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On Generalised Closedness and Continuity in ξ-Nano Topological Spaces Via Ideal

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Abstract:- The objective of the work is to establish the notion of novel kind of set namely ξIg− closed and ξIg− open set in ξ-nano-topological spaces with an Ideal and explore its few properties. Also, we establish ξIg− continuous function in terms of ξIg− closed and ξIg− open sets. Additionally, certain characteristics and relationships among these sets are examined.

Keywords*:* ξ-nano topology, ξ - nano Ideal topological spaces, ξIg - closed sets and ξIg- continuous functions.

1. Introduction

The inclusion of ideals in a general topological spaces was originated by Vaidyanatha Swamy [14] and Kuratowski [8]. Later on, the elaborated investigation were carried out on an ideal and it produces a new topology determined by Jankovic and Hamlett [6]. The topological space (X, ℶ) is endowed with a topology induced by an ideal is referred as an ideal-topological space. When a set A ⊆ X, define A∗ (I) = {x ∈ X: W ∩A ∉ I for each neighborhood(open) W of x} and cl∗ (A) = A ∪ A∗ (I), induces a broader Topology ℶ^* which is finer than .ℶ

M.L.Thivagar [12,13] developed the concept of nano topology. Furthermore, Jenavee et al. [7] work led to the establishment of ξ-nano-topology, in terms of ξ-open set and is a broader version of nano-topology comprising all nano open sets and containing a greater number of open sets, making it finer than nano topology.

Levine [9] contributed the idea of generalised-closed set as a broader version of classical closed set in general topological spaces.

The nIg - open set and nIg- closed within a nano-ideal-topology was pioneered by Parimala et al. [10].

2. Preliminaries

Here, we review the certain definitions that will come in handy in the follow-up. In this paper, we regard H as universe and establish R as an equivalency-relation defined on H. Definition 2.1. [14] A collections of subset of H which are non-empty is an Ideal I if

1. H ∈ I, S ⊂ H imply S ∈ I (Heridity)
2. H, S ∈ I imply H ∪ S ∈ I (Finite Additivity)

Definition 2.2. [13] On the Universe H, an equivalence relation R is known as the indiscernibility relation. It is believed that the members who belongs to the same equivalence class are indistinguishable from one another. The approximation space is refereed as the

Pair (H, R). In a approximation space (H, R) and X is a subset of the universe H, then

the lower, upper-approximations and boundary-region are characterized as follows:

1. The lower-approximations

$$L_R(X)=\bigcup_{x\in H}\{R(x): R(x)\subseteq X\}$$

2. The upper-approximations

$$U_R(X)=\cup_{x\in H}\{R(x): R(x)\cap X \neq \emptyset\}$$

where R (x) represents the equivalency class indicated by x.

3. The Boundary region $B_R(x)$ is characterized by the difference between the upper and lower-approximations referred as

$$B_R(x) = U_R(X) - L_R(X)$$

Definition 2.3. [13] In the Universe H, if R is an equivalency relation, then

$ℶ(X) = \{H, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq H$ follows specific axioms.

1. $H, \emptyset \in$.ℶ (X)
2. Any sub-collection of members of .ℶ X ℶ X and whose union is also in
3. The sub-collection of finite members of ℶ X and whose intersection is also in ℶ (X).

Then ℶ X or simply ℶ is topology on H referred as the nano-topology on H so that $(H, ℶ_N)$ is a nono-topological .space. The members in $ℶ_N$ are referred to nano-open set (n - open) and its complement is nano-closed (n-closed)

Remark 2.1. .$ℶ_N$ basis is represented as $B_{(ℶ_N)}(x) = \{H, L_R(X), B_R(X)\}$

Definition 2.4. [7] The ξ -nano-open set is subset J of H and there is a non empty n-open sets Z of $ℶ_N$ for which

1. $Z \neq \emptyset$, H.
2. $J \subseteq N_{int}(J) \cup Z$.

A ξ -nano topological space is referred as a collection every ξ-open set, including ∅ and H, that satisfies the topological definitions and it is denoted as (H, ℶ, ξ) or simply

$(H, ℶ_ξ)$. In $(H, ℶ_ξ)$, the elements in $ℶ_ξ$-are called ξ - nano-open sets (ξ - open set) and whose complement is .ξ-nano -closed set (ξ - closed)

Remark 2.2. In nano-topology (H, $ℶ_ξ$ is ξ- open. Therefore 〖ℶ〗_ is $ℶ_ξ$) , it is clear that each n-open set of .$ℶ_N \subseteq ℶ_ξ ℶ_N$. That is finer than

Definition 2.5. [7] Let $(H, ℶ_ξ)$ is ξ-nano space, $H \subseteq H$ then

1. The ξ-nano-interior of a subset $H \subseteq H$ is largest ξ-open set inside H and referred as ξint (H).
2. The ξ-nano-closure of a subset $H \subseteq H$ is the smallest ξ-closed sets including H and referred as ξcl (H).
3. The ξ-nano-exterior of H is indicated by ξE (H) = ξint (H − H).
4. The ξ-nano-frontier of H is represented by ξF (H) = ξcl (H) − ξcl (H − H).

Remark 2.3. The basis for ℷξ is represented by $B(ℶ(ξ)) = \{H, L_R(X), B_R(X)\} = B_{(ℶ_N)}$.

Definition 2.6. A ξ-nano topology (H, ,ℶ (ξ)) induced by I on H is a ξ-nano-ideal -topology

represented as (H,).$ℶ_{(ξ)}$, ℶ (ξ),I). We just express ξI - space for (H

Definition 2.7 [15]. Let $(H, ℶ_{(ξ)})$-be a ξ- nano-topological space and (.)∗ known to be a kuratowski set operator ,from P (H) → P (H) is a power set of H. For $H \subseteq H$

$H^{*}ξ(I, ℶ_{(ξ)}) = \{x \in H : V \cap H \notin I$, for each (open) neighborhood V of x}. is denoted as

ξ -local function of H with respect an ideal I and $ℶ_{(ξ)}$. We shortly express as H*ξ for

$H^{*}ξ(I, ℶ_{(ξ)})$

Definition 2.8. [15]. The ξ*-closure of $H \subseteq H$, defines ξ*cl (H) = H ∪ H∗ξ . The ξ*-closure will produces a new topology called ξ - nano * topology given by

$$〖ℶ^*〗_{(ξ)}(I, ℶ_{(ξ)}) = \{W \subseteq H : ξ\_cl^*(H-W) = H-W$$

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and simply we write .$\beth_{(\xi)}$ 〖$\beth^*$〗 $_{(\xi)}$ is finer than 〖$\beth^*$〗 $_{(\xi)}$ (I,$\beth_{(\xi)}$) and 〖$\beth^*$〗 $_{(\xi)}$ for

Remark 2.4. If (H, ,$\beth_{(\xi)}$,I) is ξI-space, H ⊆ H. Then

1. The members of 〖$\beth^*$〗 (ξ) are termed as ξ - nano *- open sets (ξ*-open set) and their complement of ξ*-open set are called ξ - nano *- closed set (ξ*- closed).
2. The ξ-nano interior (respectively ξ-nano closure) of a subset H ⊆ H in 〖$\beth^*$〗 $_{(\xi)}$are termed as ξ_int^* (H) (respectively ξ_cl^* (H)).
3. If I = ∅ then $H_\xi^* = \xi cl\ (H) = \xi\_cl^*\ (H)$ and . 〖$\beth^*$〗 $_{(\xi)}$=∅ if I = P (H)

Proposition 2.1. [15].Let (H, $\beth_{(\xi)}$,I) is ξI-space include an ideal I and H,S be a subset

of H. Then,

1. $H \subseteq S \Rightarrow H_\xi^* \subseteq S_\xi^*$.
2. $I \subseteq J \Rightarrow H_\xi^*\ (J) \subseteq H_\xi^*\ (I)$.
3. $H_\xi^* = \xi cl\ (H_\xi^*) \subseteq \xi cl\ (H)$.
4. $(H_\xi^*)_\xi^* \subseteq H_\xi^*$.
5. $H_\xi^* \cup S_\xi^* =$ (〖H∪S)〗 $_\xi^*$.
6. $H_\xi^* - S_\xi^* =$ (〖H∪S)〗 $_\xi^* - S_\xi^* \subseteq$ (〖H∪S)〗 $_\xi^*$ ..
7. $V \in \beth_{(\xi)} \Rightarrow V \subseteq H_\xi^* = V \cap (V \cap H)_\xi^* \subseteq (V \cap H)_\xi^*$.
8. $T \in I \Rightarrow$ (〖H∪T)〗 $_\xi^* = H_\xi^* =$ (〖H-T)〗 $_\xi^*$.

3. ξI-Generalised Closed set

Definition 3.1. [10] Let H ⊆ H in nano-Ideal space (H, $\beth_{(N)}$,I) is called nano-Ideal generalised closed set $\beth_{(\xi)}$. Also the complement of the closed set nIg is ∈ (simply nIg -closed) if $H_n^* \subseteq V$ provided H ⊆ V, V .called the nIg - open

Definition 3.2. Let H of ξ-nano space (H, $\beth_{(\xi)}$) -is called ξ- generalised closed (simply

ξ g -closed set) if ξcl (H) ⊆ V provided H ⊆ V and V ∈ $\beth_{(\xi)}$-open. Also the complement of ξ g -closed is).$\beth_{(\xi)}$, called the ξ g - open in (H

Definition 3.3. Let H⊂ H in a ξI− space, (H, $\beth_{(\xi)}$,I) is called ξI- generalised closed (simply ξ Ig -closed) if). $\beth_{(\xi)}$,I, $\beth_{(\xi)}$ in (H ∈ $H_\xi^* \subseteq V$ provided H ⊆ V and V

Example 3.1. Let H = {ζ1, ζ2, ζ3, ζ4 } with H / R = {{ζ1 }, {ζ3 }, {ζ2 , ζ4 }} and

X = {ζ1 , ζ2 } then $\beth_{(N)}$ = {∅, H, {ζ1 }, {ζ1 , ζ2 , ζ4 }, {ζ2 , ζ4 }}. Let ξ = {ζ2 , ζ4 } then

$\beth_{(\xi)}$= {∅, H, {ζ1 }, {ζ2 }, {ζ4 }, {ζ1 , ζ2 }, {ζ1 , ζ4 }, {ζ2 , ζ4}, {ζ1 , ζ2 , ζ4 }} with

I = {∅, {ζ1}}

1. For H = {ζ2 , ζ3 }, H_ξ^* = {ζ2 , ζ3 } and it is clear that $H_\xi^* \subseteq V = H$, provided H ⊆ V = H. Here H is ξI g -closed sets in H.
2. For S = {ζ1 , ζ2 }, S_ξ^* = {ζ2 , ζ3 } and S ⊆ V = {ζ1 , ζ2 }but $S_\xi^* \not\subseteq V$ and so S is not ξI g - closed.

Theorem 3.1. [10] A subset H of a nano-ideal space (H, $\beth_{(N)}$,I) is Ng - closed then H is

NIg -closed.

Proposition 3.1. In (H, .$\beth_{(\xi)}$),any ξ- closed set is ξ g - closed

Proposition 3.2. Any N g - closedset in (H,).ב_(ξ), ב_N) is ξ g -closed in (H

Theorem 3.2. A subset H in ξ I-space (H, .ב_(ξ),I) is ξ I g - closed provided H is ξ g - closed

Proof. Assume K is ξ g - closed, V ∈ ב_(ξ)), such that K ⊆ V. Since, K is ξ g - closed then ξcl (K), ב_(ξ) in (H .V and so K_ξ^* ⊆ . Hence K is ξ I g - closed ⊆

Remark 3.1. The Example 3.2 demonstrates, the reverse statement in the Theorem 3.1 is not necessarily true.

Example 3.2. In Example 3.1 Let H = {ζ1}, H_ξ^* = {∅} ⊆ V provided H ⊆ V. Here H is

ξ I g -closed but H = {ζ1} is not ξ g - closed as ξcl (H) = {ζ1, ζ3}⊈ V = {ζ1 }

where H ⊆ V = {ζ1}.

Remark 3.2. The consequence of the above theorems, leads to the logical Implication Diagram shows the relationship among the closed sets in the topologies ב_ξ ב_N and

n-closed =⇒ n g − closed =⇒ nI g − closed

⇓ ⇓ ⇓

ξ − closed =⇒ ξ g − closed =⇒ ξ I g -closed

Theorem 3.3. If H,S be two ξ I g - closed subsets in (H, .ב_(ξ),I) then H ∪ S is also ξ I g -closed

Proof. Let H and S are ξ I g - closed sets in (H, ב_(ξ),I) and let H ⊂ V, S ⊆ W where V,W are ξ-open sets in ב_(ξ) . Since H and S are ξ I g -closed then H_ξ^* ⊆ V and S_ξ^* ⊆ W which ∈ ב_(ξ) . Then H ∪ S ⊆ V ∪ W implies (H ∪ S)_ξ^* ⊆ V ∪ W . Therefore, H ∪ S is ξ I g – closed

in (H,). ב_(ξ),I

Corollary 3.1. The ξ I g - closed sets are closed under arbitrary union.

Proof. In accordance with Theorem 3.3

Remark 3.3. Finite intersection of ξ I g -closedness may not always be closed under ξ I g

in (H, .ב_(ξ),I) as illustrated in Example

Example 3.3. Let H = {ζ1 , ζ2 , ζ3 , ζ4 , ζ5 , ζ6 } with H / R = {{ζ1 , ζ2 }, {ζ3 }, {ζ4 },

{ζ5 , ζ6 }} and X = {ζ1 , ζ2 , ζ6 }. Then ב_(N) = {H, ∅, {ζ1 , ζ2 }, {ζ1 , ζ2 , ζ5 , ζ6 }, {ζ5 , ζ6 }} and let ξ ={ζ1 }. ζ2,

Then ב_(ξ) = {H, ∅, {ζ1 }{ζ2 }, {ζ1 , ζ2 }, {ζ5 , ζ6 }, {ζ1 , ζ5 , ζ6 }, {ζ2 , ζ5 , ζ6 }, {ζ1 , ζ2 , ζ5 , ζ6 }}with I = {∅, {ζ5 }}. Let H = {ζ1 , ζ3 }, H_ξ^* = {ζ1 , ζ3 , ζ4} and so H_ξ^* ⊆ V = H as

H ⊆ V =H. Let S = {ζ1 , ζ4 }, S_ξ^* = {ζ1 , ζ3 , ζ4 } and so S_ξ^* ⊆ V = H as S ⊆ V = H. Hence, H and S are ξ I g - closed sets but H ∩ S = {h1 } is not ξ I g - closed as (H ∩ S)_ξ^* = {ζ1 ,

ζ3 , ζ4}⊈ V = {ζ1 } provided H ∩ S ⊆ V.

Theorem 3.4. If H ⊆ S ⊆ ξ*cl (H) where H is ξ I g -closed in (H, ב_(ξ),I),then S is

ξ I g -closed.

Proof. Let H is ξ I g -closed and H ⊆ S ⊆ ξ*cl (H). Suppose S ⊆ , where V belongs

to ב_(ξ). then H ⊆ V. Since, H is ξ I g -closed, so we have H_ξ^* ⊆ V.Also S ⊂ ξ*cl (H) and so

S_ξ^* ⊆ H_ξ^* ⊆ V implies S_ξ^* ⊆ V. Hence S is ξ I g -closed.

Theorem 3.5. If H ⊆ H , ξ I- space (H, ∈ ב_(ξ),I) then H is ξ I g -closed iff ξ*cl (H). ⊆ V, where H ⊆ V and V . ב_(ξ)

Proof. Suppose, H is ξ I g -closed then $H_\xi^* \subset V$, provided $H \subseteq V$ and $V \in \beth_{(\xi)}$.

Now ξ*cl (H)= $H \cup H_\xi^* \subseteq V$.

Conversely, suppose ξ*cl (H)⊆ V where H ⊆ V, that is $H \cup H_\xi^* \subseteq V$ and so $H_\xi^* \subseteq V$, provided H ⊆ V. Hence H is ξ I g -closed.

Theorem 3.6. Any subset in ξ I -Space (H, $\beth_{(\xi)}$,I) along with an ideal I is ξ I g - closed iff each ξ-open in $\beth_{(\xi}$ ^* .$\beth_{(\xi)}$ is ξ^* closed in)

Proof. Every subset V in (H, ⊆^* $\beth_{(\xi)}$) is ξ I g -closed, V ∈ $\beth_{(\xi)}$) then V - ξ I g -closed. and we have V_ξ .V, implies V is ξ ∗ -closed

On the other-hand, each ξ-open V ∈) $\beth_{(\xi)}$, $\beth_{(\xi)}$ is ξ^* -closed then $V_\xi^* \subseteq V$. Let H be any subset of (H .and H ⊆ V,since V is ξ-open then $H_\xi^* \subseteq V_\xi^* \subseteq V$ and so H is ξ I g -closed

Theorem 3.7. Any ξ^* - closed set in ξ I - topological space (H, .$\beth_{(\xi)}$,I) with respect to an ideal I is ξ I g -closed

Proof. If H ⊆ H in (H, $\beth_{(\xi)}$,I) and also H is ξ^* - closed set then $H_\xi^* \subseteq H$. If H ⊆ V, where V is ξ-open .implies $H_\xi^* \subseteq V$. Hence H is ξ I g -closed

Remark 3.4. The statement of Example 3.4 proves that the reverse of the above theo- rem is not always true.

Example 3.4. In Example 3.3, we have H = {ζ1 , ζ3 }, which is closed under ξ I g but not closed under ξ^* .Since, H_ξ^* = {ζ1 , ζ3 , ζ4 } does not fully belong to H = {ζ1 , ζ3 }.

Theorem 3.8. If (H, .$\beth_{(\xi)}$,I) be ξI- space along an Ideal I ,H ⊆ H.Then, the statements are equivalent

1. H is ξ I g –closed.
2. 2.ξ_{cl}^* (H) ⊆ V where H ⊆ V and V ∈ $\beth_{(\xi)}$.
3. For each h ∈ ξ_{cl}^* (H) then ξcl ({h}) ∩ H = ∅.
4. ξ_{cl}^* (H) - H has no non-empty ξ - closed.
5. H_ξ^* - H has no non-empty ξ - closed.

Proof. (1) ⇒ (2): Proof is trivial using Theorem 3.6

(2) ⇒ (3): Suppose ξcl ({h}) ∩ H = ∅. where h ∈ ξ_{cl}^* (H) then H ⊆ H − ξ ({h}). By (2)

ξ_{cl}^* (H) ⊆ H − ξcl({h}) and so ξ_{cl}^* (H) ∩ {h} = ∅, it contradicts to h ∈ ξ_{cl}^* (H).

Hence, ξcl ({h}) ∩ H ≠ ∅.

(3) ⇒ (4): Suppose that K ⊆ ξ_{cl}^* (H) − H where K is ξ - closed. Let h ∈ K, Since K ⊆ H − H and so K ∩ H = ∅ implies ξcl ({h}) ∩ H = ∅, as K is ξ- closed and h ∈ K.. This is a contradiction to (3). Hence ξ_{cl}^* (H) − H has no non-empty ξ-closed set.

(4) ⇒ (5): ξ_{cl}^* (H) − H = (H ∪ H_ξ^*) − H = H_ξ^* − H.. Hence H_ξ^* − H has no nonempty

ξ - closed set.

(5) ⇒ (1): Let H ⊆ V where V ∈ $\beth_{(\xi)}$ then H − V ⊆ H − H and so H_ξ^* ∩ (H − V) ⊆

H_ξ^* ∩ (H − H) = H_ξ^* ∩ H c = H_ξ^* − H. Therefore H_ξ^* ∩ (H − V) ⊆ H_ξ^* − H = ∅ and so

H_ξ^* ∩ (H − V) = ∅ implies $H_\xi^* \subseteq V$. Hence, H is ξ I g -closed set.

4. ξI-Generalised open set

Definition 4.1. Subset H in ξI-space, (H, $\beth_{(\xi)}$,I)is referred be ξ I-generalised open (or ξ I g − open) iff H -H is .ξ I g -closed

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Theorem 4.1. Let H be a subset of a ξI-Space (H, ℶ_(ξ),I) along with ideal I then H is

ξ I g -open iff K ⊆ ξ_int^* (H) provided K ⊆ H and K is ξ-closed.

Proof. If H is ξ I g -open, then H -H is ξ I g -closed. Let K is ξ-closed and K ⊆ H then

H − H ⊂ H − K is ξ-open then by Theorem [3.4] 〖ξ^*〗_(cl)(H − H) is a subset of H − K implies K ⊆ ξ_int^* (H).

Conversely, Suppose that K ⊆ ξ_int^* (H) , where K ⊆ H and let V be ξ-open in (H, ℶ_(ξ)) in a way that H − H V then H − V ⊆ H and so H − V ⊆ ξ_int^* (H) which implies that ⊆

H-ξ_int^* (H) ⊆ V. Therefore, 〖ξ〗_cl^* (H − H) ⊆ V and by Theorem [3.4], H − H is ξ I g -closed. Therefore, H is ξ I g - open.

Theorem 4.2. In a ξI- topological space (H, ℶ_(ξ)) based on an ideal I, if H ⊆ H, the following statements are .equivalent

1. H is ξ I g - closed.
2. H ∪ (H − H_ξ^*) is ξ I g - closed.
3. H_ξ^* − H is ξ I g - open.

Proof. (1)⇒ (2): let H be ξ I g - closedset, If V ∈ ℶ_(ξ) such that ℶ_(ξ) is ξ-openset in

H ∪ (H − H_ξ^*) ⊆ V then H − V is a subset of H − (H ∪ (H − H_ξ^*)) = H_ξ^* − H. Since,

H is ξ I g - closed by Theorem [3.7] H_ξ^* − H has no non-empty ξ- closed set then H − V = ∅ implies H = V .Hence H ,only open-set having H ∪ (H − H_ξ^*) implies H ∪ (H − H_ξ^*) is

ξ I g - closed.

(2)⇒ (1): Suppose that H ∪ (H − H_ξ^*) is ξ I g - closed. Let K be any ξ - closed such that

K ⊆ H_ξ^* − H. Then H ∪ (H − H_ξ^*)⊆ H − K which implies

(H ∪ (H - H_ξ^*))_ξ^* ⊆ H_ξ^* ∪ (H - H_ξ^*)_ξ^* ⊆ H − K and so K ⊆ H − H_ξ^* . Since K ⊆ H_ξ^* , it follows that K = ∅. Hence H is ξ I g - closed.

(2)⇒ (3): Suppose that H ∪ (H − H_ξ^*) is ξ I g - closed but H − (H_ξ^* − H) = H ∪ (H − H_ξ^*) and so H − (H_ξ^* − H) is ξ I g - closed then H − (H − (H_ξ^* − H)) is ξ I g -open. Hence, H_ξ^* − H is ξ I g - open.

(3)⇒ (2):It is obvious.

5. ξI-Generalised continuous Function

Definition 5.1. [2] A function f : (H, ℶ_(N)^') is referred Nano generalized continuous, ℶ_(N)) → (V .(abbreviated as ng-continuous) when the preimage of each n-openset in V is n g -open in H

Definition 5.2. Let (H, ℶ_(ξ, ℶ_(ξ)^') be two ξ-Nano topological space. Then a function f_ξ : (H, ℶ_(ξ)) and (V ℶ_(ξ)^')is called ξ-nano generalised continuous (abbreviated as ξ g – continuity) if the pre-image of, V→ ( )) .every ξ-openset in V is ξ g -open in H

Definition 5.3. A function f_ξ : (H, ℶ_(ξ)^') - is said to be ξ I - generalised continuous function, ℶ_(ξ)) → (V .(abbreviated as ξ I g - continuity) if the pre-image of every ξ-open in V is ξ I g -open in H

Example 5.1. Let H = {ζ1 , ζ2 , ζ3 } with H / R = {{ζ1 }, {ζ2 }, {ζ3 }} and X = {ζ1 , ζ2 } then , ℶ_(N) = {∅, H {ζ1 , ζ2 }} and let ξ = {ζ1 , ζ2 } then, ℶ_(ξ) = {∅, H, {ζ1 }, {ζ2 }, {ζ1 , ζ2 }} and I = {∅, {ζ1 }}. Then ξ-open sets are {{ζ1 }, {ζ2 }, {ζ1 , ζ2 }} and ξ I g -closed sets are {{ζ1 }, {ζ3 }, {ζ1, ζ3 }, {ζ2 , ζ3 }}. Therefore, ξ I g -ℶ_(ξ)) be the identity, ℶ_(ξ)) → (H, open sets are {{ζ1 }, {ζ2 }, {ζ1 , ζ2 }, {ζ2 , ζ3 }}. Now define fξ : (H .function then the pre-image of each ξ -open in V is ξ I g -open in H. Therefore, f_ξ is ξ I g -continuous

Theorem 5.1. A function f_ξ : (H, -ב_(ξ)^') is ξ I g - continuous iff the preimage of each ξ, ב_(ξ ,) I) → (V .ב_(ξ ,)) along an ideal I, ב_(ξ)^') is ξ I g - closed in (H, closedset in (V

Proof. Suppose that f_ξ: (H, ב_(ξ)^') is ξ I g -continuous. If F is a ξ-closed in V, then V -F is ξ-, ב_(ξ ,) I)→ (V open belongs to V. As f_ξ is ξ I g -continuous, the pre-image f_ξ^(-1) (V − F) is ξ I g -open belongs to H. This .means, H − f_ξ^(-1) (F) becomes ξ I g -open, indicating that f_ξ^(-1) (F) is ξ I g -closed set in H

Conversely, Suppose that pre-image of any ξ-closed belonging V is ξ I g -closed in H.

Let W be ξ-open in V then V -W is ξ -closedset in V. Since, f_ξ^(-1) (V − W) is ξ I g – closed belonging H and so H − f_ξ^(-1) (W) is ξ I g -closed which implies f_ξ^(-1) (W) is ξ I g -open

in H. Hence, f_ξ^(-1) is ξ I g - is continuous.

Theorem 5.2. Following are equivalent, for a function f_ξ : (H, ^')ב_(ξ), ב_(ξ ,) I) → (V

1. If f_ξ is ξ I g - continuous.
2. For each h ∈ H and every ξ-open S containing f_ξ (h), there exists a ξ I g -openset H containing h such that f_ξ (H) ⊆ S.
3. For each h ∈ H and each ξ-openset S containing f_ξ (h), (f^(-1) (S))_ξ^* is a neighborhood of h.

Proof. (1)⇒(2): Suppose f_ξ is ξ I g - continuous. Since S is a ξ-open set containing f_ξ (h), then by (1) f_ξ^(-1) (S) is ξ I g - open set in (H, ב_(ξ ,) I). By taking H = f_ξ^(-1) (S) which

containing h. Therefore f_ξ (H) ⊆ S.

(2)⇒(3): Suppose (2) is true and since, S is ξ-open in (V, ב_(ξ)^') having f_ξ (h) then by (2), there exists an ξ I g - open H having h with f_ξ (H) ⊆ S. So, h ∈ H ⊆ ξint (H_ξ^*) ⊆ ξ_int ((f_ξ^(-1) (S))_ξ^*) ⊆ (f_ξ^(-1) (S))∗ξ .Hence (f_ξ^(-1) (S))_ξ^* is a neighborhood of h.

(3)⇒(1): Let S is ξ-open set in (V, ב_(ξ)^')and each s ∈ S, f_ξ^(-1) (s) ∈ f_ξ^(-1) (S) ⊆ H. Then By (3), (f_ξ^(- (S))_ξ^* is a neighborhood of f_ξ^(-1) (s) and so f_ξ^(-1) (S) ⊆ ξ_int (f_ξ^(-1) (S))_ξ^* . Hence f_ξ is)1

ξ I g -continuous.

Remark 5.1. Let f_ξ : (H, ב_(ξ)^'')be, ב_(ξ)^',I') → (W, ב_(ξ)^') and g_ξ:(V, ב_(ξ ,) I) → (V

ξ I g -continuous then (g_ξ of_ξ) not necessarily be ξ I g -continuous as illustrated from the

Example 5.2.

Example 5.2. Let H = {ζ1, ζ2 , ζ3 }, H / R = {{ζ1 }, {ζ2 }, {ζ3 }}, X = {ζ1 , ζ2 , }.

ב_N = {H, ∅, {ζ1 , ζ2 }} . Let ξ = {ζ1 , ζ2 } then ב_(ξ ,) = {H, ∅, {ζ1 }, {ζ2 }, {ζ1 , ζ2 }} with

I = {∅, {ζ3 }} on H. Hence, the ξ I g - closed sets in H are {{ζ3}, {ζ1 , ζ3 }, {ζ2 , ζ3}}.

Let V = {η1 , η2 , η3 , η4 } with V / R' = {{η1 }, {η3 }, {η2 , η4 }} and Y = {η1 , η2 }. Then

〖ב'〗 _N = {V, ∅, {η1 }, {η2 , η4 }, {η1 , η2 , η4 }} and let ξ = {η2 , η4 }, then

〖ב'〗 _ξ = {V, ∅, {η1 }, {η2 }, {η4 }, {η1 , η2 }, {η1 , η4 }, {η2 , η4 }, {η1 , η2 , η4 }} with

I' = {∅, {η1 }} on V. The ξ I g - closed sets in V are {{η1 }, {η3 }, {η1 , η3 }, {η2 , η3 },

{η3 , η4 }}. Let W = {η_1^' , η_2^' , η_3^' , η_4^' } with W/ R'' = {{η_1^' }, {η_3^' }, {η_2^' , η_4^' }}, and

Z = {η_1^' , η_2^' }. Then '' = 〖ב''〗 _N= {W, ∅, {η_1^' }{η_1^' , η_2^' , η_4^' }, {η_2^' , η_4^' }} and let ξ {η_2^' , η_4^' }

then 〖ב''〗 _ξ = {W, ∅, {η_1^'}, {η_2^' }, {η_4^' }, {η_1^' , η_2^' }, {η_1^' , η_4^' }, {η_2^' , η_4^' }{η_1^' , η_2^', η_4^' }} Define f_ξ : (H, ב_(ξ)^') by f_ξ (ζ1) = η2 , f_ξ (ζ2) = η1 , f_ξ (ζ3) = η3 and, ב_(ξ ,) I) → (V g_ξ:(V, = ב_(ξ)^") defined by g_ξ (η1) = η_3^' , g_ξ (η2) = η_4^' = g_ξ (η4) andg(η3), ב_(ξ)^',I') → (W ^' η_1^' . It is clear that f_ξ and g_ξ are ξ I g - continuous function but (g_ξ o f_ξ) is not ξIg- continuous as {η_3 .is ξ-closed in W but (g_ξ o f_ξ)^(-1) (η_3^') = {ζ2 } is not ξ I g -closed in H}

References

[1] ME Abd El-Monsef, EF Lashien, and AA Nasef. On i-open sets and i-continuous functions. Kyungpook mathematical journal, 32(1):21–30, 1992.

[2] K Bhuvaneswari and K Mythili Gnanapriya. On nano generalized continuous function in nano topological space. International Journal of Mathematical Archive 6(6):182–186, 2015.

[3] TR Hamlett. Ideals in topological spaces and the set operator ψ. Boll. Un. Mat Ital., 7:863–874, 1990.

[4] Eiichi Hayashi. Topologies defined by local properties. Mathematische Annalen, 156(3):205–215, 1964.

[5] D Jankovic. Compatible extensions of ideals. Boll. Un. Mat. Ital., 7:453–465, 1992.

[6] Dragan Jankovi´c and TR Hamlett. New topologies from old via ideals. The American mathematical monthly, 97(4):295–310, 1990.

[7] KS JENAVEE, R ASOKAN, and O NETHAJI. ζ-nano topological space. Indian journal of Natural Science, 74(1):21–30, 2022.

[8] Kazimierz Kuratowski. Topology: Volume I, volume 1. Elsevier, 2014.

[9] Norman Levine. Generalized closed sets in topology. Rendiconti del Circolo Matematico di Palermo, 19:89–96, 1970.

[10] M Parimala, S Jafari, and S Murali. Nano ideal generalized closed sets in nano ideal topological spaces. In Annales Univ. Sci. Budapest, volume 60, pages 3–11, 2017.

[11] P Samuels. A topology formed from a given topology and ideal. Journal of the London Mathematical Society, 2(4):409–416, 1975.

[12] M Lellis Thivagar and Carmel Richard. On nano continuity. Mathematical theory and modeling, 3(7):32–37, 2013.

[13] M Lellis Thivagar and Carmel Richard. On nano forms of weakly open sets. Inter- national journal of mathematics and statistics invention, 1(1):31–37, 2013.

[14] R Vaidyanathaswamy. The localisation theory in set-topology. In Proceedings of the Indian Academy of Sciences-Section A, volume 20, pages 51–61. Springer India, 1944.

[15] R. Venkatesan and R. Alagar. ξ-Nano ideal topological space(submitted)