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Binary Supra R^G Closed Sets and Binary Supra R^G- Continuous Functions

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Abstract— In this article we introduce a new class of Binary supra closed sets namely Binary Supra R^G closed sets in Binary Topological spaces. Also We examine some of its characteristics. Further we introduce a strong form of Binary Supra continuity called Binary Supra R^G-continuous and Binary Supra R^G- Irresolute functions. We present some basic and comparative properties of these functions.

Index Terms— Binary Supra R^G-closed set, Binary Supra R^G-open set, Binary Supra R^G-Continuous function, Binary Supra R^G- Irresolute function, Binary Supra R^G-$T_{1/2}$ Space.

In 1983, Mashhour et al[5] introduced the concept of Supra topological spaces and studied Supra Continuous maps. Since the advent of these notions, many researchers extended their studies in Supra topological spaces and found new class of Supra closed sets. The authors D.Savithiri, C.Janaki, [8] introduced the concept of R^G-closed sets in 2013. In 2017, M. Lellis Thivagar and J. Kavitha [4] constructed a new structure by merging binary and Supra topology namely Binary Supra topological Spaces.

As an extension, in this article, we introduce a new class of Binary Supra R^G closed sets and Continuous Functions with concrete examples. Moreover some of its basic characterizations and relationships with other Binary Supra closed sets is discussed.

Let X be a nonempty set. The subclass $\mu \subseteq P(X)$ where P(X) is a power set of X is called a supra topology on X if $X,\phi \in \mu$ and μ is closed under arbitrary union. The pair (X,μ) is called a supra topological space. The members of μ are called supra open sets and some of the properties are discussed in [3]. Let $(X,\tau\}$ be a topological space and μ be an associated supra topology on X. A function f: $X\rightarrow Y$ is supra continuous functions if the inverse image of each open set in Y is supra open in X.

A single structure which carries the subsets of X and Y for studying the information about the ordered pair (A,B) of $X\times Y$. Such a structure is called a binary structure from X to Y is given in [2].

2. Preliminaries

Definition 2.1: [3] (i) The supra closure of a set A is denoted by $cl^{\mu}(A)$ and is defined as $cl^{\mu}(A) = \cap\{B$: B is a supra closed and $A \subseteq B\}$.

(ii) The supra interior of a set A is denoted by $int^{\mu}(A)$ and is defined as $int^{\mu}(A) = \cup\{B$: B is a supra open and $A \supseteq B\}$.

Definition 2.2: [2] Let X and Y be any two non empty sets. A binary topology from X to Y is a binary structure $\mu_b \subseteq P(X) \times P(Y)$ that satisfies the following axioms:

(i) $(\phi,\phi) \in \mu_b$ and $(X,Y) \in \mu_b$.

(ii) $(A_1\cap A_2, B_1\cap B_2) \in \mu_b$ whenever (A_1,B_1) and $(A_2,B_2) \in \mu_b$

(iii) If $\{(A_\alpha,B_\alpha) : \alpha\in\Delta\}$ is a family of members of μ_b, then $(\cup A_\alpha,\cup B_\alpha) \in \mu_b$.

If μ_b is a binary topology from X to Y then the triplet (X, Y, μ_b) is called a binary topological space and the members of μ_b are called binary open sets.

The complement of an element of P(X) × P(Y) is defined component wise. That is the binary complement of (A, B) is (X – A, Y – B).

Definition 2.3:[6] Let (X, Y, μ_b) be a binary topological space and A ⊆ X , B ⊆ Y. Then (A, B) is called binary closed if (X – A, Y – B) is binary open.

Definition 2.4: [7]Let (A,B), (C,D) ∈ P(X) × P(Y). Then

(i) (A, B) ⊆ (C, D) if A ⊆ C and B ⊆ D. (ii) (A, B) ∪ (C, D) =(A ∪ C, B ∪ D).

(iii) (A, B) ∩ (C, D) =(A ∩ C, B ∩ D).

Definition 2.5:[9] A subset (A,B) of a binary topological space (X, Y, μ_b) is said to be a binary R^G-closed set if μ_bgCl(A,B) ⊆ (U,V) whenever (A,B) ⊆ (U,V) and (U,V) is binary regular open in (X, Y, μ_b).

Definition 2.6:[4]A Binary Supra Topology from X to Y is a structure B_μ ⊆ P(X) ×P(Y) that satisfies the following axioms. (i) If (X,Y) ∈ B_μ and (ϕ,ϕ)∈ B_μ.

(ii) If $\{(A_\alpha,B_\alpha) : \alpha\in\Delta\}$ is a family of members of B_μ, then $(\cup A_\alpha,\cup B_\alpha) \in B_\mu$.

If B_μ is a binary Supra topology from X to Y then the triplet (X, Y, B_μ) is called a binary supra topological space and the members of B_μ are called Binary supra open sets. The complement of Binary supra open set is called as a Binary supra closed set.

Definition 2.7: [4] Let (X, Y, B_μ) be a binary supra topological space and A ⊆ X, B ⊆ Y. Let $(A, B)^{1*} = \cap\{A_\alpha :(A_\alpha, B_\alpha)$ be binary supra closed and $(A, B) \subseteq (A_\alpha, B_\alpha)\}$, $(A, B)^{2*} = \cap\{B_\alpha :(A_\alpha, B_\alpha)$ be binary supra closed , $(A, B) \subseteq (A_\alpha, B_\alpha)\}$. Thus the pair $((A, B)^{1*},(A, B)^{2*})$ is called the Binary Supra closure of (A,B) and denoted by B_μCl(A,B).

Definition 2.8: [4] Let (X, Y, B_μ) be a binary supra topological space and A ⊆ X, B ⊆ Y. Let $(A, B)^{1^\circ} = \cup\{A_\alpha : (A_\alpha, B_\alpha)$ be binary supra open , $(A_\alpha, B_\alpha) \subseteq (A, B)\}$, $(A, B)^{2^\circ} = \cup\{B_\alpha : (A_\alpha, B_\alpha)$ is binary supra open , $(A_\alpha, B_\alpha) \subseteq (A, B)\}$. Thus the pair $((A, B)^{1^\circ},(A, B)^{2^\circ})$ is called the Binary Supra interior of (A,B) and denoted by B_μInt(A,B).

Definition 2.9:[4] Let (X, Y, B_μ) be a Binary Supra topological space and *(x, y)* ∈ X × Y, then a subset (A, B) of (X, Y) be called a Binary Supra neighborhood of *(x, y)* if there exists a Binary Supra open set (U, V) such that *(x, y)* ∈ (U, V) ⊆ (A, B).

Definition 2.10:[4] Let (X, Y, B_μ) be a binary supra topological space. Let (A,B) ⊆ (X,Y). Define $(A,B,\mu_{A,B}) = \{(A\cap U, B\cap V) : (U,V) \in B_\mu\}$. Then $\mu_{A,B}$ is a binary supra topology from A to B. The binary supra topological space $(A,B,\mu_{A,B})$ is called a binary supra subspace of (X, Y, B_μ).

Definition 2.11: [4] A subset (A,B) of (X,Y, B_μ) is called a

(i) binary supra α-open **(In short $B_\mu\alpha$O)** set if (A,B) ⊆ B_μInt(B_μCl(B_μInt(A,B))).

(ii) binary supra semi-open **(In short B_μSO)**set if (A,B) ⊆ B_μCl(B_μInt(A,B)).

(iii) binary supra preopen **(In short B_μPO)** set if (A,B) ⊆ B_μInt(B_μCl(A,B)).

(iv) binary supra regular open **(In short B_μRO)** set if (A,B) = B_μInt(B_μCl(A,B)).

(iv) binary supra regular semi-open **(In short B_μRSO)** set if (U,V) ⊆ (A,B) ⊆ B_μCl(U,V) where (U,V) is a B_μRO set.

Definition 2.12:[4] Let (X,Y, B_μ) be a binary supra topological space, let (X,τ) be a supra topological space. Let f: Z → X×Y be a function, then f is called a

(i) binary supra continuous if f^{-1}(A,B) is supra open set in Z for every binary supra open set (A,B) of X×Y.

(ii) binary supra α-continuous if $f^{-1}(A,B)$ is supra α-open set in Z for every binary supra open set (A,B) of X×Y.

(iii) binary supra semi-continuous if $f^{-1}(A,B)$ is supra semi-open set in Z for every binary supra open set (A,B) of X×Y.

(iv) binary supra pre-continuous if $f^{-1}(A,B)$ is supra pre-open set in Z for every binary supra open set (A,B) of X×Y.

3. Binary Supra R^G-Closed Sets

Definition 3.1: Let (X,Y,B_μ) be a binary supra topological space and $(A,B) \subseteq (X,Y)$. Then (A,B) is called a binary supra regular ^ generalized closed **(In short B_μR^G-closed)** set if there exists a binary supra regular open set (U,V) such that $B_\mu gcl(A,B) \subseteq (U,V)$ whenever $(A,B) \subseteq (U,V)$.

Definition 3.2: A subset (A,B) of a binary supra topological space (X,Y,B_μ) is said to be a binary supra R^G-open **(In short B_μR^G-open)** if $(A,B)^C$ is B_μR^G-closed.

Definition 3.3: (i) The binary supra R^G closure of a set (A,B) is denoted by B_μR^GCl(A,B) and is defined as B_μR^GCl(A,B) = $\cap$ {(U,V) : (U,V) is a binary supra R^G- closed and (U,V) $\subseteq$ (A,B)}.

(ii) The binary supra R^G interior of a set (A,B) is denoted by B_μR^GInt(A,B) and is defined as B_μR^GInt(A,B) = $\cup$ { U,V) : (U,V) is a binary supra R^G- open and (U,V) $\supseteq$ (A,B)}.

Theorem 3.4: In a binary supra topological space (X,Y, B_μ), if $(A,B) \subseteq (X,Y)$ then

(i) B_μR^GCl(A,B) is the smallest B_μR^G-closed set containing (A,B) .

(ii) (A,B) is binary supra R^G-closed in (X,Y,B_μ) iff (A,B) = B_μR^GCl(A,B).

Proof: (i) Let $\{(A_i,B_i) ; i \in\Delta\}$ be the collection of all binary supra R^G-closed sets containing (A,B). Then (C,D) = $\cap \{(A_i,B_i) ; i \in\Delta\}$ is a binary supra R^G-closed set. Now each (A_i,B_i) is a superset of (A,B) means that (A,B) is contained in their intersection. Hence $(A,B) \subseteq (C,D)$, that is $(C,D) \subseteq (A_i,B_i)$ for each $(x,y) \in\Delta$ and therefore (C,D) is the smallest B_μR^G-closed set containing (A,B).

(ii) Let (A,B) be a B_μR^G-closed set. Since $(A,B) \subseteq B_\mu$R^GCl(A,B) is the smallest B_μR^G closed set containing (A,B). Conversely, let B_μR^GCl(A,B) = (A,B) then (A,B) is binary supra R^G-closed because by definition B_μR^GCl(A,B) is the smallest binary supra R^G-closed set containing (A,B) and B_μR^GCl(A,B) = (A,B) follows that (A,B) is B_μR^G-closed.

Theorem 3.5: Let (X,Y,B_μ) be a binary supra topological space and (A,B) a subset of (X,Y). Then

(i) B_μR^GInt(A,B) is the largest binary supra R^G-open set contained in (A,B).

(ii) (A,B) is binary supra R^G-open if and only if B_μR^GInt(A,B) = (A,B).

Proof: (i) Let (C,D) be any B_μR^G-open subset of (A,B) and $(x,y) \in (C,D)$ that is $(x,y) \in (C,D) \subseteq (A,B)$. Since (C,D) is binary supra R^G-open, (A,B) is a R^G-neighborhood of $(x.y) \in (C,D)$ and consequently (x,y) is a binary supra R^G interior of (A,B). Since $(x,y) \in (C,D) \Rightarrow (x,y) \in B_\mu$R^GInt(A,B). Therefore $(C,D) \subseteq B_\mu$R^GInt(A,B) . Since (C,D) is B_μR^G-open, thus B_μR^GInt(A,B) is the largest B_μR^G-open subset of (A,B).

(ii) Let (A,B) = B_μR^GInt(A,B), B_μR^GInt(A,B) is B_μR^G-open. Then (A,B) is also B_μR^G-open. Conversely, since (A,B) is a B_μR^G-open, then B_μR^GInt(A,B) is the largest B_μR^G-open subset of (A,B). Hence B_μR^GInt(A,B) = (A,B).

Theorem 3.6: Every (i) binary supra closed (ii) binary supra g closed (iii) binary supra g*-closed set is binary supra R^G-closed set .

Proof: (i) Let $(A,B) \subseteq (X,Y,B_\mu)$ be a binary supra closed set and $(A,B) \subseteq (U,V)$ where (U,V) be a B_μ-regular open. Since (A,B) is binary supra closed and every B_μ-closed set is B_μg-closed set, $B_\mu gcl(A,B) \subseteq B_\mu cl(A,B) = (A,B) \subseteq$

(U,V). Hence (A,B) is a B_μR^G-closed set.

(ii) Let (A,B) ⊆ (X,Y,B_μ) be a B_μg-closed set and (A,B) ⊆ (U,V) where (U,V) is a B_μ-regular open. Since every B_μRO set is B_μ-open and (A,B) is B_μg-closed, $B_\mu$cl(A,B) ⊆ ((U,V). Every B_μ-closed set is B_μg-closed set means $B_\mu$gcl(A,B) ⊆ $B_\mu$cl(A,B) ⊆ (U,V) and therefore (A,B) is B_μR^G-closed.

(iii) Let (A,B) ⊆ (X,Y,B_μ) be a B_μg*-closed set and (A,B) ⊆ (U,V) where (U,V) is a B_μ-regular open. Since every B_μRO set is B_μg-open and (A,B) is B_μg*-closed, $B_\mu$cl(A,B) ⊆ ((U,V). Every B_μ-closed set is B_μg-closed which implies $B_\mu$gcl(A,B) ⊆ $B_\mu$cl(A,B) ⊆ (U,V) implies that (A,B) is B_μR^G-closed.

Remark 3.7: The following example makes clear that the converse of the Theorem 3.6 need not be true.

Example 3.8: Let X = {a,b}, Y= {1,2}, B_μ = { (ϕ,ϕ) ,({a},ϕ), ({b},ϕ), (X,ϕ), (X,{1}), (X,{2}), (X,Y)) be a binary supra topology. In the binary supra topological space (X,Y,B_μ), binary supra subset ({a},{1}) is a B_μR^G-closed set, but it is not a B_μ-closed , B_μg-closed and B_μg*-closed set.

Theorem 3.9: The finite union of B_μR^G-closed set is B_μR^G-closed.

Proof: Assume that (A,B) and (C,D) are B_μR^G-closed sets in (X,Y,B_μ). Let (A,B) ∪ (C,D) ⊆ (U,V) where (U,V) is a B_μRO. Then (A,B) ⊆ (U,V) and (C,D) ⊆ (U,V). Since (A,B) and (C,D) are B_μR^G-closed, $B_\mu$gcl(A,B) ⊆ (U,V) and $B_\mu$gcl(C,D) ⊆ (U,V) implies $B_\mu$gcl[(A,B) ∪ (C,D)] = $B_\mu$gcl(A,B) ∪ $B_\mu$gcl(C,D) ⊆ (U,V), hence (A,B) ∪ (C,D) is a B_μR^G-closed set.

Remark 3.10: The intersection of two B_μR^G-closed set in (X,Y, B_μ) need not be a B_μR^G-closed set as shown in the following example.

Example 3.11: In the binary supra topological space (X,Y,B_μ) where X = {a,b}, Y = {1,2} and B_μ = {(ϕ,ϕ), ({a},ϕ), ({b},ϕ), (X,ϕ), (X,{1}, (X,{2}), (X,Y)}. The binary supra subsets (A,B) = ({a},{1}) and (C,D) = ({a},{2}) are binary supra R^G-closed sets, but (A,B) ∩ (C,D) = ({a},ϕ) is not a B_μR^G-closed set.

Theorem 3.12: Let (X,Y,B_μ) be a binary supra topological space, A ⊆ X, B ⊆ Y. If (A,B) is a binary supra R^G-open in (X,Y,B_μ), then A^C is μ_bR^G-closed set in X and B^C is μ_bR^G-closed set in Y.

Proof: By definition, μ_X = {A ⊆ X; (A,B) ∈ B_μ for some B ⊆ Y) is a supra topology on X and μ_Y = {B ⊆ Y; (A,B) ∈ B_μ for some A ⊆ X) is a supra topology on Y. Since (A,B) is a binary supra R^G-open, then it is B_μ-open in (X,Y,B_μ) implies A ∈ μ_X and B ∈ μ_Y. That is A is supra open in X and A^C is supra closed set in X. Similarly B^C is supra closed set in Y. Every supra closed set is supra R^G-closed set. Therefore A^C and B^C are supra R^G-closed sets in (X, μ_X) and (Y, μ_Y) respectively.

Remark 3.13: The following example shows that the converse of the above theorem need not be true from the fact that every supra R^G closed set need not be a supra closed set.

Example 3.14: Let X = {a,b}, Y = {1,2}, μ_X = {ϕ,{a},X} μ_Y = {ϕ,{2},Y} and B_μ = {(ϕ,ϕ),(ϕ,{2}) ,(ϕ,Y),({a},ϕ},({a},{2}),({a},Y), (X,ϕ), (X,{2}),.(X,Y)}. In the supra topological space (X, μ_X), {b} is μ_bR^G-closed set and {2} is μ_bR^G-closed set in (Y, μ_Y), but ({a},{1}) is not a binary supra open set in (X,Y,B_μ).

Theorem 3.15: Let (A,B)be an B_μR^G closed set in a binary supra topological space (X,Y,B_μ). Then $B_\mu$gcl(A,B) ⊆ (A,B) contains no non-empty B_μ- regular closed set in X.

Proof: Let (U,V) be a B_μ- regular closed set such that (U,V) ⊆ $B_\mu$gcl(A,B) – (A,B). Then (U,V) ⊆ (X,Y) – (A,B) implies (A,B) ⊂ (X,Y) – (U,V). Since (A,B) is B_μR^G-closed set and (X,Y) – (U,V) is B_μ-regular open, then $B_\mu$gcl(A,B) ⊂ (X,Y) – (A,B). That is (U,V) ⊂ (X,Y) – $B_\mu$gcl(A,B). Hence (U,V) ⊂ $B_\mu$gcl(A,B) ∩ (X,Y) – $B_\mu$gcl(A,B)) =(ϕ,ϕ). Thus (U,V) = (ϕ,ϕ), whence $B_\mu$gcl (A,B) – (A,B) does not contain nonempty B_μ-regular closed set.

Theorem 3.16: Let (A,B) be a binary supra R^G-closed set in a supra binary topological space (X,Y,B_μ) and (A,B) ⊆ (C,D) ⊆ $B_\mu$gcl(A,B). Then (C,D) is a binary supra R^G- closed set.

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Proof: Since (A,B) is a binary supra R^G-closed set, there exists a binary supra regular open set (U,V) such that $B_\mu gcl(A,B) \subseteq (U,V)$. Since $(C,D) \subseteq B_\mu gcl(A,B)$, $B_\mu gcl(C,D) \subseteq B_\mu gcl(B_\mu gcl(A,B))$ i.e., $B_\mu gcl(C,D) \subseteq B_\mu gcl(A,B) \subseteq (U,V)$. Therefore (C,D) is also a binary supra R^G- closed set.

Theorem 3.17: Let (X,Y,B_μ) be a binary supra topological space and $A \subseteq X$, $B \subseteq Y$. Suppose that $(A,B,\mu_{A,B})$ is a binary supra subspace of (X,Y,B_μ). Suppose (C,D) is a B_μR^G-closed set in (X,Y,B_μ) and $(C,D) \subseteq (A,B)$. Then (C,D) is B_μR^G-closed set in $(X,Y,B\mu_{A,B})$.

Proof: Since (C,D) is a B_μR^G-closed set in (X,Y,B_μ), we have $B_\mu gcl(C,D) \subseteq (U,V)$ where (U,V) is a B_μRO in (X,Y,B_μ) and hence it should be in B_μ. By definition of binary supra subspace, $(U \cap A, V \cap B) \in \mu_{A,B}$. Let (U,V) be a $B\mu_{A,B}$-RO set and $(C,D) \subseteq (U,V)$. $B\mu_{A,B}\, gcl(C,D) = B\mu_{A,B}\, gcl(C \cap A, D \cap B) \subseteq B\mu_{A,B}\, gcl(C \cap U, D \cap V) \subseteq (U,V)$. Therefore (C,D) is a B_μR^G –closed set in $(X,Y,B\mu_{A,B})$.

Remark 3.18: The following example makes clear that the converse of the above theorem need not be true.

Example 3.19: Let X = {a,b,c}, Y = {1,2}, Clearly, B_μ = {(ϕ,ϕ),(ϕ,{2}),({b},{1}),({b},Y},({c},{2}),({b,c},Y), (X,{2}),(X,Y)} is a binary supra topology from X to Y. Let (A,B) = ({a,b}, {1,2}) be a subset of (X,Y). Let $\mu_{A,B}$ = {(ϕ,ϕ), ({a},ϕ), ({b},ϕ), (X,ϕ),(X,{1}),(X,{2}),(X,Y)} be a binary supra topology from A to B. Consider (C,D) = ({b},{1}) $\subseteq$ (A,B) which is a $B\mu_{A,B}$ R^G-closed set in$(X,Y,B\mu_{A,B})$ but (C,D) is not a B_μR^G-closed set in (X,Y,B_μ).

Remark 3.20: The following example shows that the concept of binary supra R^G-closed set is independent with the concepts of

1. binary supra semi-closed sets

2. binary supra pre-closed sets

3. binary supra semipre-closed sets

4. binary supra wg-closed sets

5. binary supra α-closed sets

Example 3.21: Let X = [a,b,c}, Y = {1,2} , B_μ = {(ϕ,ϕ),(ϕ,{2}),({b},{1}), ({b},Y},({c},{2}),({b,c},Y), (X,{2}),(X,Y)}. In the binary supra topological space (X,Y,B_μ),

1. Let A = ({a},{1}),B = ({b},{1}) then A is a B_μR^G-closed set but it is not a $B_\mu$semi-closed set and B is a $B_\mu$semi-closed set but it is not a B_μR^G-closed set.

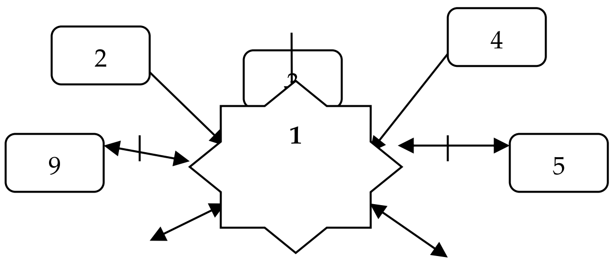
2. Let A = (ϕ,Y),B = ({b},ϕ) then A is a B_μR^G-closed set but it is not a $B_\mu$pre-closed set and B is a $B_\mu$pre-closed set but it is not a B_μR^G-closed set.

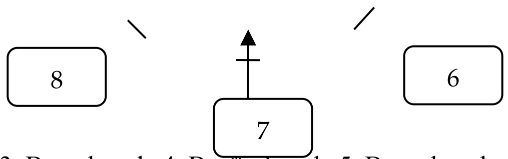
3. Let A = ({a},Y),B = ({c},ϕ) then A is a B_μR^G-closed set but it is not a $B_\mu$semipre-closed set, but B is a $B_\mu$semipre-closed set but it is not a B_μR^G-closed set.

4. Let A = ({a},Y),B = ({b},ϕ) then A is a B_μR^G-closed set but it is not a $B_\mu$wg-closed set and B is a $B_\mu$wg-closed set but it is not a B_μR^G-closed set.

5. Let A = {a},{2}),B = ({c},ϕ) then A is a B_μR^G-closed set but it is not a B_μα-closed set and B is a B_μα-closed set but it is not a B_μR^G-closed set.

The above discussions are implicated in the following diagram.





1.B_μR^G-closed 2. $B_\mu$closed 3. B_μg-closed 4. B_μg*-closed 5. $B_\mu\alpha$-closed 6. $B_\mu$semi-closed 7. $B_\mu$pre-closed 8. $B_\mu$semipre-closed 9. $B_\mu$wg-closed

A $\longrightarrow$ B means that A implies B not in reverse, A $\nleftrightarrow$ B means that A and B are independent.

4. Binary Supra R^G-Continuous and Irresolute functions

Definition 4.1: Let (X,Y,B_μ) be a binary supra topological space. Let (Z,τ) be a supra topological space and μ be a supra topology associated with τ. Let f: $Z \to X \times Y$ be a function. Then f is called a binary supra R^G-continuous **(Shortly B_μR^G-continuous)** function if the inverse image of every binary supra closed subset (A,B) of X×Y is supra R^G- closed in Z.

Definition 4.2: Let (X,Y,B_μ) be a binary supra topological space. Let (Z,τ) be a supra topological space and μ be a supra topology associated with τ. Let f: $Z \to X \times Y$ be a function. Then f is called a binary supra R^G-irresolute **(Shortly B_μR^G-Irresolute)** function if the inverse image of every binary supra R^G-closed subset (A,B) of X×Y is supra R^G-closed in Z.

Theorem 4.3: Let (X,Y,B_μ) be a binary supra topological space and (X,τ) be a supra topological space. Then a function $f:Z\to X\times Y$ is binary supra R^G-continuous iff the inverse image under f of every binary supra open set (A,B) in (X,Y,B_μ).

Proof: let (U,V) be a binary supra open subset of (X,Y,B_μ). If $f^{-1}(U,V) = \phi$, then ϕ is B_μR^G-open. Suppose $f^{-1}(U,V) \neq \phi$, then let x be an arbitrary element of $f^{-1}(U,V)$ so that $f(x) \in (U,V)$. Since f is binary supra R^G-continuous, there is a supra R^G-open set G containing x such that $f(G) \subseteq (U,V)$ or $G \subseteq f^{-1}(U,V)$ corresponding to a binary supra open set (U,V) in X×Y. Hence $x \in G \subseteq f^{-1}(U,V)$. It is clear that $f^{-1}(U,V)$ is a supra neighborhood of x implies $f^{-1}(U,V)$ is supra open. Therefore $f^{-1}(U,V)$ is supra R^G-open.

Conversely, let (U,V) be any binary supra open set containing f(x) so that $x \in f^{-1}(U,V)$ where $f^{-1}(U,V)$ is B_μR^G-open. Put $f^{-1}(U,V) = A$ where A is a binary supra R^G-open set containing x. Also $f(A) = f(f^{-1}(U,V)) \subseteq (U,V)$. Hence by definition, f is binary supra R^G-continuous at x, but x is arbitrary, it follows that f is B_μR^G-continuous at every point x of Z. Thus f is binary supra R^G-continuous.

Theorem 4.4: Let (X,Y,B_μ) be a binary topological space and f: $Z \to X\times Y$ be a function such that $Z\backslash f^{-1}(A,B) = f^{-1}(X\backslash A, Y\backslash B)$ for all $A \subseteq X$, and $B \subseteq Y$. Then f is binary supra R^G-continuous iff $f^{-1}(A,B)$ is supra closed in Z for all binary supra closed sets (A,B) in (X,Y,B_μ).

Proof: Assume that f is binary supra R^G-continuous. Let $(A,B) \in X\times Y$ be binary supra closed. Therefore $(X\backslash A, Y\backslash B)$ is a binary supra open set. Since f is binary supra R^G-continuous, by Theorem 4.3, we have, $f^{-1}(X\backslash A, Y\backslash B)$ is supra R^G-open in Z. Hence $f^{-1}(A,B)$ is supra R^G-closed in Z.

Conversely, assume that $f^{-1}(A,B)$ is supra R^G-closed in Z for all binary closed set (A,B) in (X,Y,B_μ). Let $(A,B) \in X\times Y$ be a binary supra open set. Since $(A,B) \in B_\mu$, $(X\backslash A, Y\backslash B)$ is binary supra closed in X×Y, by assumption, $f^{-1}(X\backslash A,X\backslash B)$ is supra R^G-closed in Z. Thus $Z\backslash f^{-1}(A,B)$ is supra R^G-closed in Z. Hence $f^{-1}(A,B)$ is supra R^G-open in Z. Therefore f is binary supra R^G-continuous.

Theorem 4.5: a) Every binary supra continuous function is B_μR^G-continuous.

b) Every B_μg-continuous function is B_μR^G-continuous.

c) Every B_μg*-continuous function is B_μR^G -continuous.

Proof: Obvious from the Theorem 3.6.

Remark 4.6: The converse of the Theorem 4.5 need not be true as shown in the following example.

Example 4.7: Let X = [a,b}, Y = {1,2} , Z= {a,b,c,d} and B_μ = {(ϕ,ϕ),({a},{2}),({b},Y),(X,Y)} be a binary supra topology from X to Y and τ = {ϕ,{a},{b,c},{a,b}, {a,b,c},,Z} be a supra topology on Z.. Define a function f: Z → X×Y as f(a) = ({b},ϕ), f(b) = (ϕ,{1}), f(c) = (ϕ,{2}), f(d) = ({a},ϕ). Then f is B_μR^G-continuous but it is not a binary supra continuous, B_μg-continuous and B_μg*-continuous since f^{-1} ({b},{1}) = {a,b} is not supra closed, μ_bg-closed, μ_bg*-closed set in (Z,τ).

Theorem 4.8: Let (Z,τ) be a supra R^GT^$_{1/2}$ space and f: Z→X×Y be a function. Then f is B_μR^G-continuous iff f is B_μg-continuous.

Proof: Let f be a B_μR^G-continuous function and (A,B) a binary supra closed set of X×Y. Then by hypothesis, f^{-1}(A,B) is B_μR^G-closed in X×Y. Since Z is a μ_bT^$_{1/2}$ space, f^{-1}(A,B) is a supra g-closed set in Z and hence f is binary supra g-continuous.

Conversely, let f be a B_μg-continuous function, then f^{-1}(A,B) is B_μg-closed for every binary supra closed set (A,B) of X×Y. Every B_μg -closed set is B_μR^G -closed which implies that f is B_μR^G-continuous.

Remark 4.9: B_μR^G-irresolute function need not be a B_μR^G-continuous function which is shown by using the following example.

Example 4.10: Let X = {a,b}, Y = {1,2}, Z = {a,b,c,d} B_μ = {(ϕ,ϕ),({a},ϕ), ({b},ϕ),(X,ϕ),(X,{1}, (X,{2}),(X,Y)} be a binary supra topology of (X,Y,B_μ) and τ = {ϕ,{c},{a,c},{b,d}, {b,c,d},Z} be a supra topology of Z. Define f: Z → X×Y as f(a) = ({b},ϕ), f(b) = (ϕ,{1}), f(c) = (ϕ,{2}), f(d) = ({a},ϕ) then f is B_μR^G – irresolute but it is not a B_μR^G-continuous function.

Definition 4.11: A map f: Z → X×Y is said to be a Binary supra contra R^G-continuous function **(In short B_μ contra R^G-continuous)** if the inverse image of a B_μ-open set (A,B) of X×Y is supra R^G-closed set in Z.

Example 4.12: Let X = {a,b}, Y = {1,2}, Z = {a,b,c,d} B_μ = {(ϕ,ϕ), ({a},{2}),({b},Y), (X,Y)} be a binary supra topology of (X,Y,B_μ) and τ = {ϕ,{a,b},{b,c},{a,b,c}} be a supra topology of Z. Define f: Z → X×Y as f(a) = (ϕ,{2}), f(b) = ({a},ϕ), f(c) =(ϕ,{1}), f(d) = ({b},ϕ) then f is B_μ contra R^G –continuous function.

Definition 4.13: A function f: Z →X×Y is said to be a

(i) binary supra almost contra continuous **(shortly B_μ almost contra - continuous)** function if the inverse image of every binary supra regular open set (U,V) of X×Y is a supra closed set in Z.

(ii) binary supra almost contra regular^generalized continuous **(shortly B_μ almost contra R^G-continuous)** function if the inverse image of every binary supra regular open set (U,V) of X×Y is supra R^G- closed set in Z.

Theorem 4.14: Every binary supra almost contra continuous function is binary supra almost contra R^G-continuous function.

Proof: Straight forward from the definition.

Remark 4.15: The converse of the above theorem need not be true as seen in the following example.

Theorem 4.16: Suppose that supra R^G closed sets of Z is closed under arbitrary unions. The following statements are equivalent for a given function f: (Z,η) → X×Y.

(i) f is binary supra almost contra R^G- continuous function.

(ii) For every binary supra regular closed subset (A,B) of X×Y,f^{-1}(A,B) ∈ R^G$^\mu$O(Z,η).

(iii) For each x∈Z and each binary supra closed set (A,B) in X×Y containing f(x), there exists a supra R^G- open set U in Z containing x such that f(U) ⊆ (A,B).

Proof: (i) ⇒ (ii): Let (A,B) be a binary supra regular closed set. Then (X-A,Y-B) is a binary supra regular open set. Since f is binary supra almost contra R^G- continuous function, the inverse image of (X-A,Y-B) ∈ $R\hat{}G^{\mu}C(Z,\eta)$. Hence $f^{-1}(A,B) \in R\hat{}G^{\mu}O(Z,\eta)$.

(ii) ⇒ (i) and (iii) ⇒ (i) are obvious.

(ii) ⇒ (iii): Let (A,B) be a binary supra regular closed set in X×Y containing f(x). $f^{-1}(A,B) \in R\hat{}G^{\mu}O(Z,\eta)$ and $x \in f^{-1}(A,B)$. Taking $U = f^{-1}(A,B)$, $f(U) \subset (A,B)$.

(iii) ⇒ (ii): Let $(A,B) \in B_{\mu}RC(X\times Y)$ and $x \in f^{-1}(A,B)$. From (iii), there exists a supra R^G open set U in Z containing x such that $U \subset f^{-1}(A,B)$. We have $f^{-1}(A,B) = \cup \{U: x \in f^{-1}(A,B)\}$. Thus $f^{-1}(A,B)$ is supra R^G-open in Z.

Definition 4.17: A map f: Z → X×Y is called a

(i) binary supra R^G-closed **(In short B_{μ}R^G – closed)** if f(U) is B_{μ}R^G-closed in X×Y for every supra closed set U of Z.

(ii) binary supra R^G-open **(In short B_{μ}R^G-open)** if f(U) is B_{μ}R^G-open in X×Y for every supra open set U of Z.

Definition 4.18: A map f: Z → X×Y is called a

(i) binary supra contra R^G-closed **(In short contra B_{μ}R^G – closed)** if f(U) is B_{μ}R^G-closed in X×Y for every supra open set U of Z.

(ii) binary supra contra g-closed **(In short contra B_{μ}g – closed)** if f(U) is B_{μ}g-closed in X×Y for every supra open set U of Z.

Definition 4.19: A binary supra topological space (X,Y,B_{μ}) is said to be a binary supra R^G locally indiscrete **(In short B_{μ}R^G-locally indiscrete)** if every binary supra R^G -open subset of (X,Y,B_{μ}) is binary supra closed.

Theorem 4.20: a) If f: Z → X×Y is binary supra contra R^G-open map and the space (X,Y,B_{μ}) is B_{μ}R^G- locally indiscrete, then f is a binary supra- closed map.

b) If f: Z → X×Y is binary supra R^G-open map and the space (X,Y,B_{μ}) is B_{μ}R^G- locally indiscrete, then f is a binary supra contra -closed map.

c) If f: Z → X×Y is binary supra contra R^G-closed map and the space (X,Y,B_{μ}) is B_{μ}R^G- $T\hat{}_{1/2}$ space then f is a binary supra contra g- closed map.

Proof: a) Let G be any supra closed set in Z. By hypothesis, f(G) is B_{μ}R^G-open in X×Y. Since Y is B_{μ}R^G-locally indiscrete, f(G) is binary supra closed in X×Y. Therefore f is a binary supra closed map.

b) Let H be a supra open set in Z. Since f is a B_{μ}R^G-open map, f(H) is B_{μ}R^G-open and B_{μ}R^G- locally indiscrete implies that f(H) is binary supra closed in X×Y. Thus f is a binary supra contra- closed map.

c) Let V be a supra open subset of Z. By hypothesis, f(H) is B_{μ}R^G-closed in X×Y. Since (X,Y,B_{μ}) is $B_{\mu}R\hat{}GT\hat{}_{1/2}$-space, f(H) is B_{μ}g-closed which implies that f is a binary supra contra g-closed.

Conclusion: We introduce a new class of Binary Supra closed sets namely Binary Supra R^G closed sets. We then studied some fundamental properties of it. In further we introduce the concept of Binary $R\hat{}G^{\mu}$-continuous and irresolute functions and analyzed their behavior. The interrelations with other continuous functions in Binary Supra topological spaces also studied by using some concrete examples. In future, Binary supra R^G-homeomorphisms can be extended further.

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