 2C_{15}T_{11,j}$$

for $i = 1, 2, 3, \ldots.m-1$,

$$\begin{aligned}&-[A - B_i]w_{1i-1,j+1} + [1 + 2A + Z_3]w_{1i,j+1} - [A + B_i]w_{1i+1,j+1}\\&+Cb_{i-1,j+1} - D_ib_{i,j+1} - Cb_{i+1,j+1} - Z_1T_{1i,j+1}\\&= [A - B_i]w_{1i-1,j} + [1 - 2A - Z_3]w_{1i,j} + [A + B_i]w_{1i+1,j}\end{aligned}$$

$$\begin{aligned}&-[E - F_i]b_{i-1,j+1} + [1 + 2E - H_i]b_{i,j+1} - [E + F_i]b_{i+1,j+1}\\&-Iw_{1i-1,j+1} + Iw_{1i+1,j+1} = [E - F_i]b_{i-1,j} + [1 - 2E + H_i]b_{i,j}\end{aligned}$$

$$\begin{aligned}&-[C_5 - C_{6i}]T_{1i-1,j+1} + (1 + 2C_5)T_{1i,j+1} - [C_5 + C_{6i}]T_{1i+1,j+1}\\&= [C_5 - C_{6i}]T_{1i-1,j} + (1 - 2C_5)T_{1i,j} + [C_5 + C_{6i}]T_{1i+1,j}\\&+C_7\left\{\left[w_{1i+1,j} - w_{1i-1,j}\right]^2 + \left[w_{1i+1,j+1} - w_{1i-1,j+1}\right]^2\right\}\\&+C_8\left\{\left[\frac{w_{1i+1,j} - w_{1i-1,j}}{2h} + 2b_{i,j}\right]^2 + \left[\frac{w_{1i+1,j+1} - w_{1i-1,j+1}}{2h} + 2b_{i,j+1}\right]^2\right\}\\&+C_{9i}\left\{b_{i,j}\left(b_{i+1,j} - b_{i-1,j}\right) + b_{i,j+1}\left(b_{i+1,j+1} - b_{i-1,j+1}\right)\right\}\\&+C_{10}\left\{\left[b_{i+1,j} - b_{i-1,j}\right]^2 + \left[b_{i+1,j+1} - b_{i-1,j+1}\right]^2\right\} + C_{11i}\left\{\left(b_{i,j}\right)^2 + \left(b_{i,j+1}\right)^2\right\},\end{aligned}$$

Region-II: Newtonian fluid region ($i = m+1, m+2, \ldots..l-1$)

$$\begin{aligned}&-[J - L_i]w_{2i-1,j+1} + [1 + 2J + Z_4]w_{2i,j+1} - [J + L_i]w_{2i+1,j+1} - Z_2T_{2i,j+1}\\&= [J - L_i]w_{2i-1,j} + [1 - 2J - Z_4]w_{2i,j} + [J + L_i]w_{2i+1,j} + Z_2T_{2i,j} + \frac{kG}{m_2}\\&-[C_{12} - C_{13i}]T_{2i-1,j+1} + (1 + 2C_{12})T_{2i,j+1} - [C_{12} + C_{13i}]T_{2i+1,j+1}\\&= [C_{12} - C_{13i}]T_{2i-1,j} + (1 - 2C_{12})T_{2i,j} + [C_{12} + C_{13i}]T_{2i+1,j}\end{aligned}$$

where, $m = \frac{l}{2}, l = \frac{2}{h}$ and h is step size in radial direction while k is the step size in temporal direction.

The discretized version of the conditions are

Initial conditions:

$$\begin{aligned}w_{1i,0} &= 0, \text{ for } 0 \le i \le m\\w_{2i,0} &= 0, \text{ for } m < i \le l\\b_{i,0} &= 0, \text{ for } 0 \le i \le m\\T_{1i,0} &= T_0 \text{ for } 0 \le i \le m\\T_{2i,0} &= T_0 \text{ for } m \le i \le l\end{aligned}$$

Boundary and interface conditions:

$$w_{11,j+1} = w_{1-1,j+1} \text{ for all } j$$
$$b_{1,j+1} = b_{-1,j+1} \text{ for all } j$$
$$w_{2l,j+1} = 0 \text{ for all } j$$
$$-Pw_{1m-1,j+1} + Qw_{1m,j+1} - Rw_{2m+1,j+1} = 0 \text{ for all } j$$
$$-Sw_{1m-1,j+1} + Sw_{1m,j+1} + b_{m,j+1} = 0 \text{ for all } j$$
$$T_{11,j+1} = T_{1-1,j+1} \text{ for all } j$$
$$T_{2l,j+1} = 0 \text{ for all } j$$
$$-T_{1m-1,j+1} + (1+K)T_{1m,j+1} - KT_{2m+1,j+1} = 0 \text{ for all } j$$

where, $A = \frac{k(1+n_1)}{2Reh^2}, B_i = \frac{k(1+n_1)}{4Rehr_i}, C = \frac{kn_1}{4Reh}, D_i = \frac{kn_1}{2Rer_i}, E = \frac{k\left(1+\frac{n_1}{2}\right)}{2\text{Reh}^2}, F_i = \frac{k\left(1+\frac{n_1}{2}\right)}{4Rehr_i}, H_i = \left[\frac{kn_1}{Re} - \frac{k\left(1+\frac{n_1}{2}\right)}{2Rer_i^2}\right], J = \frac{km_1}{2Rem_2h^2},$

$L_i = \frac{km_1}{4Rehm_2r_i}, M = \frac{k(1+n_1)}{Reh^2}, N = \frac{3k\left(1+\frac{n_1}{2}\right)}{4\text{Reh}^2}, Z_1 = \frac{kG_R}{2Re^2}, Z_2 = \frac{kG_Rm_3}{2Re^2}, Z_3 = \frac{k\left(M^2+\frac{1}{Da}\right)}{2Re}, Z_4 = \frac{k\left(M^2+\frac{m_1}{Da}\right)}{2Rem_2}, C_1 = \frac{1}{ReP_R}, C_2 = B_RC_1, C_3 = \frac{K}{ReP_Rm_2C_p}, C_4 = \frac{B_Rm_1}{ReP_Rm_2C_p}, C_5 = \frac{kC_1}{2h^2}, C_{6i} = \frac{kC_1}{4hr_i}, C_7 = \frac{kC_2}{8h^2}, C_8 = \frac{kn_1C_2}{2}, C_{9i} = \frac{k\delta_1C_2}{2hr_i}, C_{10} = \frac{k\delta_2C_2}{8h^2}, C_{11i} = \frac{k\delta_2C_2}{2{r_i}^2}, C_{12} = \frac{kC_3}{2h^2},$

$C_{13i} = \frac{kC_3}{4hr_i}, C_{14} = \frac{kC_4}{8h^2}, C_{15} = \frac{kC_1}{h^2}, P = 1 + \frac{n_1}{2}, Q = 1 + m_1 + \frac{n_1}{2}, R = m_1, S = \frac{1}{2h}.$

It is evident that, at every time level j, the difference equations are non-linear and coupled. Employing the conditions, we get $(5m+1)$ non-linear equations in terms of $(5m+1)$ unknowns in fluid velocities, microrotation and fluid temperatures. Newton's method is applied to solve the underlying non-linear system repetitively for each time level j to get the flow and temperature profiles.

Volume flow rate, Skin friction coefficient and Nusselt number

The volume flow rate, skin friction coefficient and Nusselt number are studied for the problem under consideration. The numerical results concerning various physical parameters of interest are displayed through tables (1), (2) and (3).

4. Results And Discussion

In this section, the fluid flow and heat transfer profiles that are obtained numerically are plotted. The effect of pertinent fluid parameters on fluid velocities, microrotation and fluid temperatures is also discussed.

For the sake of the grid-independent solution, a grid sensitivity analysis for fluid velocities and temperatures is performed. Three different cases of uniform grids are tested to obtain a grid-independent solution. The outcome of this study for fluid velocities is depicted through Fig. (2). It can be noted that the solution with a 100 x 100 grid is almost the same as compared to the solution with a much finer grid of 150 x 150. Hence, the uniform grid of 100 x 100 is taken for the study.

It should be noted that, the present analysis is regarding the mixed convective flow and heat transfer, where we have governing equations in flow and heat transfer variables in highly coupled form. The flow variables has some influence of heat transfer in it and the heat transfer variable has some influence of flow variables in it. In view of the same, all the parameters appearing in the problem will affect both flow and heat transfer profiles. This was not so in the case of forced convective flows in earlier cases where we have momentum equation decoupled from temperature terms.

The fluid velocities in both regions increase with time and eventually attain a steady state after a higher time level. A decreasing behaviour is seen in fluid velocities when varied with the micropolarity parameter n_1. As $n_1$increases, the vortex viscosity increases which decreases the fluid velocity. Though $n_1$is the feature of micropolar fluid alone, nevertheless, $n_1$is affecting Newtonian fluid too, given continuous fluid velocities and shear stresses. It is evident from fig. (5) that the fluid velocities are increasing with the Grashof number. The bigger the Grashof number, the more the effect of buoyancy terms leads to more free convection and hence velocity increases. It is seen that the Brinkmann number promotes the fluid velocities in both regions of the flow. The fluid velocities increase with the Reynolds number and ratio of thermal expansion coefficients and decrease with increasing values of the ratio of densities, thermal conductivities, Prandtl number and the ratio of specific heats.

It is noticed that the microrotation is increasing with time and reaching a steady state at a subsequent time. It is noted from fig (6) that the Grashof number reduces the microrotation. The microrotation is increased by a ratio of viscosities, ratio of thermal expansion coefficients and Prandtl number. The fluid temperatures enter a steady state after a certain higher time level. It is observed from fig. (4) that the fluid temperatures in both regions of flow are decreasing with increasing values of micropolarity parameter n_1. Grashof number measures the ratio of buoyancy forces to viscous forces acting on the fluid. The Grashof number is having an increasing impact on fluid temperatures. The fluid temperatures in both regions are seen to be decreasing with the increasing values of Brinkmann number (see fig. 3).

The numerical values of volume flow rate are displayed for several parameters of interest through Table 1. It is observed that the rate of volume flow is an increasing function of the Grashof number, which may be due to the dominance of buoyancy forces over viscous forces. The volume flow rate decreases with the micropolarity parameter. The Reynolds number is rendering an incrementing impact on the volume flow rate.

Table 2 displays the numerical results of the skin friction coefficient for the considered flow problem. It is observed that, at the boundary of the pipe, skin friction increases with the Grashof number and Brinkmann number, and it decreases with the increasing values of the Reynolds number and micropolarity parameter. The numerical values for the Nusselt number with various physical parameters of interest are given in table 3. It can be seen that the Nusselt number increases with that of the Brinkmann number, micropolarity parameter, and Grashof number.

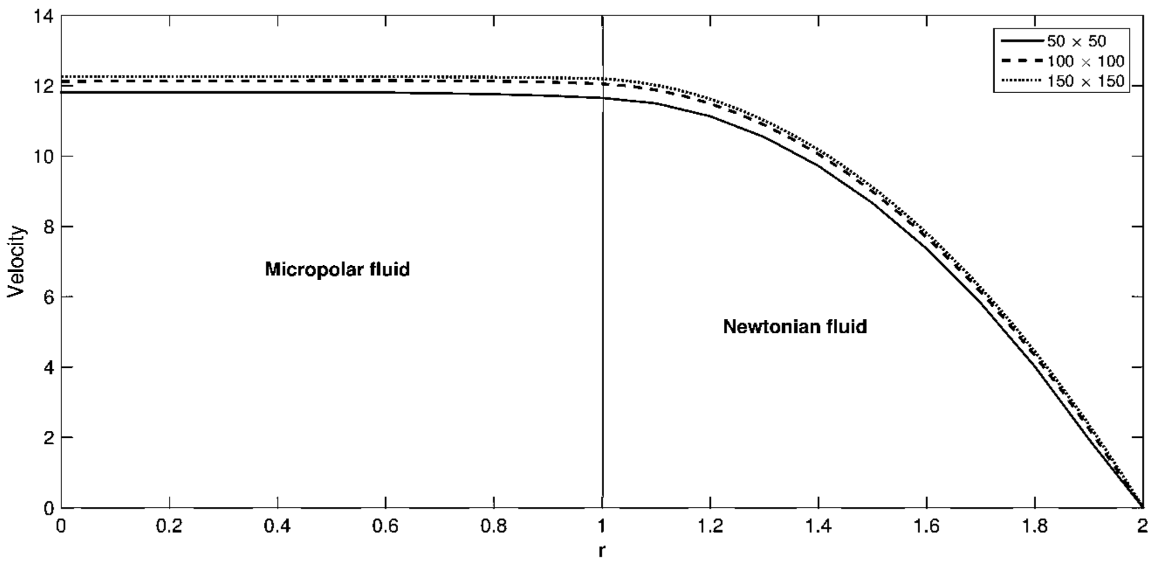

Figure 2 Effect of mesh on fluid velocity profiles

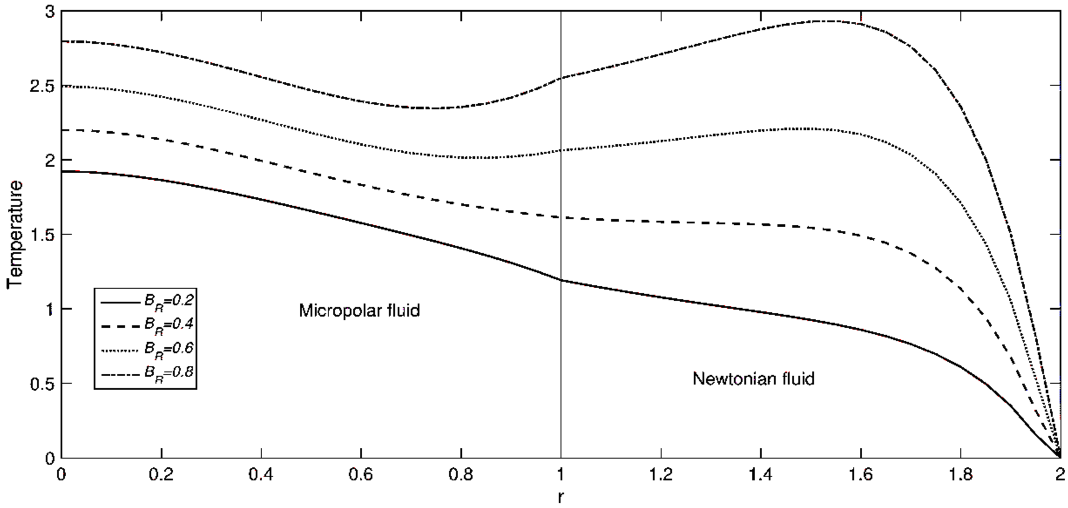

Figure 3 Fluid temperatures with varying Brinkmann number

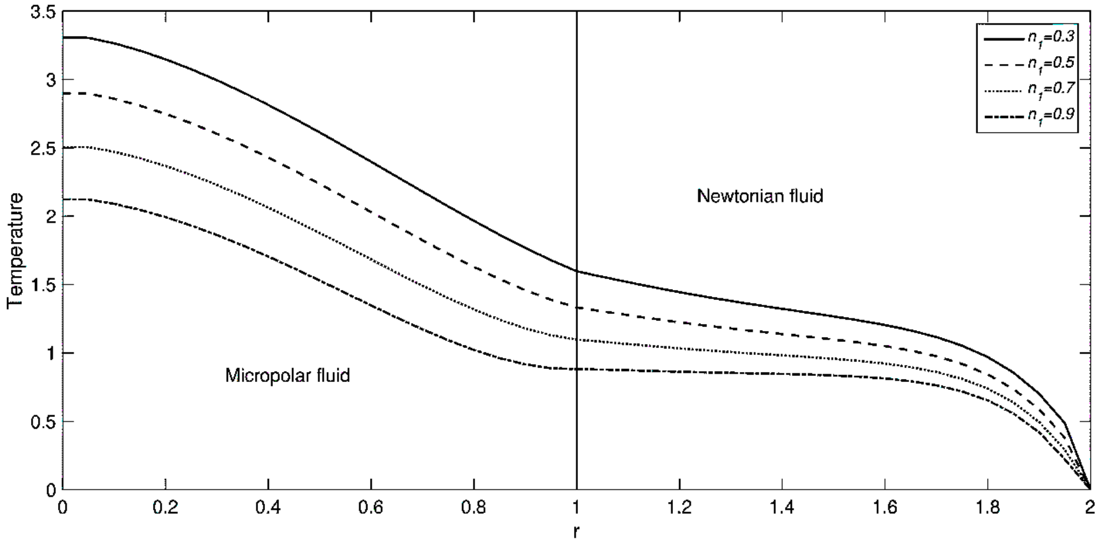

Figure 4 Fluid temperatures with varying micropolarity parameter

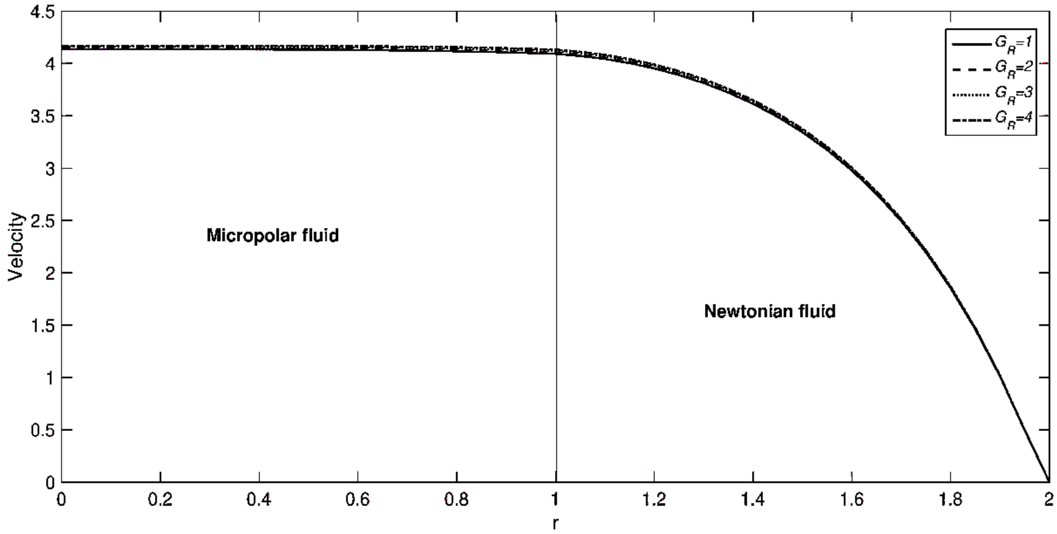

Figure 5 Fluid velocities with varying Grashof number

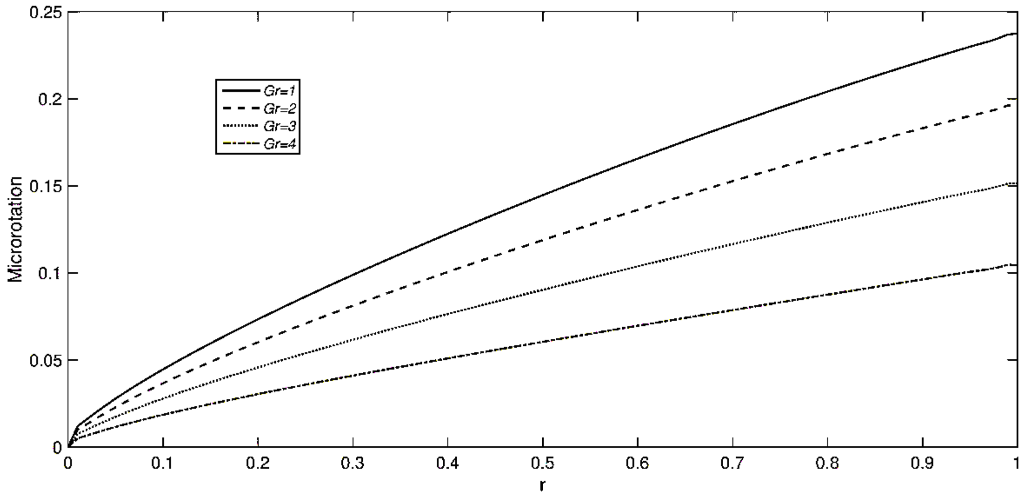

Figure 6 Microrotation with varying Grashof number

Table 1: Volume flow rate for various values of fluid parameters

Gr	*Q*	*Re*	*Q*	*BR*	*Q*	*n*1	*Q*
1	10.5086	1	10.5086	0.2	10.2234	0.3	10.8140
2	12.1045	2	12.2153	0.4	10.5086	0.5	10.5086
3	14.6025	3	13.4376	0.6	10.7828	0.7	10.2304
4	18.6483	4	14.1438	0.8	11.0296	0.9	9.9779

Table 2: Skin friction coefficient for various values of fluid parameters

Gr	$(Cf)r=s$	*Re*	$(Cf)r=s$	*BR*	$(Cf)r=s$	*n*1	$(Cf)r=s$
1	27.1661	1	27.1661	0.2	26.3072	0.3	27.7491
2	30.5084	2	19.4610	0.4	27.1661	0.5	27.1661
3	37.0376	3	16.4155	0.6	28.1708	0.7	26.7039
4	50.7425	4	14.5285	0.8	29.2451	0.9	26.3533

Table 3: Nusselt number for various values of fluid parameters

Gr	$(Nu)r=s$	*Re*	$(Nu)r=s$	*BR*	$(Nu)r=s$	*n*1	$(Nu)r=s$
1	4.2393	1	8.7795	0.2	1.9470	0.3	3.7871
2	4.4610	2	21.4166	0.4	4.2393	0.5	4.2393
3	4.6959	3	33.2185	0.6	7.3354	0.7	5.2790
4	4.9380	4	45.1128	0.8	11.7605	0.9	6.9495

5. Conclusions

This section studies the unsteady mixed convective flow of two immiscible micropolar and Newtonian fluids through the vertical pipe. The governing non-linear, coupled partial differential equations are solved numerically using the Crank-Nicolson and Newton's methods. The outcomes of the study are presented hereunder:

- Fluid velocities in the case of mixed convective flow are found to be more than forced convective flow.
- The obtained numerical solutions for fluid velocities and temperatures are found to be grid-independent.

- Unlike in the case of forced convective flows, there is an impact of heat transfer parameters like Brinkmann number and Prandtl number, not only on temperature profiles but also on the fluid velocity profiles.
- Volume flow rate is increasing with Reynolds number while it is reduced by an increase of microrotational effects.
- The skin friction coefficient at the boundary of the pipe decreases with the Reynolds number.

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