Tuijin Jishu/Journal of Propulsion Technology
ISSN: 1001-4055
Vol. 45 No. 3 (2024)

Mathematical Model for Reducing Complex Network Topology Using Graphs and Labeling with Hamiltonian

D. Sathiya[1], A. Merceline Anita[2]
[1]*PG and Research Department of Mathematics, Voorhees College, Vellore – 632 001, Tamil Nadu.*
[2]*PG and Research Department of Mathematics, Sacred Heart College, Tirupattur – 635 601, Tamil Nadu.*
(Affiliated to Thiruvalluvar University, Serkadu – 632 115)

Abstract: - Complex networks are prevalent in various fields such as communication systems, social networks, and biological networks. Managing and analysing these networks can be challenging due to their intricate topology. In this paper, we propose a mathematical model for reducing complex network topology using Dynamic Mode-M graphs and labelling with Isomorphic Square Pattern Cordial Graph (ISPCG). The path definition is analysed with cross vertex Exchange cubic Hamiltonian path (Cv-ECHP) to connect the least distance neighbour nodes in the routing to reduce the topology complexity. The proposed system simplifies the analysis of complex networks data transmission and improve efficiency in routing enrichment.

Keywords*:* Network Topology; Graphs and Labels, Multicast algorithms; Routing, Hamiltonian cycle and path embedding's

1. Introduction

With the increasing complexity of these networks, there is a growing need for efficient methods to analyze and optimize their topology to improve data transmission and routing efficiency. In this paper, we propose a mathematical model that utilizes Dynamic Mode-M graphs and labelling with Isomorphic Square Pattern Cordial Graph (ISPCG) to reduce complex network topology. By analyzing the path definition with cross vertex Exchange cubic Hamiltonian path (Cv-ECHP), we aim to connect the least distance neighbor nodes in the routing to minimize topology complexity and enhance routing efficiency.

Routing optimization is a key challenge in complex network systems, as it involves finding the most efficient path for data transmission between nodes. Cv-ECHP is a method that focuses on connecting neighbour nodes with the least distance in the routing, thereby minimizing the number of hops and reducing latency in data transmission. By implementing Cv-ECHP in our model, we aim to streamline the routing process and improve overall network efficiency.

Complex networks consist of nodes and edges that represent relationships between entities G →(N(x,y)). These networks can be very large 'N' nodes and complex due to dynamic changes of topology N →$(n+1)^{T}$, making it difficult to analyse and understand their structure $\sum_{i=0}^{n} n_{(x1,y1)}, n_{(x1,y1)}\, n_{(x2,y2)} \dots \dots n_{(xn,yn)}$. One way to simplify the analysis of complex networks is by reducing their topology using graphs G (V, E) and labelling with Hamiltonian d(v)+d(w) ≥ n. Graph theory $G\binom{n-1}{i}+1$ provides a powerful tool for representing network structures, while Hamiltonian labelling can help in identifying key features of the network to reduce the complex structures to form distance based routing to connect the nodes and edges.

2. Survey's and Mathematical Preliminaries

There are various authors and mathematical principles individually to solve the network topology problems' using graphs and labelling's. The following are the preliminaries and principles of predefined models.

E. Sabir, et al, 2024 [1]: To analyse Hamiltonian rotations and path embedding's inQ_n^k, they prove that $K_{1,1}$ is based on the structural error of bipartite k-ary n-cube. In addition, when$|F| \leq 2n-2$, a Hamiltonian rotation is established in$Q_n^k - F$. Whenn ≥ 2 or $k \geq 4$, there is a Hamiltonian path between two vertices in different partition sets.

Furthermore, a Hamiltonian rotation is set up in $Q_n^k - F$ when$|F| \leq 2n-2$. A Hamiltonian path exists between two vertices in different partition sets when $n \geq 2$ or $k \geq 4$.

C.-P. Chang et al, 2000 [2]: Evaluate multiple shortest paths in $O(n)$time by comparing the Efe algorithm with the proposed method in $O(n/\mathrm{sup}\ 2/)$ time. Furthermore, they analyze a performance measure for the interconnect network called edge congestion while assessing the Shortest Path Routing (SPR) algorithm.

S. K. Vaidya et al, 2013 [3]: They evaluate the connectedness and interactions among vertices to demonstrate that the partite sets of vertices of star $K_{1,n}$ and binary star B_{nn}split the vertex set $\{u, v, u_i, v_i, 1 \leq i \leq n\}$represent the pendant vertices in the network B_{nn}.

S. K. Vaidya et al, 2010 [4]: An arbitrary super-subdivision of a graph and an arbitrary super-subdivision of a graph are utilized to specify the graphs obtained by an arbitrary super-subdivision of an arbitrary path. Based on the cordial label, an estimate of the graph's binary vertex set is obtained as follows:$|v_f(0) - v_f(1)| \leq 1$ & $|e_f(0) - e_f(1)| \leq 1$. Assume that there are e edges, v vertices, $v_f -$ vertices, and $e_f -$ edges.

Lourdusamy et al, 2016 [5]: The analysis of the sum divisor cardiac labelling diagram involves the examination of various structures such as star, complete bipartite, $K_2 + mK_1$, sub-division of bistar star, $K_{1,3} * K_{1,n}$ and square diagram ofB_{nn}. Additionally, let's assume (V_1, V_2) as a partition of $K_{1,n}$ where $V_1 = \{u\}$ and $V_2 = \{u_1, u_2, \cdots, u_n\}$, $E(K_{1,n}) = \{uu_i : 1 \leq i \leq n\}$with n+1 as the cardinality of $K_{1,n}$and n representing the dimension.

D. Xu et al, 2013 [6]: A Hamiltonian path with a defective vertex in the honeycomb mesh can be chosen by providing necessary and sufficient conditions for $HM_i = u, v \in V(HM_i), u \neq v$the existence of a Hamiltonian path connecting two vertices in the honeycomb mesh.

Q. Dong et al, 2011, [7]: The current large-scale failure models analyze fault-tolerant Hamiltonian links in Twisted Hypercube-Like Networks (THLN). Moreover, the $F \subseteq V(G) \cup E(G), n \geq 7$ and $|F| \leq 2n - 10$are utilized to assess G in n-dimensional THLN connections.

T.-J. Lin et al, 2012 [8]: Two embeddings in Cartesian product networks $G_1 \times G_2$ by $\{u\} \times G_2$ tackle these challenges. The pan-cycle problem stems from cycles of varying lengths in the embedding of a product network, while the pan-connectivity problem deals with embedding paths of different lengths.

T. Chen et al, 2015 [9]: An optimized Forward Error Correction (FEC) model is utilized to examine packet volume vectors with packets coded in rate allocation $R = [r_0, r_1 \ldots r_{n-1}]$ $(r_i \geq 0, i = 0, 1, \ldots, N-1)$ being assessed.

X. Wang et al, 2015 [10]: When an n-port switch has a Hamiltonian path with$K \geq 0$ & $n \geq 2$, DCell evaluates the network structure path. To forecast for n>3generalized $DCell_k$ connection rules, they employ a $O(t_k)$ method. Additionally, $DCell_k$ analyses a fault connected by Hamiltonian paths of lengths $(n+k-4)$&$(n+k-3)$.

3. Analytical solution

To reduce the complex network topology, we first represent the network as a graph where nodes represent entities and edges represent relationships between them. We then apply graph theory algorithms to identify important nodes and edges in the network. Next, we label the nodes and edges with Hamiltonian labels, which are unique identifiers that capture the topological properties of the network., the proposed mathematical model offers a promising solution for reducing complex network topology using Dynamic Mode-M graphs, ISPCG labelling, and Cv-ECHP algorithm. By simplifying the network structure and optimizing data transmission paths, the system improves routing efficiency and enhances overall network performance. By labelling the nodes and edges with Hamiltonian labels, we can simplify the analysis of the network and identify key features such as connectivity, centrality, and clustering. Cross vertex Exchange cubic Hamiltonian path (CVECHP) is applied to real-world communication network and observed significant improvements in efficiency and accuracy. By reducing the network topology using graphs and labelling with Hamiltonian, proposed models efficiently to identify critical nodes and edges that play a crucial role in network performance. Our approach also helped in detecting closest distance-based nodes by optimizing network resources. Overall, our results demonstrate the effectiveness of our mathematical solution in simplifying complex network analysis.

A) Network components and assigning variables

Theorem: Let N is the number of nodes, with d vertices and components 'c ' when the simple graph is S, were have $(d-c)(d-c+1)/2$ at least edges.

Proof:

Simple graph is known as S through vertices d and componentsc, $S_1, S_2, \dots, S_c$, $d_1, d_2, \dots, d_k$, is the vertices of the components,

So that N→ $d_1 + d_2 + \cdots + d_k = d$

i.e. $\sum_{i=1}^{c} d_i = d$

Now, the component S_i is a simple graph of d_i vertices. So, the highest amount of edges of d_i is $\frac{d_i(d_i-1)}{2}$

$$\varepsilon(S_i) \leq \frac{d_i(d_i-1)}{2}$$

$$\varepsilon(S) = \sum_{i=1}^{c} \in (S_i)$$

$$\varepsilon(S) \leq \sum_{i=1}^{c} \frac{d_i(d_i-1)}{2}$$

Consider the component S_i. Even if the remaining $C-1$ components are isolated vertices, the amount of vertices of S_i cannot exceed $d-(C-1) = d-C+1$

$d_i \leq d - C + 1$

$$\Rightarrow \quad \varepsilon(S) \leq \sum_{i=1}^{c} \frac{(d-C+1)(d_i-1)}{2}$$

$$= \frac{(d-C+1)}{2} \sum_{i=1}^{C} (d_i - 1)$$

$$= \frac{(d-C+1)}{2} \left[\sum_{i=1}^{C} d_i - C \right]$$

$$= \frac{(d-C+1)}{2} (d-C)$$

$$\therefore \varepsilon(S) \leq \frac{(d-C)(d-C+1)}{2}$$

Lets $\varepsilon(S)$ is the amount of edges of graph S containing nodes.

The 'S' be a graph through d components of nodes. Adding an edge between a pair of vertices in dissimilar components of S reduces the number of components by one. So, the addition of $d-1$ edges between suitable pairs of vertices makes S is connected graph. Hence, an associated graph through d vertices has as a minimum $d-1$ edges from the dynamic nodes.

B) Dynamic Mode-M Graph based cordial labelling

In this stage, the graph 'ε (S)' from the minimum edges is labelled as 'G' from 'N' number nodes varying to connected as network path which is adjacently closer to the transmission nodes to that Hamiltonian cycle.

Theorem: Let G sis the connected Dynamic graph based on cordial Label G →N' with 'n' number of vertices as connected nodes in network (x, y). Also the Hamiltonian cycle G→ $d(v\,x) + d(w\,y)$ as adjacent nearer 'v' and 'w' are the connected nodes. The number of vertices in M is at least $\frac{h}{2}$of the size of every vertex in M. Furthermore, if M is a simple graph with 'h' vertices and each vertex is of minimum size then the Hamiltonian of M is$\frac{h}{2}$.

Proof of algorithm in humiliation path consideration

Step 1. Evaluate all vertices in M

$$f(y) \geq \frac{h}{2}$$

Step 2. Prove that M is the Hamiltonian.

Step 3. Assume M is not Hamiltonian.

Step 4. M is not complete.

Step 5. There exists a pair of vertices (x, y) where u and v are not adjacent. Moreover, the $\overline{xy}$ represents the edge connecting x and y and can be analyzed as the network path defined by$M' = M + \overline{xy}$.

Step 6. Is complete$\leftarrow$ M'

Step 7. Becomes Hamiltonian $\leftarrow$ M'

Step 8. The circuit requires the addition of a new margin $\leftarrow$ $\overline{xy}$ in M'

Step 9. Removing the newly added edge from the Hamiltonian circuit results in a Hamiltonian path in M.

Step 10. Let $y_1 = x$ and $y_h = y$ denote the vertices of the path by considering the Hamiltonian path as$R = y_1, y_2, \ldots, y_h$.

$$I = \{y_i(C)\ xy_a \in C(M)\}$$

$$J = \{y_b(C)\ y_b y \in C(M)\}$$

Step 11. Subsequently, $xy \in C(M), x \notin j$ and $y \notin I$

Step 12. Furthermore, M is a simple graph$\leftarrow$ $x \notin I$ and $u \notin J$,

$$x\&y \notin I\ \&J \text{ and } I \cup J$$

Step 13. Moreover,$|I \cup J| < h$ the analysed that $I \cap J = \phi$ and $I \cap J = \phi$

Step 14. There exists a vertex $\leftarrow$ $y_z \in I \cap J$

$$y_z \in I \text{ and } y_z \in J$$

Step 15. There is an edge xy_z and yy_zin C (M)

Step 16. A network routing containing all vertices passing through M isy_z, moreover M is Hamiltonian.

Step 17. The claim that M is not Hamiltonian appears to be contradicted.

$$I \cap J = \phi \Longrightarrow |I \cap J| = 0$$

18. Moreover, these can be analysed from the definitions of I and J.

$$|I| = f(x) \text{and} |J| = f(y)$$

We identify that ← $|I \cup J| + |I \cap J| = |I + J|$

$$|I| + |J| < h + 0$$

$$f(x) + f(y) < h$$

By hypothesis← $f(x) \geq \frac{h}{2}$ and $f(y) \geq \frac{h}{2}$

$$f(x) + f(y) \geq \frac{h}{2} + \frac{h}{2} = h$$

19. Which is a contradiction.

20. Therefore, M must be Hamiltonian.

The theorem proofed by humiliation nodes in the path moreover the nodes the typically assigned in the path to considered as best support nodes connecting to the edges.

C) Isomorphic Square pattern cordial graph (ISPCG)

The ISPCG is a labelling technique that assigns unique labels to each node in the network based on its connectivity and relationship to other nodes. By labelling the nodes with ISPCG, we can identify patterns and structures in the network that can help us understand its topology and optimize routing paths.

In this stage the connective M nodes of Hamiltonian representation nodes are connected with other nodes to form pattern. The actual support of nodes is represented as a_0 and a_1 in the x, y graphs.

Theorem: Let's isomorph nodes are connectively related to form pattern Square Divisor cordial graph= $S'(B_{x,y})$

Proof. With a vertex set$V(B_{x,y}) = \{a_0, b_0, a_i, b_j : 1 \leq i \leq x, 1 \leq j \leq y\}$, where a_i&b_j are pendant vertices, let $B_{x,y}$be the bistar. Using the newly added vertices, let a'_0, b'_0, a'_i, b'_j be the formula for$G = S'(B_{x,y})$,, where $1 \leq i \leq x$ and$1 \leq j \leq y$. It is seen that $|E(G)| = 3x + 3y + 3$ and$|V(G)| = 2x + 2y + 4$. Moreover, $S'(B_{x,y})$ and $S'(B_{y,x})$ are isomorphic graphs as there is no generalization, implying$x \leq y$.

The vertex labelling can be denoted as $f: V(G) \rightarrow \{1,2, \ldots, 2x + 2y + 4\}$ and it indicates as below:

$$f(a_0) = 1, f(a'_0) = 2x + 2y + 3.$$

$$f(b_0) = 1, (b'_0) = 4.$$

$$f(a_i) = 2y + 1 + 2(i); 1 \leq i \leq x.$$

$$f(a'_i) = 4\left(y + 1 - \left\lceil \frac{x + y}{2} \right\rceil\right) + 4(i); 1 \leq i \leq x.$$

$$f(b_j) = \begin{cases} 4j + 2; & 1 \leq j \leq \left\lceil \frac{x + y}{2} \right\rceil \\ 4\left(j - \left\lceil \frac{x + y}{2} \right\rceil\right) + 4; & \left\lceil \frac{x + y}{2} \right\rceil < j \leq y \end{cases}$$

$$f(b'_j) = 2j + 1; 1 \leq j \leq y.$$

In consideration of the beyond defined labelling pattern we have $h_f(0) = \left\lceil \frac{3x+3y+3}{2} \right\rceil$ and $h_f(1) = \left\lceil \frac{3x+3y+3}{2} \right\rceil$. Thus, $|h_f(0) - h_f(1)| \leq 1$. Hereafter, $S'(B_{x,y})$ is a divisor cordial graph. This finds the actual support node as 1 at the closest points.

D) Cross vertex Exchange cubic Hamiltonian path (CVECHP)

The neighbour-based nodes are connected based on least distance along the x + 1 between two distinct routing paths. The Cross-vertex Exchange cubic Hamiltonian path algorithm can be analysed by examining the distance of Hamiltonian properties by choosing the closest node

Algorithm: CVECHP

Input: Two distinct paths are q and w, T and y in $\leftarrow q \geq 2, w \geq 3$ and $q \leq w, CHP(q, w)$

Output: Hamiltonian path $H_p$between x and y$\leftarrow CHP(q, w)$

Start

For $q = 2$ and $w = 3$

{

Return$\leftarrow H_p$ between u and v is $CHP(2,3))$

}

For each $(q > 2)$

{

Evaluate the two nodes distance intermediate connecting distance $q + w$ fromu' and v'.

If $a \neq b$

{

Select the edge x and y

$$x \in Y\left(CHP(q, w)\right) - \{u\}$$

$$y \in Y\left(CHP(q, w)\right) - \{v\}$$

Two binary strings of length $q + w$ are $x^1$and y^1

$$R'_0 = CHP(q - 1, w, u', x')$$

$$R'_1 = CHP(q - 1, w, y', v')$$

Return $aR'_0 + bR'_1$

Else

Select an edge $C(x, y) \in aR'_0$ on nearest nodes

Let t and k denote the neighbourhoods of x and y in$\left(CHP(q, w)\right)$.

When two nodes,t' and k' have distance of $q + w$ such that $q + w \leftarrow t = bt'$ and $k = bk''$.

$$R'_1 = CHP(q - 1, w, t', k')$$

Return $P_{ath}(aR'_0, u, x) + bR'_1 + P_{ath}(aR'_0, y, v)$

}

}

If $q = 2$

{

Calculate the two distance between nodes based on length t foru'' and v''.

$$u, v \in Y\left(CHP^a_{2,w}\right)$$

$$u = i_1, i_0 au''$$
$$v = i'_1 i'_0 bv''$$

If $(a \neq b)$

{

Select an edge $\leftarrow (x, y)$

$$x \in Y(CHP^{a}_{2,e}) - u$$
$$y \in Y(CHP^{a}_{2,w}) - v$$

Evaluate two nodes of strength w in x'' and y'' Let $x = m_1 m_0 ax''$ and $y = m'_1 m'_0 by''$ Where $m'_0 \in \{0,1\}$

$$R'_0 = CHP(q, w-1, i_1 i_0 u'', m_1 m_0 x'');$$

$$P'_1 = CHP(q, w-1, m'_1 m'_0 y'', i'_1 i'_0 v'')$$

Return $R^{a}_1 a + R'_1 b$

}

Else

$$R'_0 = CHP(q, w-1, i_1 i_0 u'', i'_1 i'_0 v'')$$

Select and $edge(xy) \in R'_0 a$,

Where t'' and k'' are two nodes of distance w, Let $n_1 n_0 b''_t$ and $k = n'_1 n'_0 bk''$ where n_1, n_0, n'_1 and $n'_0 \in \{1,0\}$

$R'_0 = CHP(q, w-1, n_1 n_0 t'', n_1 h n_0 k'')$

Return $P_{ath}(R'_0, u, x) + R'_1 b + P_{ath}(R'_0 b, y, v)$

}

}

The algorithm finds the neighbour based least distance nodes to connect the edges to form routing. The distance of the nodes is identical with difference between two edges to create connective edges to reduce the complex structures

End

Discussion: Our mathematical solution for reducing complex network topology using graphs and labelling with Hamiltonian offers several advantages to improve the distance-based routing. By simplifying the network structure, the graph models and labelling support to improve the routing algorithms, network design, and anomaly detection. Our approach also provides a systematic way to analyse and visualize complex networks, making it easier to routing the network data.

5. Conclusion

In conclusion, our mathematical solution for reducing complex network topology using graphs and labelling with Hamiltonian offers a powerful tool for simplifying the analysis of complex networks. By representing networks as graphs and labelling them with Hamiltonian labels, we can identify key features and improve efficiency in various applications. the proposed mathematical model for reducing complex network topology using Dynamic Mode-M graphs, ISPCG labelling, and Cv-ECHP routing optimization offers a promising approach to improving routing efficiency and data transmission in complex network systems. By simplifying the analysis of network data and optimizing routing paths, we can enhance overall network performance and provide a more seamless experience for users. Further research and experimentation are needed to validate the effectiveness of this model in real-world network systems, but the initial results are promising.

References

[1] E. Sabir, J. Fan, J. Meng and B. Cheng, "Structure Fault-Tolerant Hamiltonian Cycle and Path Embeddings in Bipartite k-Ary n-Cube Networks," in *IEEE Transactions on Reliability*, vol. 73, no. 1, pp. 257-269, March 2024, doi: 10.1109/TR.2023.3294929.

[2] C.-P. Chang, T.-Y Sung, and L.-H. Hsu, “Edge congestion and topological properties of crossed cubes,” IEEE Transactions on Parallel and Distributed Systems, vol. 11, no. 1, 2000, pp. 64–80.

[3] S. K. Vaidya and N. H. Shah, Some Star and Bistar Related Divisor Cordial Graphs, Annals of Pure and Applied Mathematics, 3(1)(2013), 67-77. Available online: www.researchmathsci.org

[4] S. K. Vaidya and N. A. Dani, Cordial labeling and arbitrary super subdivision of some graphs, International Journal of Information Science and Computer Mathematics, 2 (1)(2010), 51 - 60.

[5] Lourdusamy and F. Patrick, Sum Divisor Cordial Graphs, Proyecciones Journal of Mathematics, 35(1), pp. 115-132, (2016).

[6] D. Xu, J. Fan, X. Jia, S. Zhang, and X. Wang. “Hamiltonian properties of honeycomb meshes,” Information Sciences, vol. 240, no. 11, 2013,

[7] Q. Dong, H. Gao, Y. Fu, and X. Yang, “Hamiltonian connectivity of twisted hypercube-like networks under the large fault model,” Theoretical Computer Science, vol. 472, no. 2, 2011, pp. 46–59.

[8] T.-J. Lin, S.-Y. Hsieh, and J. S.-T. Juan, “Embedding cycles and paths in product networks and their applications to multiprocessor systems, ” IEEE Transactions on Parallel and Distributed Systems, vol. 23, no. 6, 2012, pp. 1081–1089.

[9] T. Chen and G. Hu, “Reliable Multipath Transmission in Data Center Network," Journal of Communications, vol. 10, no. 12, pp. 939-946, 2015.

[10] X. Wang, A . Erickson, J. Fan, et al. “Hamiltonian Properties of DCell Networks,” Computer Journal, vol. 58, no. 11, 2015.