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Study on Polarization Phase Shifting Interferometry with Quarter Wave Plate Different from Monitoring Wavelength

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Abstract: Polarization Phase Shifting Interferometry is a relatively new technique in which phase shifted interferogram are generated using polarization optics. The presented interferometer is providing a easy learning on phase shifting interferometer with not so accurate components making realization of setup easy and leaning phase distribution. The present study introduce a novel polarization phase-shifting interferometer that acquires four phase-shifted fringe patterns without stringent requirement of thickness of quarter wave plate (QWP) to be exactly quarter wave at that wavelength of the Laser source used with the experiment which is reported for the first time. In this study interferometer is in a Twyman-Green configuration and the interference pattern were acquired with a Charged Coupled Device (CCD) camera using LabVIEW IMAQ interface and processed with MATLAB. Concept of Jones matrix is applied to evaluate effects of changes in phase calculation due to non-exact quarter wave plate. The experimental results showing the 2-D phase and its spatial variations are also presented.

Keywords: Non-exact wavelength Quarter wave plate, Interference, Polarization Phase Shifting Interferometry, Phase Analysis, Phase Distributions

1. Introduction

Phase shifting, often known as phase stepping, is one of the most effective trending techniques for extracting phase information from fringe patterns. Phase-shifted fringe patterns are recorded and by resolving an equation system for three or more phase shifted interferograms recorded at equal phase steps, the phase of the object can be recovered by using algorithms[1].A phase-shifting interferometer has applications in various fields including[2] high-accuracy optical measurements[3], digital holography[4], speckle metrology[5], and other[6].In a typical phase-shifting interferometer, minimum of three phase-shifted interferograms are needed to calculate the phase distribution because the interferogram's irradiance has Bias, modulation, and fringe phase are three unidentified variables[7]. Among phase extraction algorithms, employing four interferograms has proven to be extremely helpful as they provide a decent contrast and acceptable signal-to-noise ratio [8].Various variation of phase shifting using concept of polarization has been reported earlier[9–12] but all have used QWP exactly at monitoring laser wavelength. The method of polarization phase shifting interferometry can be used to determine material property like refractive index[13], surface profile[14], displacement[15] etc.It also offers some advantages such as easy operation, quick measurement and a straightforward optical setup. To estimate the thickness of transparent thin films, an alternate polarization phase-shifting technique was proposed with a cyclic interferometer to keep the operation stable over external vibrations[16].Zurita G. *et al* suggested a single-shot interferometer using four interferograms featuring the use of phase vortices of unitary topological charge. Using polarization phase shifting and field replication methods, the optical wrapped phase was determined using a four-step algorithm[17].Recently ,measurement inaccuracy in dynamic interferometer generated by a quarter wave plate was demonstrated[18].

The present study shows retrieval of 2-D phase map using polarization and phase shifting interferometery concept using non exact wavelength quarter wave plate (a quarter wave plate not true at measurement wavelength but at a nearby wavelength). Although 2D image acquisition and processing makes this kind of device a little bit slow, but the amount of information and its ease far outweighs this loss. Also this technology is still improving and the region of interest (ROI) approach available with VISION tools can bring faster image and data processing.

Therefore the primary objective of the present work is to automatically calculate the 2-D phase distribution linked to the recorded fringe pattern with ease and to look for minute deviations in the phase distribution cost effectively. Non –exact wavelength quarter wave plate is used for phase retrieval thereby providing better tolerance on this critical component. Complete Jones-matrix formulation is developed and it is suitable for the present kind of setup. Minimum components in setup for similar or better results have been targeted and a new approach has been tried to achieve the results.

2. Theoretical background

2.1 Quarter wave plate

A linearly polarized beam given by $E = xE_{ox}e^{i(\kappa z-\omega t)} + yE_{0y}e^{i(\kappa z-\omega t)}$ is used for polarization phase shifting interferometry incident normally at x-axis-aligned non-exact quarter wave plate then the transmitted wave equation would be

$$E = xE_{0x}e^{i(\kappa z-\omega t)} + yE_{0y}e^{i(\kappa z-\omega t+\zeta)} \tag{1}$$

Where E_{0x} and E_{0y} component of electric field in x & y direction .κ , ω are wave vector, angular frequency of propagating wave and ζ is phase shift in transmitted wave due to non- exact quarter wave plate.This takes following form if it is exact quarter wave plate[19]

$$E = xE_{0x}e^{i(\kappa z-\omega t)} + yE_{0y}e^{i(\kappa z-\omega t\pm\frac{\pi}{2})} \tag{2}$$

2.2 Phase-shifting interferometry

In a conventional interferometric apparatus, a laser beam is enlarged and collimated using lenses, and it is then split into two beams by a beam splitter. Such split beams are designed to pass across two various optical arm of interferometer before being recombined to create an interference pattern. Here, it is specifically stated that the two overlapping wavefields, E_1 and E_2 which gives interference pattern can be written as[11]

$$E_1(x,y) = E_1(x,y)e^{i\phi_1(x,y)} \text{ and } E_2(x,y) = E_2(x,y)e^{i[\phi_2(x,y)+\psi]} \tag{3}$$

Whereφ$_1$(x, y), φ$_2$(x, y) are the localized phases of the fields and ψ is an extra phase controlled by optical setup. The interference pattern's light intensity distribution, I(x, y), is influenced by the comparative phase of the two wavefields[20].

$$I(x,y) = I_1(x,y) + I_2(x,y) + \sqrt{I_1 I_2}\cos(\phi+\psi) \tag{4}$$

I_1, I_2 are the individual field's intensities and $\varphi = \varphi_2 - \varphi_1$ is the local phase difference.

When performing unknown phase-shift, it is set to assume the values Δ, 2Δ, and 3Δ in that order. In order to produce intensities I_I, I_{II}, I_{III}, and I_{IV} in accordance with[21]

$$I_I = I_1 + I_2 + \sqrt{I_1 I_2}\cos(\phi), \qquad I_{II} = I_1 + I_2 + \sqrt{I_1 I_2}\cos(\phi+\Delta) \tag{5}$$

$$I_{III} = I_1 + I_2 + \sqrt{I_1 I_2}\cos(\phi+2\Delta) \text{ and } I_{IV} = I_1 + I_2 + \sqrt{I_1 I_2}\cos(\phi+3\Delta).$$

After that, the phase recovery procedure is applied by computing[21]

$$\delta(x,y) = \tan^{-1}\left\{\tan\beta\frac{[I_{II}-I_{III}]+[I_I-I_{IV}]}{[I_{II}+I_{III}]-[I_I+I_{IV}]}\right\} \tag{6}$$

Where(*x*, *y*) is distribution [10] of phase and β is

$$\beta = \tan^{-1}\sqrt{\frac{3[I_{II}-I_{III}]-[I_I-I_{IV}]}{[I_{II}-I_{III}]+[I_I-I_{IV}]}} \quad (7)$$

As can be seen from Eq. (6), the phase-shift algorithm is especially efficient and reliable. It is free from gain effects, which are eliminated by taking the ratio between the resulting differences, as well as offset effects of the detected signal, which cancel out by subtraction. In fact, the phase-shift algorithm is presently the primary approach for data collecting and processing in interferometry equipment and is available in a variety of implementations.

2.3 Jones Matrix and vector formalism

It is possible to explain for propagation inside interferometric optical setups using the Jones matrix and vector formalism, which is typically used to investigate polarized [22][23]. The electric field vector can be written as

$$E = \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

Where Ex and Ey are the scalar Cartesian elements of E. When the source field intensity is normalized to unity as is customary, the light entering the interferometer is shown to be linearly polarized at an angle of 45 degrees to the x-axis as

$$E = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

By returning from the crystal to the laboratory coordinate system, the Jones vector of the polarization state of the exiting beam in the xy coordinate system due to retardation plate is obtained as

$$\begin{pmatrix} E_x' \\ E_y' \end{pmatrix} = R(\theta)W_0R(-\theta)\begin{pmatrix} E_x \\ E_y \end{pmatrix} \quad (8)$$

Where R (θ) is rotation matrix and W_0 is Jones matrix for wave plate.

$$R(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

And

$$W_0 = e^{-i\phi}\begin{pmatrix} e^{\frac{-i\tau}{2}} & 0 \\ 0 & -e^{\frac{-i\tau}{2}} \end{pmatrix}$$

Where τ is phase retardation(measurement of relative change in phase).Usually, the phase factor $e^{-i\varphi}$ can be omitted[22].A retardation plate is represented by the product of three matrices and is defined by its phase retardation τ and azimuth angle θ as given below

$$W = R(-\theta)W_0R(\theta) \quad (9)$$

Generally, Jones matrix for quarter wave plate is given by

$$W = \begin{pmatrix} \cos(\tau/2) & -i\sin(\tau/2) \\ -i\sin(\tau/2) & \cos(\tau/2) \end{pmatrix} \quad (10)$$

The optical components of importance in this case are a quarter-wave plate (λ/4) at 45° and linear polarizer's (LPx), (LPy) at 0° and 90° with axis, respectively. The matrices describe how they operate under ideal circumstances.

$$LP_x = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad LP_y = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{and } W = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}$$

The non-polarizing beam splitter can be calculated with the formula (BS) = 1/√2.

The wavefield is obtained as follows at the quarter-wave plate's output:

$$\begin{pmatrix} E'_x \\ E'_y \end{pmatrix} = W\left[(BS)(LP_y)e^{i\phi}(LP_y)(BS)+(BS)(LP_x)(LP_x)(BS)\right]\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{11}$$

The gap in optical paths between the interferometer's two arms is explained by the phase factor $e^{i\varphi}$. The output field vector acquired when the abovementioned processes are completed is:

$$\begin{pmatrix} E'_x \\ E'_y \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 1+ie^{i\phi} \\ i+e^{i\phi} \end{pmatrix} \tag{12}$$

This result will be obtained when exact quarter wave plate used for analysis.

The field $E_\theta = E_x\cos\theta + E_y \sin\theta$ is then transmitted via the final linear polarizer, which is at an angle θ to the x-axis. Using Euler's cosine and sine formulas, one may derive

$$E_\theta = \frac{1}{4}\left[e^{i\theta} + e^{i\left(\phi-\theta+\frac{\pi}{2}\right)}\right] \tag{13}$$

3. Experimental configuration

Fig.1. depicts the optical setup for Polarization Phase Shifting Interferometry. A linearly polarized light from a 1.5mW red He-Ne laser with a wavelength of λ = 633 nm from NEWPORT serves as the light source. In order to have the identical polarization along the x and y axes, the laser is oriented so that the plane of polarization is at 45° to the x-axis (horizontal). The incoming beam is enlarged and collimated using two lenses and then passed into Twyman- Green interferometer.

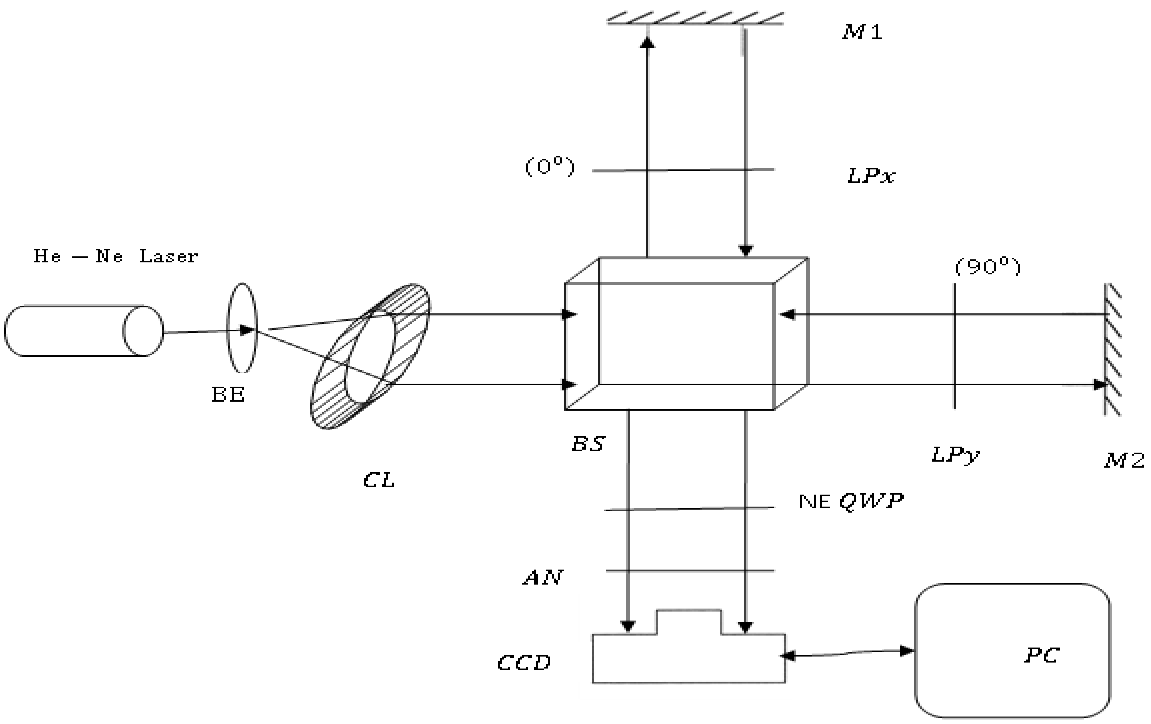

Fig.1. The configuration of proposed PPSI with Twyman-Green interferometer . BE, Beam Expander; CL, Collimator; BS, Non- polarizing beam splitter ; M1,M2, mirrors; LPx,LPy, Linear Polarizer; NE QWP, Quarter wave plate at 780nm; AN, Analyzer;

The beam is then split in two by a non-polarizing beam splitter (BS), which directs each half to one of the interferometer's two arms. The arms are constructed by two plane mirrors, Ml, M2, with linear polarizers (Polaroid sheets), LPx, and LPy, placed in front at 0° and 90° angles to the x-axis. The reflected beams were guided to a non-exact quarter wave plate (λ/4) at 633nm, as it is a quarter wave plate (λ/4) at wavelength 780nm, at an arbitrary angle to the x-axis after being recombined at the beam splitter.

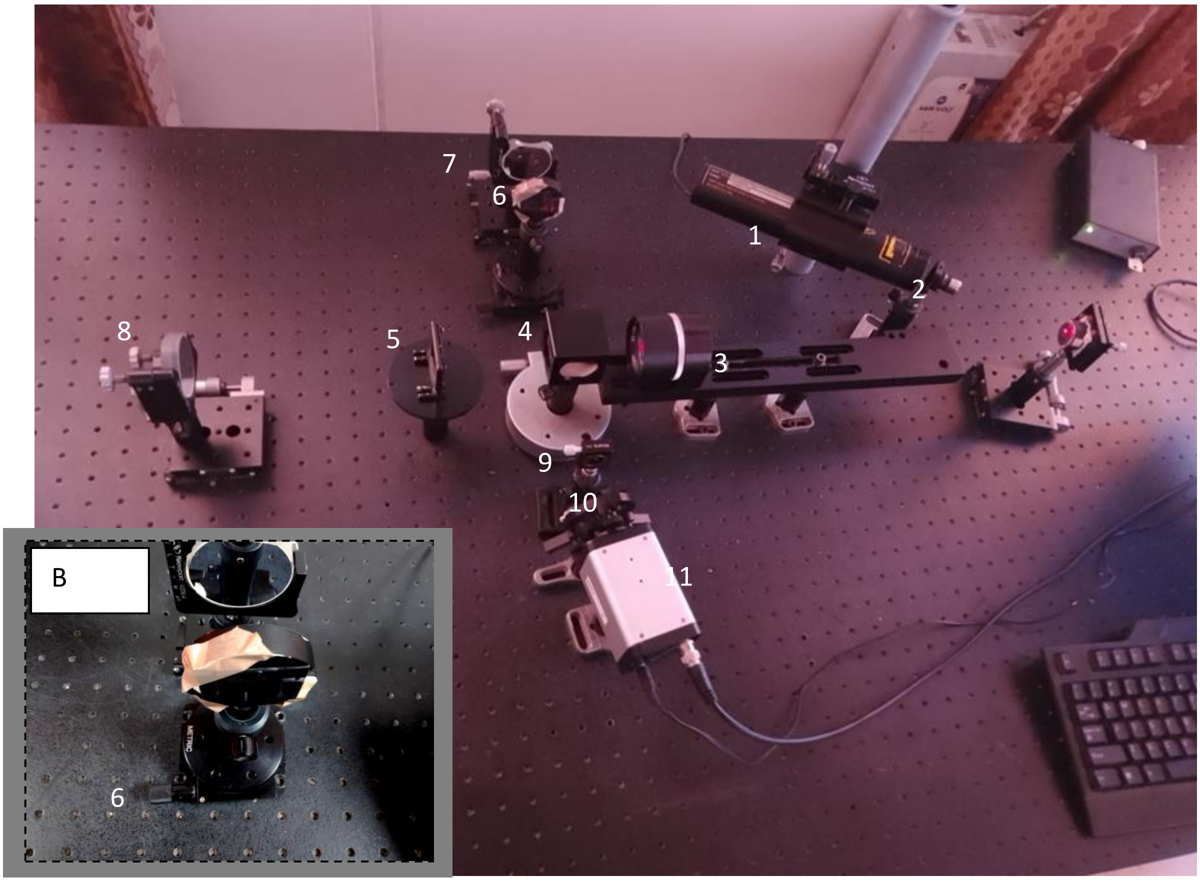

Fig.2. Experimental setup with Twyman- Green interferometer 1.Laser source;2,Beam expander;3,Collimator;4,Non-Polarising Beam splitter; 5,6,Linear polarizers LPx, LPy;7,8,Mirrors;9, Non-exact wavelength Quarter wave plate;10,Analyzer;11,CCD.Dashed box B is the top view of phase shifting device

The last linear polarizer acts as an analyzer receiving the transmitted light, which is then projected onto a CCD .Then the data obtained by CCD transferred to a personal computer through IMAQ card with LabVIEW/VISION (National Instrument Inc.) software and interference pattern were captured.

The main characteristic of the configuration used for phase-shifting interferometry is the cross-polarization of the light beams from the two arms, which causes one part of each beam to become right- handed elliptically polarized and the other part to become left- handed elliptically polarized when they pass through the quarter-wave plate. The linear polarizer LPx placed on a rotating stage used to introduce unknown phase shift in interferogram.

4. Experiment and results

On a vibration free table in the lab, the optical arrangement of Fig.1. has been realized. The mirrors, beam splitting cube and the beam expander's lenses are all interferometric grade optical parts from NEWPORT that were used. The polarizing devices used as linear polarizer made up of Polaroid sheets. Despite the fact that a sample is typically included in demonstrative sets of polarization optics among the retardation sheets, the non-exact wavelength quarter-wave plate is the most complex part. A digital camera with 24 bit RGB video format, which has a resolution of 640 x 480 pixels used to record fringe pattern in monochromatic mode.

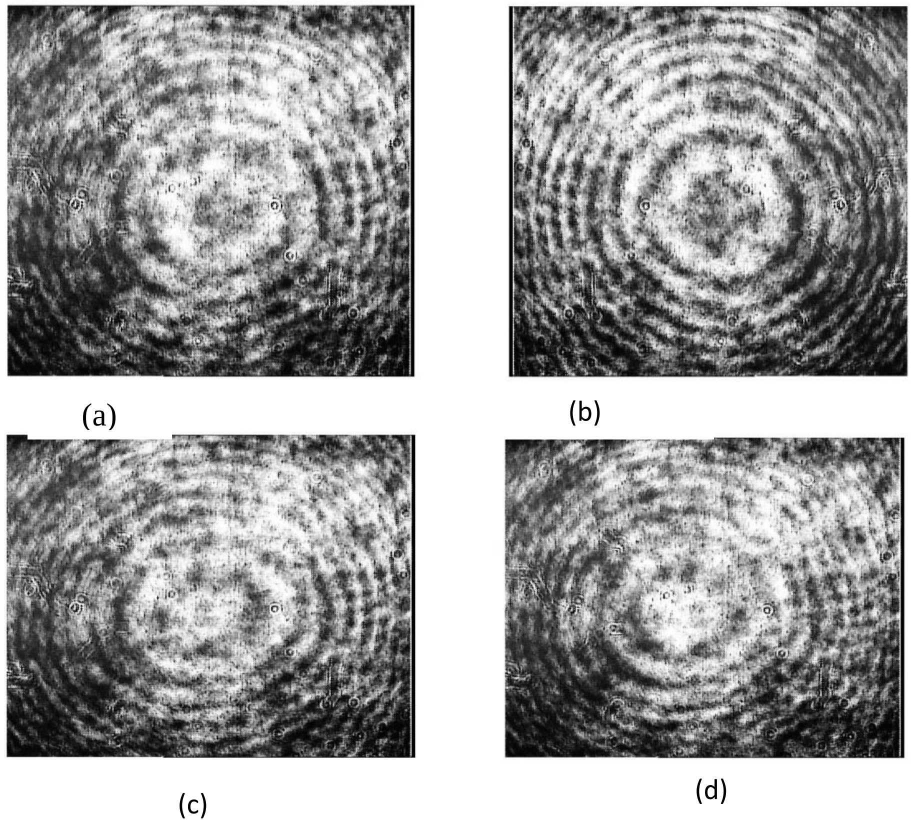

Fig.3. Four phase shifted interferogram captured by CCD

(a) Unshifted interferogram **(b) ∆ phase shifted interferogram**

(c) 2∆ phase shifted interferogram **(d) interferogram with phase shift 3∆**

To prevent disturbances and vibrations that will also obscure the fringe pattern, special care was taken to firmly attach the optical components to the table. The interferometer is then adjusted to allow for phase-shift functioning before the polarizers are fitted. Finally, the camera records the four images by rotating the linear polarizer LPx at different rotation by inserting unknown phase shift in fringe pattern. The same fringe patterns transferred to a computer through CCD are shown in Fig.3. The software MATLAB (MathWorks INc) used to analyze the resulting fringe patterns. The MATLAB program of Carrie's algorithm that reads pixels by pixels has been created four matrices which are used to store the original intensity data from the fringe pattern. The phase-shift technique of Eq. (4) is then applied to the same (i, j) element of the matrices. All of the pixels are subjected to the procedure, producing a new matrix that contains the phase map. The generated phase map is wrapped, Phase unwrapping is carried out using MATLAB programming (Fig.4). The 2-D phase variation distribution is shown in Fig.5.

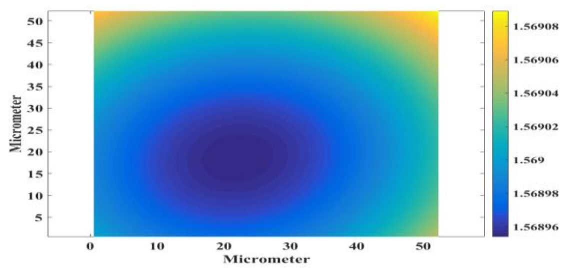

Fig.4. Unwrapped phase map of four phase – shifted interferogram shown in **Fig.3**.

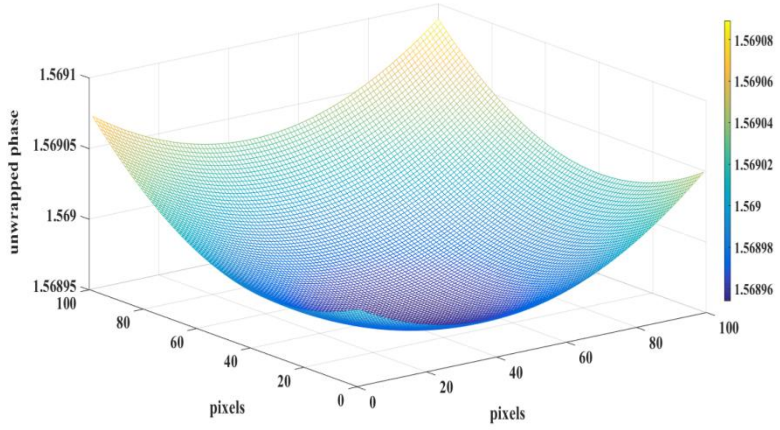

Fig.5. The 2-D phase variation distribution

The optical path difference (OPD) map gets modified when the optical configuration changes[20]. As an illustration, a piece of the glass plate is placed between the mirror and the linear polarizer in one arm of the interferometer. In the course of acquisition, processing, and subtraction, the OPD map is obtained due to double pass through the glass plate and is shown in Fig.6.

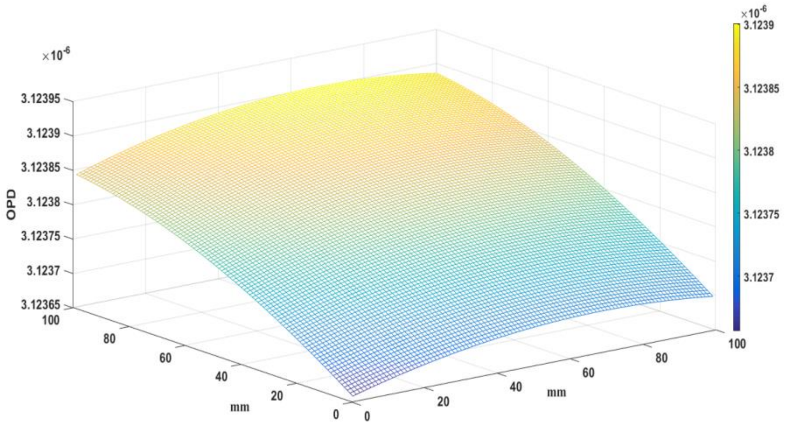

Fig.6. Optical path difference induced by BK7 glass plate

5. Discussion

In this study main sources of errors are inaccurate phase shift in data frames, vibrations, detector non-linearity, stray reflections, quantization errors, frequency stability, and intensity variation. The non-exact quarter plate which was meant at 780nm and is used at 633nm in the present study. It introduces a phase shift of 110º instead of desired 90º.Therefore the beam after passing through QWP is not circularly polarized but it is elliptical polarized. The W for that QWP becomes

$$W=\begin{pmatrix}\cos\left(55.45^\circ\right) & -i\sin\left(55.45^\circ\right)\\ -i\sin\left(55.45^\circ\right) & \cos\left(55.45^\circ\right)\end{pmatrix}$$

And Eq.(12) reduced to

$$\begin{pmatrix}E'_x\\ E'_y\end{pmatrix}=\frac{1}{2\sqrt{2}}\begin{pmatrix}0.567-i\left(0.823\right)e^{i\phi}\\ -i\left(0.823\right)+e^{i\phi}\left(0.567\right)\end{pmatrix}$$

This shows the used QWP was 23% off from exact quarter wave plate for the light used in that experiment. The phase accuracy of phase-shifting interferometry and the polarization-mixing error may have an impact on the error Δφ. As phase shifting interferometry is completely exploited, the minima and maxima of the interferograms have grey levels of 0 and 255, respectively. Consequently, this method gives theoretical resolution as $180^{o}/256=0.703^{o}$[24].Vibration errors are conquered by retrieving frames faster. In a well-designed system, detector nonlinearity does not play significant role. Finally, outcome of the present study with non-exact wavelength QWP introduces an additional phase shift of 20.703^{o} as compared to exact wavelength QWP and the results thus obtained are similar as that for exact QWP used.

6. Conclusion

A polarization phase-shift interferometry with a non-exact wavelength QWP phase stepping has been used. When light passes through non-exact QWP, the incident light beam's polarization might change linear to elliptical. Using Jones calculus, this impact is theoretically calculated and practically examined. The phase shift analyzed with non-exact wavelength (780nm) QWP introduced an additional phase steps of 20.703^{o}. The final results thus obtained are similar to those reported in literature and are within acceptable limits of 25% deviation from exact QWP.

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