Tuijin Jishu/Journal of Propulsion Technology
ISSN: 1001-4055
Vol. 45 No. 3 (2024)

Teaching Geometry in Kindergarten Through Information and Communications Technology

Zaranis Nicholas[1], Linardakis Michalis[2], Valla Vasiliki[3]
[1] *Professor, University of Crete, Rethymno, Greece*
[2] *Associate Professor, University of Crete, Rethymno, Greece*
[3] *Ph.D. Candidate, University of Crete, Rethymno, Greece*

Abstract:- The aim of the current work is to explore whether Information and Communication Technologies (ICT) can enhance preschool students' proficiency in geometric shapes. This research employs the "GeoHieleICTshapes" model, an educational software designed to introduce an innovative educational framework and foster the development of geometric thinking among kindergarten students. The study compares the academic performance of students in the experimental group, instructed using the "GeoHieleICTshapes" model, with that of students in the control group receiving traditional teaching aligned with the kindergarten curriculum. Central to this investigation is the creation of a novel classroom environment that integrates ICT activities with hands-on experiences (blended learning) to teach geometry to kindergarten children. Participants in the study were kindergarten students divided into experimental and control groups. Both groups underwent pre-tests and post-tests to assess their proficiency in geometric shapes. The results demonstrate that integrating ICT into teaching and learning cultivates an engaging and interactive learning environment for kindergarten students. Furthermore, this approach positively impacts the acquisition of geometric knowledge compared to traditional teaching methods.

Keywords*:* Geometry shapes, Information and Communications Technology, kindergarten students.

1. Introduction

Incorporating Information and Communications Technology (ICT) into education can significantly enhance the achievement of curriculum objectives across various subjects and grade levels, as long as it is facilitated by suitable software applications. Extensive research has demonstrated the benefits of using digital technologies in developmentally appropriate ways within mathematics education [1]. For over three decades, digital technologies have been included in the repertoire of mathematics teachers as tools to enhance engagement and understanding in both learning and teaching [2]. Research in Early Childhood Education (ECE) has shown that incorporating ICT can greatly enhance engagement, motivation, persistence, curiosity, and attention in kindergarten mathematics [3], [4], [5], [6].

Numerous early-year educators and researchers argue that the interactive environment provided by tablets in a kindergarten setting is more effective at sustaining children's interest in digital activities [7], [8]. This environment encourages children to become more engaged and involved in digital mathematical activities. The rapid advancement of digital technologies and devices has significantly expanded the tools available to teachers and students, including those in kindergarten education [9], [10]. Usability studies indicate that kindergarten children quickly, independently, and confidently learn to use tablets, enabling them to explore these devices freely [11]. Tablets offer several new features [12]: these devices are portable and lightweight, eliminating the necessity for separate input devices like a mouse and keyboard. Additionally, they feature a wide range of apps designed with child-friendly and intuitive interfaces. Regarding the pedagogical use of tablets in early childhood, recent studies suggest that tablets can be valuable educational tools, particularly for learning mathematical concepts [2], [15].

As discussed above, the use of ICT in mathematics education has been a focus of international research in recent years. Specifically, in the area of Geometry, numerous studies have examined the integration of ICT into teaching, highlighting the positive outcomes for students who have been exposed to these methods [16], [17]. Concurrently, researchers worldwide have explored the impact of ICT on the development of students' geometric thinking [18], [19], [1].

Pierre Marie van Hiele extensively studied Geometry and developed a theoretical model in 1957 that describes the gradual understanding of geometric concepts. The van Hiele model outlines five levels of geometric thinking: Visualization, Analysis, Informal Deduction, Formal Deduction, and Rigor. According to this model, children’s geometric thinking progresses hierarchically from the lowest to the highest level [20]. These levels emphasize how we think about geometric concepts rather than the amount of knowledge we possess at each stage, focusing on the differentiation of objects between levels. Numerous studies have validated the van Hiele model of geometric thought [21], [22].

2. Present Study

In this study, we utilized the van Hiele model to explore the impact of Information and Communication Technology (ICT) on kindergarten students' geometry achievement, focusing on shape recognition and understanding. Our intervention included activities tailored to the first two levels of the van Hiele model: Visualization and Analysis. Moreover, we integrated Alan Hoffer's five geometric skills: visual, verbal, drawing, logical, and applied skills [23].

Building on the theoretical framework that integrates the kindergarten mathematics curriculum with ICT, we developed a new model, referred to as the “Geometry van Hiele ICT shapes” model (GeoHieleICTshapes), which comprises four levels. While numerous previous studies have explored various teaching methods for geometry, few have specifically investigated the use of tablet computers to teach kindergarten students about squares, circles, rectangles, and triangles.

Our study sought to explore the following question: Will kindergarten students who learn geometric concepts through an educational intervention based on the “GeoHieleICTshapes” model show significant improvement in understanding squares, circles, rectangles, and triangles compared to those taught using the standard educational curriculum?

3. Methodology

The present study was conducted in three phases. Pre-tests and post-tests were administered during the first and third phases, respectively. The second phase consisted of the teaching intervention. Conducted during the 2022–2023 school year, the research took place in five public kindergarten schools located in the town of Heraklion, Crete, Greece. It was an experimental study that compared the effectiveness of using tablet computers in teaching against traditional methods based on the kindergarten curriculum. The sample consisted of 256 kindergarteners, including 127 females and 125 males, aged 4 to 6 years. The study had two groups: a control group (n = 126, six classes) and an experimental group (n = 126, seven classes). In the control group, no computers or tablets were available for student use. In the experimental group, tablet computers were available daily as part of the teaching process. To ensure consistency, instructions were provided to the six kindergarten teachers in the experimental group and the seven kindergarten teachers in the control group, all of whom had similar qualifications. Researchers trained the teachers in the experimental group specifically on the two levels of Realistic Mathematics Education.

Stringent ethical considerations concerning privacy and other relevant issues in social research were rigorously adhered to throughout this study. Procedures for informing participants, obtaining informed consent, ensuring confidentiality, and managing data usage were meticulously followed, communicated both verbally and in writing. Prior to their participation, kindergarten staff, children, and guardians were comprehensively briefed on the study's objectives and their entitlement to abstain from involvement if they chose to do so.

A. First Phase

In the initial phase, the pre-test was administered to both the experimental and control groups' classes at the beginning of October 2022. This was done to identify any initial disparities in geometric achievement potential between the two groups and to establish a baseline for measuring the effects of the treatment. For this study, we developed an evaluation test consisting of closed-ended questions to assess students' knowledge and skills related to geometric shapes. The test utilized Alan Hoffer's modified version of the van Hiele model [23]. Additionally, it was based on the first two levels of the van Hiele model as employed in the research by Clements, Swaminathan, Zeitler-Hannibal, and Sarama [24] (see Fig. 1).

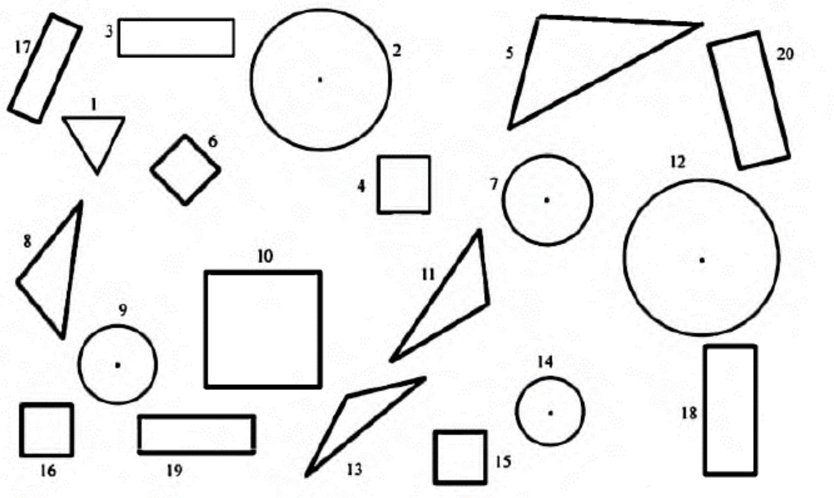

Fig.1. The van Hiele test [23].

The assessment aims to determine if students have mastered Hoffer's five geometry skills (visual, verbal, design, logical, and application) as they relate to the initial two stages of the van Hiele model of geometric reasoning (Visualization and Analysis). It is structured into two parts, each aligning with one of these stages and covering questions about all five skills.

Comprising forty exercises in total, the test includes ten exercises for each of the four geometric shapes (circle, triangle, square, and rectangle), amounting to 40 exercises in total (10 exercises x 4 shapes = 40 exercises). Additionally, each shape is represented by five exercises for each of the two van Hiele levels and one exercise for each Hoffer skill (2 van Hiele levels x 5 Hoffer skills = 10 exercises).

Scoring of the test is straightforward: each exercise earns one point for a correct answer and zero points for an incorrect answer. Therefore, the maximum score attainable is 40 points (40 exercises x 1 point = 40 points).

The test does not have a time limit, allowing children to complete it at their own pace. On average, children typically finish their respective sections of the test within 30-40 minutes.

B. Second Phase

For the second phase, children in the control group received traditional instruction based on the kindergarten curriculum. Children participated in both group and individual activities daily. The teaching content followed an 8-week syllabus aligned with the kindergarten curriculum, focusing on mathematical activities involving circles, triangles, squares, and rectangles.

- In the control group additional activities were given to the students to cover the time corresponding to the tablet computer activities of the experimental group. Activities were assigned and collected daily. Below, we list some examples of activities involving circles, triangles, squares, and rectangles.

In a kindergarten classroom activity focused on circles, the teacher engaged the children in identifying circular objects within the classroom environment. Furthermore, the children were encouraged to create their paintings inspired by Wassily Kandinsky's "Squares with Concentric Circles." Additionally, the activity included tracing and counting circles in Joan Miró's painting "The Garden."

In one activity focusing on triangles, the kindergarten teacher instructed the children to close their eyes and distinguish triangles from other shapes by identifying their basic characteristics. Additionally, the teacher engaged the children in a hands-on exercise where they used their bodies to form triangles (Fig. 2). Through these activities, the children learned that a triangle is defined by having three sides and three angles.

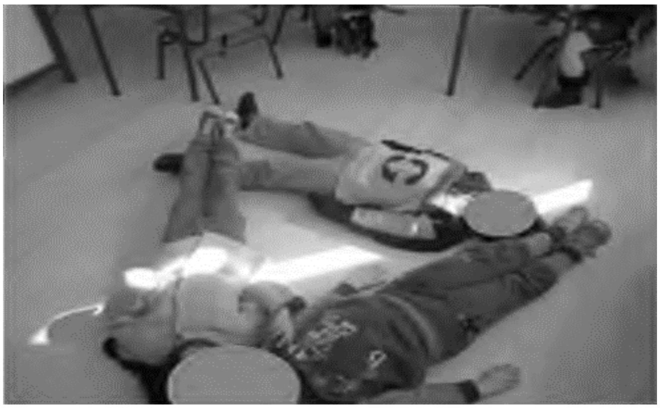

Fig.2. The children make a triangle with their bodies.

In the activities designed for learning about squares, children engage in various yard games. For instance, the kindergarten teacher challenges the children by asking, "Who can collect the square shapes within a certain time, among other shapes?" Additionally, the children use rulers to measure the sides of shapes that appear to be squares, checking if all sides are equal to confirm whether the shapes are indeed squares.

In one of the activities involving rectangles, the student joined two square pieces of paper and glued them together in the middle. Upon observing the new shape they created, they identified it as a "rectangle." Additionally, while examining Mondrian's painting "Composition with Large Red Plane, Yellow, Black, Grey, and Blue," the children recognized the presence of rectangles. Inspired by this, they proceeded to create their paintings using rectangles of various colours.

- Regarding the experimental group, the same curriculum content was covered using the "GeoHieleICTshapes" model, which spanned an 8-week syllabus divided into four levels. All instructional materials were created by the researchers using Flash CS6 Professional Edition and the App Inventor application, and these were delivered via tablet computers in the classroom. The key distinction between the "GeoHieleICTshapes" model and traditional methods lies in its use of blended learning, integrating both tablet-based and non-computer activities within the framework of Realistic Mathematics Education theory. The "GeoHieleICTshapes" model was specifically structured around activities aligned with the first two levels (Visualization and Analysis) of the van Hiele model. Furthermore, it incorporated the five geometry skills (visual, verbal, design, logical, and application) proposed by Alan Hoffer [23]. Each level of the “GeoHieleICTshapes” model corresponds to one of four shapes: circle, triangle, square, and rectangle. Each level includes activities designed for the first and second van Hiele levels of the corresponding shape. In the educational software, two characters, “Phoebus” and “Athena,” visit museums and observe paintings depicting shapes such as circles, triangles, squares, and rectangles (Fig. 3). Below are examples of tablet computer activities focusing on circles, triangles, squares, and rectangles from this educational software.

Fig.3. "Phoebus" and "Athena" visit museums in educational software of "GeoHieleICTshapes".

In the kindergarten software activity centered on circles, children assist characters Phoebus and Athena in selecting paintings that feature circles from a collection in a museum (see Fig. 4). This task aligns with the Visualization stage, the first level of the van Hiele model of geometric understanding, as described by Alan Hoffer [23].

In addition, another activity focusing on rectangles involves children aiding Phoebus in identifying the word that matches the shape displayed at the center of the screen (see Fig. 5). This verbal task further reinforces their geometric comprehension.

Fig.4. A visual activity for circles.

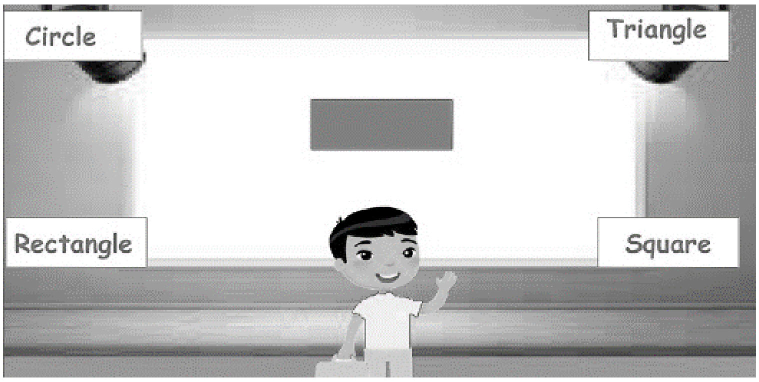

Fig.5. A verbal activity for rectangles.

Furthermore, another software activity involving squares requires kindergarten children to assist Athena in creating her painting by moving squares within the 'white' rectangle (Fig. 6). This task aligns with the first level of the van Hiele model of geometric reasoning, which focuses on developing basic logical abilities.

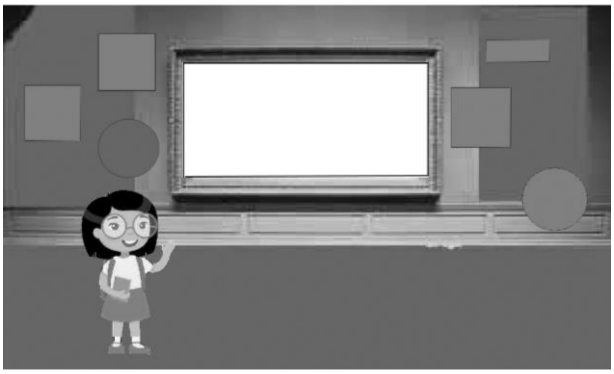

Fig.6. A logical activity for squares.

In addition, another software activity involving triangles requires kindergarten children to help Phoebus identify foods that are triangular in shape by clicking on them (Fig. 7). This activity aligns with the applied skill of visualization, integrating real-world scenarios into the learning process.

Fig.7. An applied activity for triangles.

In addition, one of the drawing activities involving circles on the tablet computer requires the kindergarten children to assist Phoebus in connecting the dots to form a circle (Fig. 8).

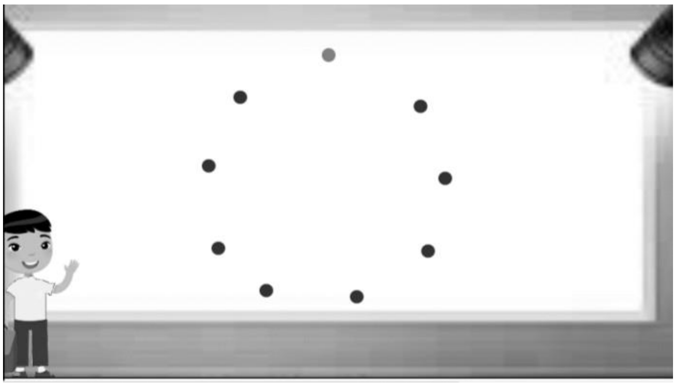

Fig.8. A drawing activity for circles.

C. Third Phase

Similarly, in the final phase of the study, following the teaching intervention, the same test was administered to all students in both the experimental and control groups as a post-test in early March 2024. This post-test aimed to evaluate their progress in geometric knowledge and skills related to geometry shapes.

4. Results

The data set analysis was performed using SPSS (version 26) statistical software. First, an independent sample t-test was executed to assess if the experimental and control groups had similar overall mathematical achievement levels prior to the teaching intervention. The independent variable differentiated between the experimental group (who used educational software) and the control group (who did not use the software), while the dependent variable was the students' pre-test scores for total geometry achievement.

The first analysis aimed to determine if both groups began at the same proficiency level in understanding circles, triangles, squares, and rectangles. This preliminary examination revealed a statistically significant difference ($t = 2.747$, $p < 0.01$) in geometry achievement between the experimental group ($M = 16.547$, $SD = 9.278$) and the control group ($M = 13.515$, $SD = 8.206$). The experimental group exhibited a higher mean score compared to the control group, with a mean difference of 3.031 in the pre-test scores. Detailed results of this test are summarized in Table I and Table II.

Subsequently, to address the first research question considering the significant pre-test differences between groups, an Analysis of Covariance (ANCOVA) was conducted on the post-test scores, taking pre-test levels into account as a covariate. The ANCOVA results indicated a statistically significant main effect of intervention type on post-test scores for total geometry achievement, $F(1, 249) = 10471.650$, $p < 0.001$, $\eta^2 = 0.486$ (Table III). Thus, after adjusting for pre-test scores, the experimental group significantly outperformed the control group in the post-test assessment of total geometry achievement following the teaching intervention.

Table I. Group statistics of pre-test scores

Group	N	Mean	Std. Dev.	Std. Error
Experimental	126	16.547	9.278	0.826
Control	126	13.515	8.206	0.731

Table III. Independent samples t-test of the pre-test scores BETWEEN CONTROL AND EXPERIMENTAL GROUP

Pre-test	t	df [a]	Mean difference	Sig. (2-tailed)
difference	2.747	246.321	3.031	0.006

Table IIIII. Comparison of student scores for total Geometry achievement in post-test: ANCOVA analysis

Sources	Type III Sum of Squares	df	Mean Squares	F	Sig.	Partial Eta Squared
Pre-test	4267.928	1	4267.928	95.814	0.000	0.278
Group	10471.650	1	10471.650	235.086	0.000	0.486
Error	11091.413	249	44.544			

To assess the significance of performance improvement within the experimental group, a paired t-test was conducted using the group's pre-test and post-test scores. The mean pre-test score was 16.547 ($SD = 9.278$), while the mean post-test score was 36.031 ($SD = 6.714$), as shown in Table IV. With α set at 0.05 and df equal to 125, the resulting p-value was less than 0.001 (Table V). This indicates that the mean post-test score was significantly higher than the mean pre-test score in the experimental group.

Table IVV. Paired samples statistics of the Experimental Group

Experimental Group	N	Mean	Std. Dev.	Std. Error
Pre-test	126	16.547	9.278	0.826
Post-test	126	36.031	6.714	0.598

Table V. Paired samples test of the Experimental Group

	t	df [a]	Mean difference	Sig. (2-tailed)
Pair 1	-23,679	125	-19,48413	<0.001

Similarly, to determine if the improvement in the control group's performance was significant, a paired t-test was conducted comparing their pre-test and post-test scores. The mean pre-test score was 13.515 (SD = 8.206), while the mean post-test score was 21.515 (SD = 8.819). The results are given in Table VI. With $\alpha = 0.05$ and df = 38, the p-value was less than 0.001 (Table VII). Therefore, the mean post-test score was significantly higher than the mean pre-test score in the control group.

Table VI. Paired samples statistics of the control Group

Experimental Group	N	Mean	Std. Dev.	Std. Error
Pre-test	126	13.515	8.206	0.731
Post-test	126	21.515	8.819	0.785

Table VII. Paired samples test of the control Group

	t	df [a]	Mean difference	Sig. (2-tailed)
Pair 1	-23,679	125	-6,800	<0.001

The findings of this study contribute to the existing research on the beneficial impacts of integrating suitable software within a computerized learning environment for teaching geometry, complemented by specially designed activities [17], [18], [16] [12], [19]. Furthermore, the results of this study have facilitated the development of a novel teaching approach that incorporates both tablet computers and non-computer-based activities.

5. Conclusions

This study underscores the importance of integrating technology, specifically tablets, alongside pedagogical shifts to enhance learning outcomes [9], [11], [8]. Moving away from a teacher-centered model, it is crucial to view technology not merely as a supplement but as a fundamental tool in education. Embracing these principles, we investigated the use of tablets and an application for teaching mathematical concepts.

The primary aim of this study was to evaluate the impact of instructional intervention using the "GeoHieleICTshapes" model on the understanding of geometric concepts—specifically circles, triangles, squares, and rectangles—compared to traditional methods at the kindergarten level. Our research revealed that students exposed to the "GeoHieleICTshapes" model showed significant improvements in overall mathematical achievement compared to those taught using traditional methods aligned with the kindergarten curriculum. These findings are consistent with previous studies [8], [12], [10], [2], [1], suggesting that ICT facilitates a deeper understanding of mathematical concepts [3], [4], [5], [6]. Consequently, our research question was affirmed positively.

The discussion above should be considered in light of several limitations of this study. Despite taking all necessary precautions, there remains uncertainty regarding whether the test accurately captured students' knowledge. Additionally, the data collected was limited to participants residing in the city of Heraklion. Furthermore, the study's sample size was relatively small, and time constraints imposed by schools limited the duration of instructional sessions, potentially impacting the depth of coverage for certain topics.

Given the small-scale and context-specific nature of this study, any application of its findings should be approached cautiously. Moreover, because the primary focus was on students' performance, data regarding the implementation effectiveness of each teaching method by teachers was not collected. Also, the study did not assess students' misconceptions before the project began, limiting certainty regarding the effectiveness of the teaching methods in addressing these.

Future research should seek to identify similarities and differences from the present study's findings. Investigations with different schedules and age groups could explore potential advantages or disadvantages of using tablets in teaching. Subsequent studies might also investigate gender differences in learning outcomes and employ additional data collection methods such as student and teacher interviews to gain deeper insights into perceptions of tablet use. Comparisons that emphasize the role of the teacher and utilize various computing devices could help determine whether observed results are attributable to the instruments or methods employed.

In terms of educational implications, stakeholders including students, teachers, researchers, and curriculum designers should take note of the findings. The developed teaching approaches warrant broader investigation to assess their effectiveness in enhancing children's understanding of geometry. As educators, we will disseminate these findings to our students, informing their future activities for children. Additionally, the research outcomes involving Realistic Mathematics Education (RME) using ICT could extend to various mathematical subjects in kindergarten and early primary education.

Despite the acknowledged limitations, the experimental data reinforce the view that tablets positively impact learning. Notably, students exhibited increased motivation and engagement, leading to favorable learning outcomes compared to alternative methods. In conclusion, considering the discussion and the limitations outlined, the integration of tablet-based computing presents an ongoing challenge for reflective teachers aiming to optimize its use at the kindergarten level.

Acknowledgments:

Funding for this research was provided by the Research Committee of the University of Crete (ELKE). More information can be found at: http://www.elke.uoc.gr/.

References

[1] K. Larkin and N. Calder, “Mathematics education and mobile technologies”, *Mathematics Education Research Journal*, 2016, 28(1), pp. 1–7.

[2] N. Calder, “Apps: Appropriate, applicable, and appealing?” in T. Lowrie, R. Jorgensen (Zevenbergen) (Eds.), *Digital games and mathematics learning. Mathematics education in the digital ERA 4*, Dordrecht: Springer Netherlands, 2015, pp. 233–250,.

[3] J. Orlando and C. Attard, “Digital natives come of age: The reality of today’s early career teachers using mobile devices to teach mathematics”, *Mathematics Education Research Journal*, 2016, 28(1), pp. 107–121.

[4] J. Schacter and B. Jo, “Improving preschoolers’ mathematics achievement with tablets: A randomized controlled trial”, *Mathematics Education Research Journal*, 2017, pp. 1–15.

[5] H. Shamir, K. Feehan and E. Yoder, “Does CAI improve early math skills?”, in *Proceedings of the 9th International Conference on Computer Supported Education, CSEDU,* Porto, Portugal, 2017, 2, pp. 285–292.

[6] H. Crompton, and D. Burke, “Research trends in the use of Mobile learning in mathematics”, *International Journal of Mobile and Blended Learning*, 2015, 7(4), 1–15. https://doi.org/10.4018/IJMBL.2015100101.

[7] Liu, N. S. H. “iPad infuse creativity in solid geometry teaching:, Turkish Online Journal of Education Technology., 2013, 12, 177–192.

[8] S. Papadakis, M. Kalogiannakis and N. Zaranis, “Educational apps from the Android Google Play for Greek preschoolers: A systematic review’, *Computers & Education*, 2018, 116, pp. 139–160. https://doi.org/10.1016/j.compedu.2017.09.007.

[9] N. Dittert, K. Thestrup and S. Robinson, “The SEEDS pedagogy: Designing a new pedagogy for preschools using a technology-based toolkit”, *International Journal of Child-Computer Interaction*, 2021, 27, pp. 1-22. https://doi.org/10.1016/j.ijcci.2020.100210. org/10.30935/cet.414808.

[10] N. Zaranis and V. Valla, “Tablet computer assisted counting and calculating activities for kindergarten children”, in L. Gσmez Chova, A. Lσpez Martνnez and I. Candel Torres (Eds.), *EDULEARN17 Proceedings, 9th International Conference on Education and New Learning Technologies,* Barcelona, Spain. 2017, pp. 9680–9689

[11] C. Liu and G. J. Hwang, "Roles and research trends of touchscreen mobile devices in early childhood education: review of journal publications from 2010 to 2019 based on the technolo-gyenhanced learning model", *Interactive Learning Environments*, 2020, pp. 1-20. https://doi.org/10.1080/10494820.2020.1855210.

[12] N. Zaranis, "Comparing the effectiveness of using ICT for teaching geometrical shapes in kindergarten and the first grade", *International Journal of Web-Based Learning and Teaching Technologies*, 2018, 13(1), pp. 50–63. https://doi.org/10.4018/IJWLTT.2018010104.

[13] M. Bourbour, "Using digital technology in early education teaching: learning from teachers' teaching practice with interactive whiteboard", *International Journal of Early Years Education*, 2020, pp. 1-18. https://doi.org/10.1080/09669760.2020.1848523.

[14] E. Fokides, and A. Mastrokoukou, "Human anatomy, tablets, and primary school students. Results from a pilot study"' *Contemporary Educational Technology*, 2018, 9(2), 154–170. https://doi.

[15] S. Papadakis, M. Kalogiannakis and N. Zaranis, "Improving mathematics teaching in kindergarten with realistic mathematical education", *Early Childhood Education Journal*, 2016, 45, pp. 369–378. https://doi.org/10.1007/s10643-015-0768-4.

[16] G. Falloon, "Young students using iPads: App design and content influences on their learning pathways", *Computers & Education*, 2013, 68, pp. 505–521.

[17] D. H. Clements, J. Sarama, N. J. Yelland, and B. Glass, "Learning and teaching geometry with computers in the elementary and middle school" in M. K. Heid and G. W. Blume (Eds.), *Research on technology and the teaching and learning of mathematics: Volume 1, research syntheses* New York: Information Age Publishing, 2008, pp. 109–154.

[18] D. H., Clements and J. Samara, "Effects of a preschool mathematics curriculum: Summative research on the building blocks project", *Journal for Research in Mathematics Education*, 2007, 38(2), pp. 136–163.

[19] N. Zaranis, and E. Synodi, "A comparative study on the effectiveness of the computer assisted method and the interactionist approach to teaching geometry shapes to young children", *Education and Information Technologies*, 2016, 22(4), pp. 1377–1393. https://doi.org/10.1007/s10639-016-9500-2.

[20] P. M. Van Hiele, "A child's thought and geometry", in Fuys, D., Geddes, D., & Tischler, R. (Eds.), *English Translation of Selected Writings of Dina van Hiele-Geldof and Pierre M. van Hiele*, Brooklyn: Brooklyn College, School of Education 1984, pp. 243–252.

[21] M. Toumasis, *Modern teaching of mathematics*, Athens: Gutenberg, 2002 [Text in Greek].

[22] J. A., Van de Walle, K. S., Karp and J. M. Bay-Williams, *Elementary and middle school mathematics: teaching developmentally*, Boston: Pearson/Allyn and bacon publishers, 2008.

[23] A. Hoffer, "Geometry is more than proof", *Mathematics Teacher*, 1981, 74, 11–18.

[24] D. H. Clements, S. Swaminathan, M. Zeitler-Hannibal, and J. Sarama, "Reviewed work(s). Journal for Research in Mathematics Education", *National Council of Teachers of Mathematics Stable*, 1999, 30(2), pp. 192-212. http://www.jstor.org/stable/749610.