Tuijin Jishu/Journal of Propulsion Technology
ISSN: 1001-4055
Vol. 45 No. 3 (2024)

Optimizing Inventory Management for Non-Instantaneously Deteriorating Items: The Synergistic Role of Preservation Technology and Greening Investments

Niketa D Trivedi
BSH Department, MPSTME, SVKM's Narsee Monjee Institute of Management Studies (NMIMS) Deemed-to-be-University, Mumbai-400056, Maharashtra, India.

Abstract: In today's competitive business landscape, sustainable practices are becoming crucial for long-term success. Organizations implementing eco-friendly measures optimize resource use through greening investments. Businesses embrace green strategies to not only enhance product marketing but also to build customer loyalty and improve their corporate reputation. Moreover, many environmentally conscious companies address product deterioration by investing in preservation technology. This paper develops an inventory model for non-instantaneously deteriorating items, exploring the combined impact of preservation technology and greening investments on inventory management. The study examines how customer demand, influenced by both price and greenness levels, affects inventory decisions under partially backlogged shortages. Theoretical results are formulated to identify optimal pricing, replenishment cycles, and preservation technology investment costs across different greenness levels. A unique aspect of this research is the use of a power function for the preservation technology investment in the numerical example. Sensitivity analysis is conducted to evaluate the model's robustness and provide practical managerial insights.

Keywords: Inventory; non-instantaneous deterioration; preservation technology investment; greenness level; partial backlogging; pricing

1. Introduction

1.1 Motivation

The quality and storage conditions of goods significantly affect their rate of deterioration, leading to value loss. Therefore, retailers need to implement sustainable inventory models. Certain products maintain their quality for a period before beginning to deteriorate, a phenomenon known as "non-instantaneous deterioration". Enhancing storage facilities can mitigate the deterioration rate, thereby decreasing overall annual inventory costs (Dye, 2013). Consumer behaviour, especially in grocery retail, plays a crucial role in inventory management. Intense competition drives retailers to adjust pricing and promotions to maximize profits(Soni and Suthar, 2019).

Increasing consumer concern for sustainability is reshaping supply chain strategies. This study focuses on a retailer handling non-instantaneously deteriorating products, with an emphasis on greening investments. In a market sensitive to price and environmental friendliness, the retailer invests in preservation technology. The mathematical model developed in this study aims to maximize total profit by optimizing price, order cycles, and preservation costs while allowing for partial backlogging.

1.2 Research question

Effective inventory management is essential in the competitive grocery retail industry, balancing sales revenue with inventory costs. This model investigates the combined impact of preservation and green technology investments for non-instantaneously deteriorating products. The study addresses key questions for retailers: 1) what are the optimal values for sales price, preservation technology investment, and ordering quantity, considering inventory shortages and deterioration? 2) How should a retailer select the appropriate greenness level, given the various costs associated with each option, to maximize total profit?

The paper is organized as follows: Section 2 reviews the relevant literature, identifying research gaps and contributions. Section 3 outlines the assumptions and notations for the inventory system. Section 4 develops the mathematical model of the inventory system. Section 5 includes a numerical example, and Section 6 presents a sensitivity analysis to evaluate parametric performance. Finally, Section 7 discusses conclusions and future research directions.

2. Literature review

The issue of product deterioration poses a significant challenge within inventory management systems. Items such as medicine, food grains, agricultural products, and blood are all susceptible to quality degradation over time. To ensure business stability and profitability, it's essential to adopt effective inventory management strategies for perishable goods. Typically, products maintain their quality for a certain period before starting to degrade, a process identified by Wu et al. (2006) as non-instantaneous deterioration. Since then, numerous researchers have focused on managing inventories of non-instantaneously deteriorating items, with notable contributions from Ouyang et al. (2009), Geetha and Uthayakumar (2010), Maihami and Abadi (2012), Tat et al. (2015), Shaikh et al. (2017), Bardhan et al. (2019), Khan et al. (2020), and Mahdavisharif et al. (2022).

Investing in preservation technology is one strategy to mitigate deterioration rates, promoting sustainable business practices by extending the lifespan of inventory and reducing losses. This technology combats deterioration caused by external factors like environmental conditions or storage setups. By incorporating preservation technology, businesses can control these factors and enhance product shelf life.

Hsu et al. (2010) were pioneers in examining the optimal replenishment policies and preservation technology investments for retailers facing a constant deterioration rate. They proposed that the cost of preservation technology remains fixed regardless of the replenishment cycle duration. Dye and Hsieh (2012) expanded on this by linking preservation costs to the replenishment cycle, considering time-varying deterioration and partial backlogging. Dye (2013) further extended this model to explore the effects of preservation technology on time-varying deterioration and partial backlogging.

Tsao (2014) took a novel approach by integrating location decisions with preservation technology investments in inventory management for non-instantaneously deteriorating items under trade credits. Yang et al. (2015) focused on optimizing trade credit, preservation technology investment, and replenishment strategies in scenarios with time-dependent demand and deterioration to maximize retailer profits over a finite period.

More recent studies, like those by Rapolu et al. (2020), Shah et al. (2021), and Mahapatra et al. (2022), have further investigated the optimization of various factors including price, advertisement frequency, and promotional efforts in conjunction with preservation technology investment. These studies have shown the significant impact of preservation technology on inventory management across different contexts and settings, as also explored by Liu et al. (2015), Singh and Rathore (2015), Singh et al. (2016), Mishra et al. (2017, 2018, 2019), Soni and Chauhan (2018), Mashud et al. (2021), and Ihsan Hishamuddin et al. (2020).

In a fiercely competitive business landscape, organizations must leverage a variety of marketing strategies to enhance product promotion. These strategies often include engaging advertisements in both print and electronic media, price discounts, free gifts, and discount coupons to stimulate sales. For products that deteriorate non-instantaneously, researchers have developed inventory models integrating different demand functions with promotional efforts.

Tsao and Sheen (2008) introduced an inventory model focusing on dynamic pricing, promotion, and replenishment policies for deteriorating items, considering permissible payment delays. Zhang et al. (2008) examined a single-item, finite horizon, periodic review model where demand is influenced by price and promotions, aiming to maximize retailer profits. Szmerekovsky et al. (2009) explored pricing decisions and dual advertising efforts between a manufacturer and a retailer, assessing how demand is affected by price and promotional activities, and evaluated the impact of a linear contract in a decentralized system with the manufacturer as the leader.

Li et al. (2013) studied pricing, ordering, and advertising coordination within a single-manufacturer, single-retailer supply chain, assuming stochastic market demand influenced by retail prices and advertising efforts. Maihami and Karimi (2014) developed a model to optimize pricing and replenishment policies for non-instantaneously deteriorating items under stochastic demand and promotional efforts. Barrón and Sana (2015) proposed a multi-item inventory model in a two-layer supply chain where demand is sensitive to promotional activities.

Geetha and Udayakumar (2016) created an Economic Order Quantity (EOQ) model where demand depends on price and advertising. Sundara Rajan and Uthayakumar (2017) analysed an EOQ model incorporating promotional efforts and permissible payment delays, allowing for shortages. Palanivel et al. (2017) established a two-warehouse system considering promotional efforts, inflation, and partially backlogged shortages over a finite time horizon.

Rabbani et al. (2018) explored cooperative advertising in a manufacturer-retailer supply chain using noncooperative game theory, including Nash, Stackelberg retailer, and Stackelberg manufacturer games, to capture pricing and advertising strategies. Soni and Suthar (2019) investigated an inventory model with partial backlogging, time-dependent deterioration, and stochastic demand influenced by price and promotional efforts. Mahapatra et al. (2019) formulated an EOQ model for deteriorating items with fuzzy demand, influenced by promotional efforts and allowing full backordering within a specific time horizon.

Finally, Rapolu and Kandpal (2020) developed an inventory model where demand is affected by both selling price and advertisement frequency. Their study identified optimal replenishment policies, integrating preservation technology investments for non-instantaneously deteriorating items.

Global warming and the greenhouse effect have become major concerns due to the increasing frequency of extreme weather events, including glacier melting, coastal flooding, hurricanes, and the desertification of fertile regions. Human activities such as burning fossil fuels (coal, oil, and gas) for industrial purposes, intensive agriculture, and transportation have elevated greenhouse gas levels in the atmosphere, particularly carbon dioxide and methane. Given that economies heavily rely on industrial activities, completely eliminating carbon emissions is impractical. However, industries face the significant challenge of reducing these emissions and mitigating global warming, which can be partially addressed through investments in green technology (Mishra et al., 2020).

Investing in green technology helps reduce carbon emissions and conserves natural resources, promoting sustainable development without harming the environment. Many governments, both in developed and developing countries, are encouraging green technology by offering fiscal incentives. Technologies such as solar, wind, and ocean energy, energy conservation methods, and bioremediation are designed to protect the environment and conserve resources (Gangadhar and Ramakrishna, 2017). Businesses committed to sustainability can use these technologies to make better inventory decisions under various carbon emission policies.

Numerous researchers have explored the integration of green technology investments into inventory models. Hsu et al. (2013) discussed how green technology supports organizational sustainability by enhancing product marketing. Toptal et al. (2014) examined the joint decisions of inventory replenishment and green technology investment for retailers under three carbon emission regulation policies: cap, tax, and cap-and-trade. Their findings indicated that while green technology investments significantly reduce costs under all policies, the cap policy does not lower annual emissions levels, unlike tax and cap-and-trade policies.

Lou et al. (2015) investigated the effects of cap-and-trade regulations on pricing and investment decisions in a two-stage supply chain. Lin (2018) developed an optimal inventory policy aimed at reducing transportation emission costs through green technology investment. According to an Accenture study cited by Hong et al. (2019), about 80% of customers consider the environmental friendliness of a product a significant factor in their purchasing decisions.

Mashud et al. (2020) explored the relationship between preservation technology, green technology investments, and discounts on defective items. Paul et al. (2022) developed an inventory model utilizing green technology to determine optimal replenishment times and green concern levels, maximizing profit with demand sensitive to both price and environmental impact. Hasan et al. (2021) created an inventory model showing how green technology investments and promotions positively influence demand.

Maihami et al. (2021) studied the impact of various greenness levels on inventory systems for non-instantaneously deteriorating items during partially backlogged shortages. Yadav and Khanna (2021) developed a green inventory model for perishable items with time-varying deterioration rates, expiration dates, price-sensitive demand, and carbon emission taxes. Mishra et al. (2021) were the first to explore sustainable inventory management with controllable carbon emissions from greenhouse farms, finding that investments in preservation and green technology reduce deterioration and emission rates during various backorder scenarios. Marchi et al. (2023) propose a mathematical model for integrated inventory management in a single-vendor, single-buyer supply chain using a consignment stock (CS) policy. This model aims to enhance the chain's economic performance while evaluating the advantages and disadvantages of the CS policy, considering both economic and environmental aspects. San Jose et al. (2024) present a sustainable inventory management system that accounts for costs from carbon emissions due to transportation, improper product maintenance, and disposal of deteriorated items. The study's main innovation is its comprehensive and practical approach, incorporating environmental constraints to evaluate the impact of current inventory practices more accurately.

Some researchers have combined green technology and preservation technology investments in their inventory models, demonstrating significant reductions in both carbon emissions and product deterioration, thus maximizing retailer profits. Saha et al. (2017) recommended continuous investment in both technologies for maximum revenue. Mishra et al. (2019) developed an inventory policy for non-instantaneously deteriorating items, incorporating joint pricing, dynamic investments in environmental and preservation costs, and optimal replenishment times.

Further studies by Mishra et al. (2021) examined optimal replenishment strategies under different backlogging scenarios, integrating preservation and green technology investments. Mashud et al. (2021) proposed a model incorporating these technologies to reduce emissions from greenhouse operations and product deterioration within a two-warehouse system. Thomas and Mishra (2022) introduced a circular sustainable integrated model for the plastic reforming industry, involving investments in 3D printing techniques to reduce waste, emissions, and ordering costs, ultimately maximizing profit and achieving an optimal level of circularity.

3. Notations and Assumptions:

The mathematical model in this study is developed based on the following notations.

Parameters

K_1	The coefficient of the retailer's greenness investment, $K_1 > 0$
μ	Consumer greenness level elasticity
h	Holding cost ($) per unit per time unit
l	Lost sale cost per unit ($)
s	Backorder cost ($) per unit per time unit
w	Greenness Investment Level: The vendor can choose from a predefined set of greenness levels $w \in \{w_1, w_2, w_3, \ldots, w_n\}$. Level w_1 is the lowest greenness level, which is mostly required by regulations(Maihami et al. 2021)
t_d	Duration of the product's non-deterioration period
b	Maximum investment allocated for preservation technology
c	Purchasing cost ($) per unit, where $0 < c < p$
A	Ordering cost per order ($)

Variables and functions

ξ	Preservation technology cost ($) per unit (*decision variable*)
p	Selling price ($) per unit (*decision variable*)
t_1	Duration of on-hand inventory without shortages, after which inventory reaches zero (*decision variable*)
t_2	Length of the shortage period (*decision variable*)
$D(p,w)$	Thedemand function dependent on price and greenness level is expressed as, $D(p,w)=(a_0-a_1p-a_2p^2)+\mu w$ where a_0,a_1,a_2 are the parameters so chosen to best fit the demand function
$\theta(t)$	Deterioration rate proportional to time, $0<\theta(t)<1$.
$\beta(t)$	Backlogging rate that decreases with waiting time t, assuming that $\beta(t)=\frac{1}{1+\sigma t},\sigma>0$
I_0	Peak inventory level per cycle
S	Per cycle shortage quantity
Q	Per cycle ordering quantity, where $Q=S+I_0$
T	Replenishment cycle length, $T=t_1+t_2$
$I_1(t)$	Inventory level at time t, $t\in[0,t_d]$ during which the product does not deteriorate
$I_2(t)$	Inventory level at time t, $t\in[t_d,t_1]$ during which the product undergoes deterioration
$I_3(t)$	Negative inventory level at time t, $t\in[t_1,t_1+t_2]$
$P(\xi)$	Proportion of reduced deterioration rate, $0\le P(\xi)\le 1$. $P(\xi)$ is a function that is continuous, concave, and twice differentiable with respect to greenhouse retailer's investment ξ; $P(0)=0$ and $\lim_{\xi\to\infty}P(\xi)=1$.
$\Pi(t_1,t_2,p,\xi)$	For the inventory system expected total profit ($) per unit time

4. Development of the inventory model

This study examines an inventory control system for a single non-instantaneously deteriorating item, featuring partial backlogging and negligible lead time. While the replenishment rate is considered infinite, the order quantity remains finite. The item does not deteriorate for an initial fixed period t_d, after which it begins to deteriorate at a variable rate. Preservation technology is applied to reduce this deterioration. The demand for the product depends on both its selling price and greenness level. The objective is to identify the optimal positive inventory period, shortage period, and pricing to maximize expected total profit across different greenness levels.

The progression of the inventory system unfolds in the following manner: With an order arriving at time $t=0$, the product inventory level rises to I_0 and diminishes to zero by the time $t=t_1$, where $t_1\ge t_d$. The depletion of inventory during the interval $[0,t_d]$ is a consequence of demand, while over the interval $[t_d,t_1]$, it results from the combined impact of both demand and deterioration. Subsequent to time t_1, shortages start accumulating and persist until $t = t_1 + t_2$. Figure 1 illustrates the inventory level at different time points through a graphical representation.

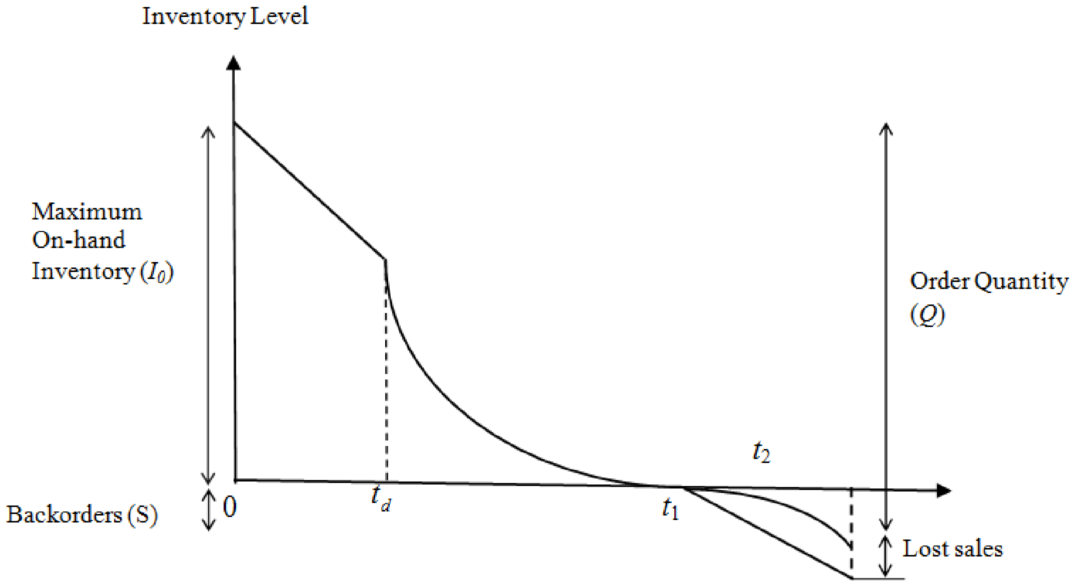


Figure 1. Visual depiction of an inventory system

Derived from the preceding description, the differential equations that illustrate the instantaneous states of the inventory level, denoted as $I(t)$, during the time intervals $t \in (0, t_1 + t_2)$, are as follows:

$$\frac{dI(t)}{dt} = \begin{cases} -D(p,w), & 0 \le t \le t_d \\ -\theta(t)(1 - P(\xi))I(t) - D(p,w), & t_d \le t \le t_1 \\ -D(p,w)\beta(t_1 + t_2 - t), & t_1 \le t \le t_1 + t_2 \end{cases} \quad (1)$$

with boundary conditions, $I(0) = I_0$ and $I(t_1) = 0$.

By solving the provided differential equations with the given boundary conditions, the inventory level is obtained as follows:

$$I(t) = \begin{cases} I_1(t), & 0 \le t \le t_d \\ I_2(t), & t_d \le t \le t_1 \\ I_3(t), & t_1 \le t \le t_1 + t_2 \end{cases} \quad (2)$$

where

$$I_1(t) = I_0 - D(p,w)t, \quad 0 \le t \le t_d \quad (3)$$

$$I_2(t) = D(p,w)e^{-(1-P(\xi))g(t)} \int_t^{t_1} e^{(1-P(\xi))g(u)} du, \quad t_d \le t \le t_1 \quad (4)$$

$$I_3(t) = -\frac{D(p,w)\left(\ln(1+\sigma t_2) - \ln\left(1+\sigma(t_1+t_2-t)\right)\right)}{\sigma} \quad (5)$$

where $\sigma > 0$ and $g(y) = \int_{t_d}^{y} \theta(x)dx$.

Utilizing the continuity of $I(t)$ at $t=t_d$, it is established that $I_1(t_d)=I_2(t_d)$. By deducing from equations (4) and (5), the maximum inventory level I_0 per cycle, is determined to be

$$I_0 = D(p,w)\left[t_d + \int_{t_d}^{t_1} e^{(1-P(\xi))g(u)}du\right] \tag{6}$$

Equation (6) substituted into equation (3) yields the inventory level

$$I_1(t) = D(p,w)\left[(t_d - t) + \int_{t_d}^{t_1} e^{(1-P(\xi))g(u)}du\right], \quad 0 \le t \le t_d \tag{7}$$

The maximum amount of demand backlogging (S) is given as

$$S = -I_3(t_1+t_2) = \frac{D(p,w)\ln(1+\sigma t_2)}{\sigma} \tag{8}$$

Thus, order quantity (Q) per cycle is

$$Q = S + I_0$$

$$= \frac{D(p,w)\ln(1+\sigma t_2)}{\sigma} + D(p,w)\left[t_d + \int_{t_d}^{t_1} e^{(1-P(\xi))g(u)}du\right] \tag{9}$$

The components contributing to the expected total profit per cycle include:

A is the ordering cost (OC).

Then the inventory holding cost (HC) per cycle is given by

$$HC = h\left(\int_0^{t_d} I_1(t)dt + \int_{t_d}^{t_1} I_2(t)dt\right)$$

$$= D(p,w)\left[\frac{ht_d^{\,2}}{2} + ht_d\int_{t_d}^{t_1} e^{(1-P(\xi))g(u)}du + h\int_{t_d}^{t_1} e^{-(1-P(\xi))g(t)}\int_{t}^{t_1} e^{(1-P(\xi))g(u)}dudt\right] \tag{10}$$

The cost of shortages (SC) per cycle resulting from backlogs is calculated as

$$SC = s\int_{t_1}^{t_1+t_2} -I_3(t)dt = \frac{sD(p,w)\left(\sigma t_2 - \ln(1+\sigma t_2)\right)}{\sigma^2} \tag{11}$$

The cost of purchases (PC) is computed as

$$PC = cQ = cD(p,w)\left[(t_1+t_2) + \left\{(t_d - t_1) + \int_{t_d}^{t_1} e^{(1-P(\xi))g(u)}du\right\} - \frac{\sigma t_2 - \ln(1+\sigma t_2)}{\sigma}\right] \tag{12}$$

The cost of lost sales (LC) opportunities per cycle is calculated as

$$LC = l\int_{t_1}^{t_1+t_2} D(p,w)\left(\frac{1}{1+\sigma(t_1+t_1-t)}\right)dt = \frac{lD(p,w)\left(\sigma t_2 - \ln(1+\sigma t_2)\right)}{\sigma} \tag{13}$$

The revenue from sales (SR) is expressed as

$$SR = p\left(\int_0^{t_1} D(p,w)dt + S\right) = pD(p,w)\left[(t_1+t_2) - \frac{\sigma t_2 - \ln(1+\sigma t_2)}{\sigma}\right] \tag{14}$$

Cost for preservation technology (PR) is given by

$$PR = (t_1+t_2)\xi \tag{15}$$

Cost for greenness investment is given by

$$GC = K_1 w^2 \tag{16}$$

Integrating the aforementioned cost and revenue elements, the anticipated total profit can be expressed as

$$P_T(t_1,t_2,p,\xi) = SR - A - HC - SC - LC - PC - PR - GC$$

Therefore, the expected total profit per unit time is

$$\Pi(t_1,t_2,p,\xi) = \frac{P_T(t_1,t_2,p,\xi)}{(t_1+t_2)}$$

Upon rearranging the terms, the expression for the expected total profit per unit time becomes:

$$\Pi(t_1,t_2,p,\xi) = (p-c)D(p,w) - \frac{D(p,w)}{(t_1+t_2)} \times \left[\frac{ht_d{}^2}{2} + c(t_d - t_1) + (ht_d + c)\int_{t_d}^{t_1} e^{(1-P(\xi))g(u)}du \right.$$
$$+ h\int_{t_d}^{t_1} e^{-(1-P(\xi))g(t)} \int_t^{t_1} e^{(1-P(\xi))g(u)}dudt$$
$$\left. + \frac{(s+\sigma(p-c+l))}{\sigma^2}\{\sigma t_2 - \ln(1+\sigma t_2)\} + \frac{A + (t_1+t_2)\xi + K_1 w^2}{D(p,w)}\right] \tag{17}$$

5. Numerical example

To showcase the practical relevance of the model proposed and the solution procedure outlined in this article, a numerical example is presented below.

Example 1: Illustrating the solution procedure involves considering an inventory situation described in Soni and Suthar (2019).

It is assumed that the time-varying deterioration rate follows a three-parameter Weibull distribution given by $\alpha\beta(t-t_d)^{\beta-1}$ with $\alpha = 0.3$ and $\beta = 2$, $t_d = 15/365$.

In previous articles on preservation technology, researchers considered an exponential form for the preservation technology investment function. In this study, the power function is chosen for the preservation technology investment $P(\xi) = 1 - \frac{1}{\xi^{\kappa}}$ is preservation investment cost and κ is the preservation technology investment efficiency parameter.

The remaining parametric values for the model are as follows. $A = 250, c = 40, h = 6, l = 15, s = 10, \sigma = 3, \eta = 400, \mu = 4, \kappa = 0.4, K_1 = 200, a_0 = 100, a_1 = 0.1, a_2 = 0.001$

Utilizing the provided data, the optimal solution is determined through the utilization of MAPLE 18 software.

The optimum results are:

$t_1^* = 1.6934,\ t_2^* = 0.0431,\ p^* = 177.09,\ \xi^* = 55.03$.

Hence, the corresponding total profit per unit time for greenness level $w = 2$ is

$\Pi\left(t_1^*, t_2^*, p^*, \xi^*\right) = 6975.12$ and $Q^* = 128.76$ units per cycle.

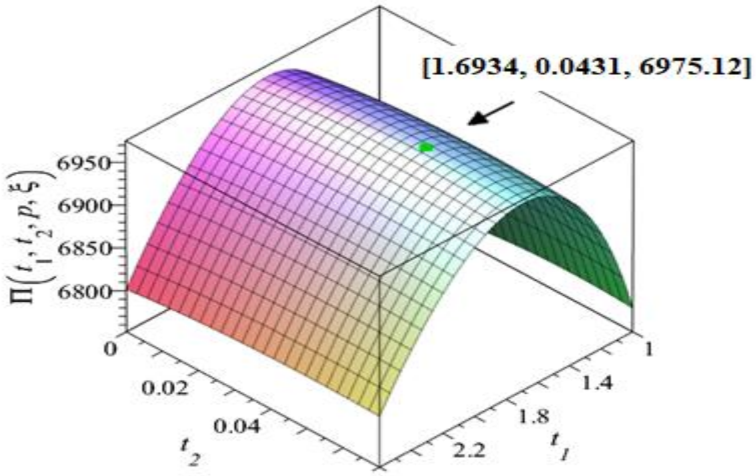


Figure 2. Profit function with respect to t_1 and t_2 for fixed p

The graph in Figure 2 depicts concave property of expected profit functionwith respect to t_1 and t_2 for a fixed $p^* = 177.09$.

Utilizing the data from the above example, the sensitivity of optimal decision variables and the corresponding total expected profit is examined across various greenness levels.

Table 1 shows the optimal results for different values of w

Table 1Computational findings for various values of w

w	p^*	t_1^*	t_2^*	Π^*	ξ^*	Q^*
1	172.66	1.1806	0.0275	6866.31	30.77	77.13
2	177.09	1.6934	0.0431	6975.12	55.03	128.76
3	181.59	2.2394	0.0621	7002.63	86.42	204.42
4	186.08	2.7724	0.0831	6987.98	122.24	306.56
5	190.51	3.2849	0.1055	6945.12	161.23	439.00
6	194.89	3.7778	0.1293	6880.72	202.61	605.71
7	199.22	4.2534	0.1544	6798.71	245.85	810.72
8	203.50	4.7147	0.1808	6701.80	290.49	1058.28
9	207.73	5.1642	0.2085	6592.04	336.15	1353.00
10	211.93	5.6049	0.2376	6471.10	382.45	1700.09

Table 1 shows that as the greenness level w increases, there is a corresponding rise in the optimal replenishment cycle time, optimal price, and optimal preservation cost. Interestingly, up to a certain point $(w = 4)$, an increase in the greenness level leads to a decrease in total profit. Additionally, it is noteworthy that the duration of on-hand inventory (t_1) significantly extends with higher greenness levels, which in turn, evidently elevates the preservation cost. These findings imply that decision-makers must carefully determine the appropriate greening level to achieve the best possible outcome.

---

6. Sensitivity analysis

6.1 Impact of some parameters

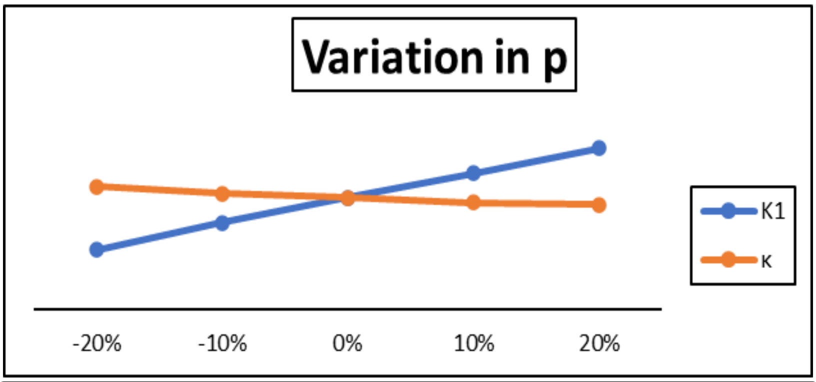
The sensitivity of the optimal decision variables and the resulting total profit was analysed by altering the values of various model parameters, using the same data as in Example 1. This sensitivity analysis involved adjusting each parameter individually within a range of -20% to +20%, while keeping all other parameters constant.

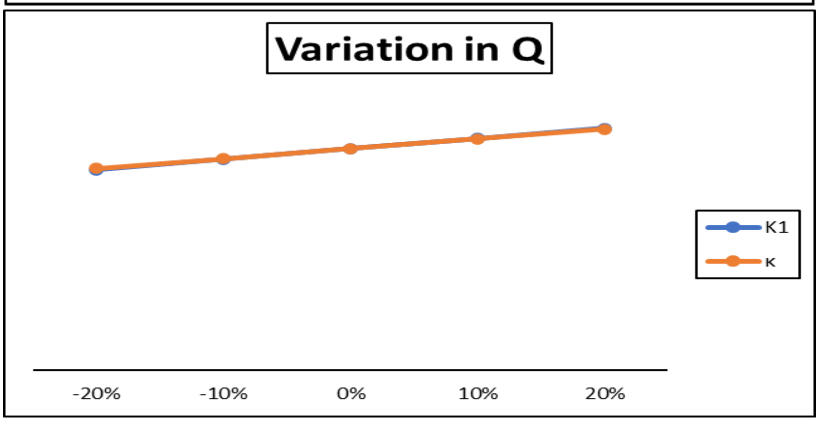
6.1.1 Analysis of greenness and preservation investment

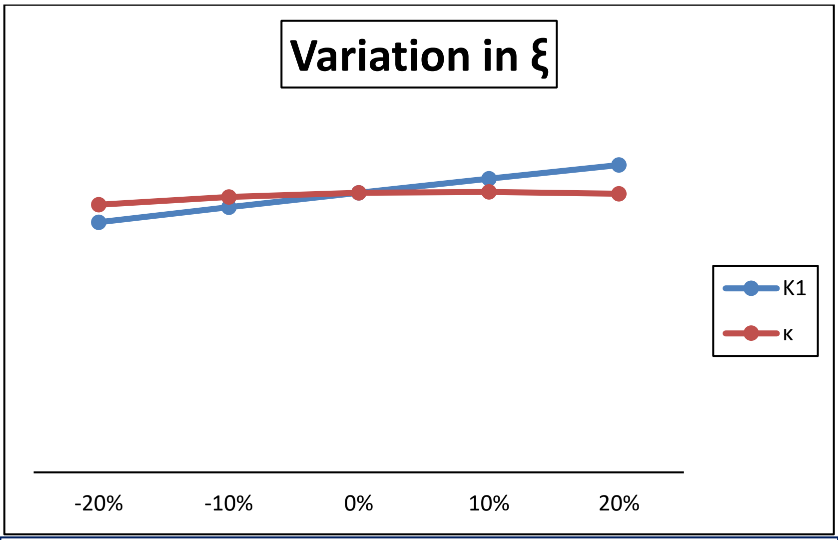
This section evaluates the impact of greenness (K_1) and preservation investments (κ) on decision variables and the profit function. The findings are summarized in Table 2, while Figure 3 illustrates the trends in selling price, order quantity, preservation costs, and their relationship to the changing parameters.

Table 2 Computational findings for various values of K_1 and κ

Parameters	Values	$p(\text{in}\,\$)$	$t_1(\text{year})$	$t_2(\text{year})$	$\Pi(\text{in}\,\$)$	$\xi(\text{in}\,\$)$	Q
K_1	160	176.86	1.5739	0.3900	7070.64	49.25	116.36
	180	176.98	1.6351	0.4107	7022.00	52.18	122.61
	200	177.09	1.6934	0.4310	6975.12	55.03	128.76
	220	177.20	1.7492	0.4507	6929.80	57.81	134.82
	240	177.31	1.8028	0.4700	6885.90	60.51	140.81
κ	0.32	177.14	1.5815	0.4545	6926.97	52.70	117.20
	0.36	177.11	1.6383	0.4420	6952.06	54.23	122.98
	0.40	177.09	1.6934	0.4310	6975.12	55.03	128.76
	0.44	177.07	1.7463	0.4211	6996.20	55.21	134.49
	0.48	177.06	1.7968	0.4124	7015.42	54.88	140.11







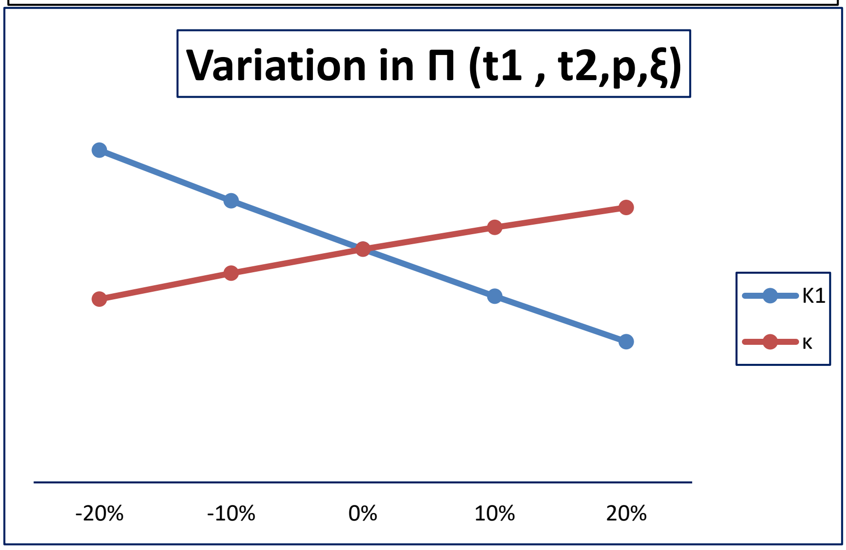


Figure 3. Variation in p, Q , ξ and Π (·) with respect to K_1 and κ.

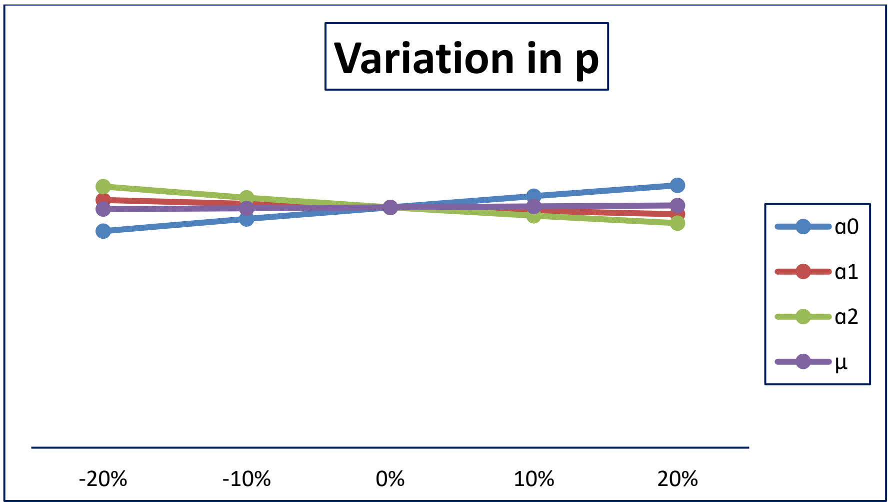
6.1.2 Analysis of demand parameters

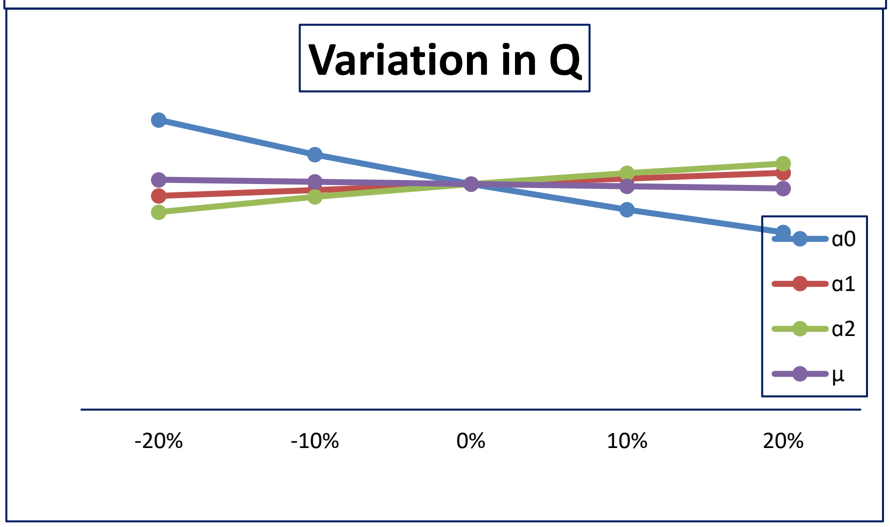
This section analyses the impact of demand parameters on the optimal solutions. The computational results are displayed in Table 3. Additionally, to illustrate the trends of the optimal selling price, order quantity, and profit concerning these parameters, the results are graphically presented in Figure 4.

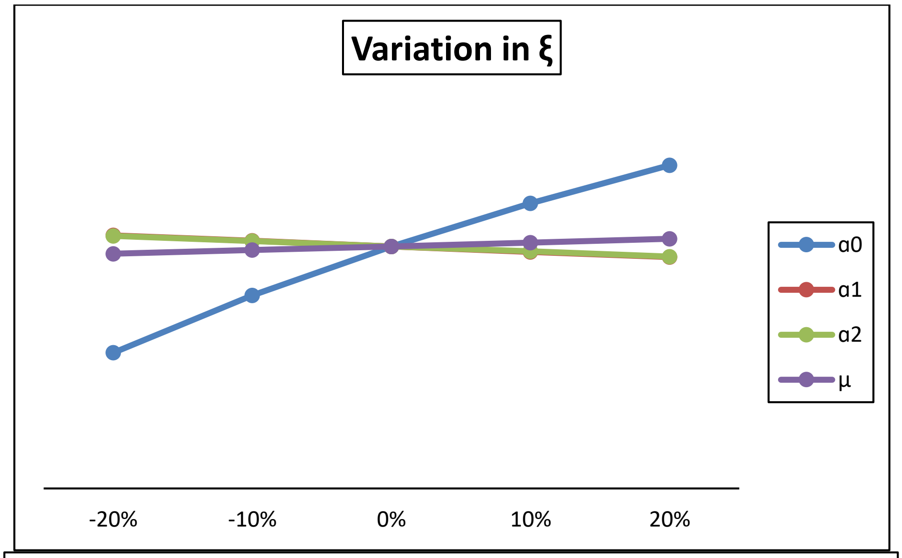
Table 3Computational findings for various values of demand parameters

Parameters	**Values**	$p(\text{in}\,\$)$	$t_1(\text{year})$	$t_2(\text{year})$	$\Pi(\text{in}\,\$)$	$\xi(\text{in}\,\$)$	Q
a_0	80	159.51	1.8442	0.5787	4566.64	52.82	165.43
	90	168.50	1.7633	0.4946	5724.42	54.01	145.61
	100	177.09	1.6934	0.4310	6975.12	55.03	128.76
	110	185.33	1.6320	0.3812	8314.18	55.93	114.14
	120	193.26	1.5776	0.3413	9737.68	56.72	101.26
a_1	0.08	182.43	1.6813	0.4096	7450.57	55.26	122.06
	0.09	179.73	1.6874	0.4202	7208.65	55.15	125.44
	0.10	177.09	1.6934	0.4310	6975.12	55.03	128.76

	0.11	174.50	1.6993	0.4420	6749.67	54.92	132.02
	0.12	171.97	1.7051	0.4533	6532.02	54.81	135.21
a_2	0.0008	192.39	1.6927	0.3873	7907.37	55.25	112.81
	0.0009	184.18	1.6929	0.4095	7408.99	55.14	121.46
	0.0010	177.09	1.6934	0.4310	6975.12	55.03	128.76
	0.0011	170.88	1.6940	0.4518	6592.69	54.93	135.03
	0.0012	165.37	1.6948	0.4721	6252.12	54.82	140.49
μ	3.2	175.74	1.7039	0.4401	6768.96	54.88	131.29
	3.6	176.42	1.6987	0.4355	6871.75	54.96	130.02
	4	177.09	1.6934	0.4310	6975.12	55.03	128.76
	4.4	177.76	1.6882	0.4265	7079.04	55.11	127.52
	4.8	178.43	1.6830	0.4221	7183.54	55.19	126.29







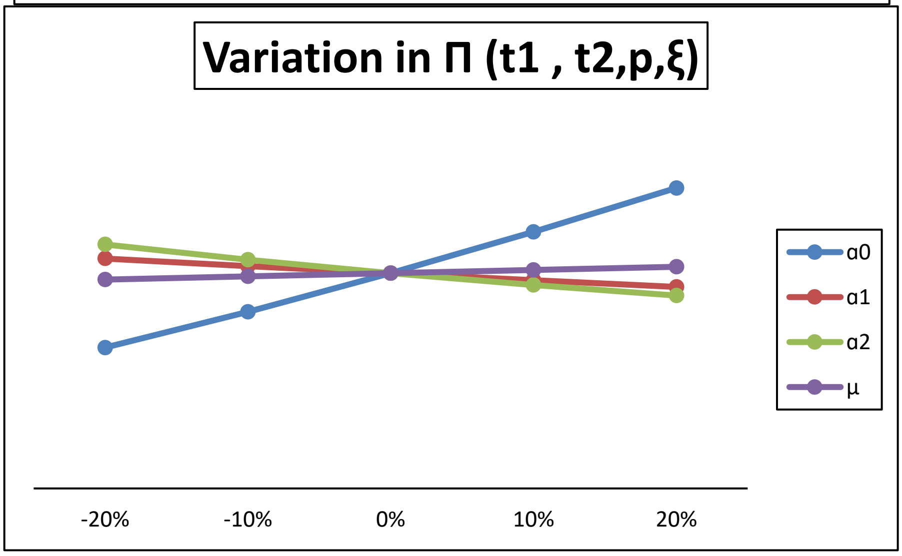


Figure 4. Variation in p, Q , ξ and Π (·) with respect to a_0, a_1, a_2 and μ .

6.2 Discussion

1. As shown in Table 2, increasing the value of parameter (K_1), results in higher selling prices (p), order quantities (Q), and preservation costs (ξ). However, these increases collectively lead to a decrease in total profit. Additionally, the decision variables and corresponding total profit exhibit moderate sensitivity to changes in both greenness and preservation investments.
2. Also, increasing the value of parameter (κ), results in lower selling prices (p), higher order quantities (Q), and changes in preservation costs (ξ). However, these increases collectively lead to an increase in total profit. The decision variables and corresponding total profit still show moderate sensitivity to changes in both greenness and preservation investments.
3. Higher values of a_0 lead to higher prices, shorter replenishment times, increased preservation costs, and higher profits. This indicates that the retailer benefits from increased market demand by setting higher prices and reducing order quantities, which helps reduce holding and deterioration costs (see Table 3).
4. Conversely, higher values of a_1, a_2 result in lower prices, longer replenishment times, larger order quantities, reduced preservation costs, and lower profits.

---

5. Similarly, higher values of (μ) push prices higher, shorten replenishment times, increase preservation costs, and boost profits. This means that the retailer benefits from the increase in market demand by setting higher prices and lowering order quantities, which helps reduce holding and deterioration costs (see Table 3).

7. Conclusions

This research advances the conventional non-instantaneous deteriorating inventory model by integrating elements such as price, demand influenced by greenness levels, preservation strategies, and investments in green technology. The primary goal is to maximize retailer profits through optimal decisions regarding pricing, replenishment cycles, and preservation technologies. The results indicate that profitability and market competitiveness can be significantly improved through a combination of preservation strategies and green technology investments. The study underscores the long-term advantages of increased investment in green technologies for both financial gains and environmental benefits. The effectiveness of preservation methods varies based on factors like product type, cost, and deterioration rate, with preservation becoming more crucial at higher levels of greenness. These insights offer a valuable and practical framework for businesses aiming to optimize their marketing and inventory strategies.

Future research directions could include examining quality degradation, relaxing assumptions about fixed greenness levels, and exploring integrated cooperative approaches involving multiple stakeholders.

References

[1] Bardhan S, Pal H, Giri B (2019) Optimal replenishment policy and preservation technology investment for a non-instantaneous deteriorating item with stock-dependent demand. Oper. Res. Int. J. 19(2), 347–368, https://doi.org/10.1007/s12351-017-0302-0

[2] Barron L.E, Sana S.S. (2015) Coordinated lot-sizing and dynamic pricing under a supplier all-units quantity discounts. Appl. Math. Model. 39(21), 6725–6737,http://dx.doi.org/10.1016/j.apm.2015.02.004

[3] De S. K, Sana S. S. (2015) An alternative fuzzy EOQ model with backlogging for selling price and promotional effort sensitive demand. International Journal of Applied and Computational Mathematics, 1(1), 69–86, https://doi.org/10.1007/s40819-014-0010-x

[4] Dye C. Y (2007) Joint pricing and ordering policy for a deteriorating inventory with partial backlogging. Omega, 35(2), 184–189, https://doi.org/10.1016/j.omega.2005.05.002

[5] Dye C.Y (2013) The effect of preservation technology investment on a non-instantaneous deteriorating inventory model. Omega (UK) 41(5):872–880,https://doi.org/10.1016/j.omega.2012.11.002

[6] Dye C.Y, Hsieh T.P (2012) An optimal replenishment policy for deteriorating items with effective investment in preservation technology.Eur J Oper Res 218(1):106–112,https://doi.org/10.1016/j.ejor.2011.10.016

[7] Dye C. Y, Yang C.T (2016) Optimal dynamic pricing and preservation technology investment for deteriorating products with reference price effects. Omega, Volume 62, 2016, 52-67, https://doi.org/10.1016/j.omega.2015.08.009

[8] Farshbaf-Geranmayeh A, Rabbani M,Taleizadeh A. A (2018) Channel coordination with cooperative advertising considering effect of advertising on willingness to pay. Journal of Optimization Theory and Applications, 176(2), 509–525, https://doi.org/10.1007/s10957-018-1217-5

[9] Ferguson M.E, Lystad E.D, Alexopoulos C (2006) Single stage heuristic for perishable inventory control in two-echelon supply chains. Working paper. Georgia Institute of Technology, http://hdl.handle.net/1853/11677

[10] Gangadhar B, Ramakrishna Naidu G (2017) Green technology vs environmental sustainability in india– an overview. International Journal of Current Advanced Research 6(3),2465-2468, https://doi.org/10.24327/ijcar.2017.2468.0029

[11] GeethaK. V, Uthayakumar R (2010) Economic design of an inventory policy for non instantaneous deteriorating items under permissible delay in payments. Journal of Computational and Applied Mathematics, 233(10), 2492–2505, https://doi.org/10.1016/j.cam.2009.10.031

[12] Geetha K.V, Udayakumar R (2016) Optimal lot sizing policy for non-instantaneous deteriorating items with price and advertisement dependent demand under partial backlogging. Int. J. Appl. Comput. Math. 2 (2), 171–193, https://doi.org/10.1007/s40819-015-0053-7
[13] Handa N, Singh S.R, Punetha N. (2022) Impact of carbon emission in two-echelon supply chain inventory decision with controllable deterioration and two-level trade-credit period, OPSEARCH 59, 1555–1586,https://doi.org/10.1007/s12597-022-00599-9
[14] Hasan M. R, Roy T. C, Daryanto Y, Wee H.M (2021) Optimizing inventory level and technology investment under a carbon tax, cap-and-trade and strict carbon limit regulations. Sustainable Production and Consumption, 25, 604–621, https://doi.org/10.1016/j.spc.2020.12.005
[15] Hatibaruah A, Saha S (2023) An inventory model for two-parameter Weibull distributed ameliorating and deteriorating items with stock and advertisement frequency dependent demand under trade credit and preservation technology, OPSEARCH 60, 951–1002, https://doi.org/10.1007/s12597-023-00629-0
[16] Hong Z, Guo X, (2019) Green product supply chain contracts considering environmental responsibilities. Omega 83, 155 to 166, https://doi.org/10.1016/j.omega.2018.02.010
[17] Hsu P.H, Wee H.M, Teng H.M (2010) Preservation technology investment for deteriorating inventory. Int J Prod Econ 124(2):388–394, https://doi.org/10.1016/j.ijpe.2009.11.034
[18] Hsu C.C, Tan K.C, Zailani S.H.M, Jayaraman V(2013) Supply chain drivers that foster the development of green initiatives in an emerging economy. Int. J. Oper. Prod. Manag. 33 (6), 656e688, https://doi.org/10.1108/IJOPM-10-2011-0401
[19] Huang Y.H, Wang C.C, Huang C.J, Dye C.Y (2011) Comments on preservation technology investment for deteriorating inventory. African Journal of Business Management, 5(11), 4636-4643 https://academicjournals.org/journal/AJBM/article-full-text-pdf/2B6E14417783.pdf
[20] IhsanHishamuddin, SitiSuzlinSupadi, Mohd Omar (2020) Improved Preservation Technology for Non-Instantaneous Deteriorating Inventory Using Boundary Condition Estimation. Applied and Computational Mathematics. Vol. 9, No. 4, pp. 118-129,https://doi.org/10.11648/j.acm.20200904.12
[21] Khan M, Shaikh A. A, Panda G. C, Bhunia A. K, Konstantaras I (2020) Non-instantaneous deterioration effect in ordering decisions for a two-warehouse inventory system under advance payment and backlogging. Annals of Operations Research, 289(2), 243-275,
[22] https://doi.org/10.1007/s10479-020-03568-x
[23] Li L, Wang Y, Yan X (2013) Coordinating a supply chain with price and advertisement dependent stochastic demand. Sci. World J. 3, 1–12, https://doi.org/10.1155/2013/315676
[24] Li, H.J(2018) Investing in transportation emission cost reduction on environmentally sustainable EOQ models with partial backordering. J. Appl. Sci. Eng. 21 (3), 291–303,https://doi.org/10.6180/jase.201809_21(3).0001
[25] Liu G, Zhang J, Tang W(2015) Joint dynamic pricing and investment strategy for perishable foods with price-quality dependent demand. Ann. Oper. Res. 226, 1, 397-416,https://doi.org/10.1007/s10479-014-1671-x
[26] Lou G. X, Xia H.Y. J, Zhang Q, Fan T. J(2015) Investment strategy of emission-reduction technology in a supply chain. Sustainability. 7 (8), 10684-10708,https://doi.org/10.3390/su70810684
[27] Lin H.J(2018) Investing in transportation emission cost reduction on environmentally sustainable EOQ models with partial backordering. J. Appl. Sci. Eng. 21 (3), 291–303.
[28] Mahapatra A. S, Sarkar B. S, Mahapatra M. S, Soni H. N, Mazumder S. K (2019) Development of a fuzzy economic order quantity model of deteriorating items with promotional effort and learning in fuzziness with a finite time horizon. Inventions, 4(3), 36, http://dx.doi.org/10.3390/inventions4030036
[29] Mahapatra A. S, Mahapatra M. S, Sarkar B. S, Mazumder S. K (2022) Benefit of preservation technology with promotion and time-dependent deterioration under fuzzy learning. Expert Systems with Applications volume 201 on page 117169,https://doi.org/10.1016/j.eswa.2022.117169
[30] Mahdavisharif M, Kazemi M, Jahani H, Bagheri F (2022) Pricing and inventory policy for non-instantaneous deteriorating items in vendor-managed inventory systems: a Stackelberg game theory

approach. International Journal of Systems Science: Operations & Logistics, 1-28, https://doi.org/10.1080/23302674.2022.2038715

[31] Maihami R, Abadi I. N. K (2012) Joint control of inventory and its pricing for non-instantaneously deteriorating items under permissible delay in payments and partial backlogging. Mathematical and Computer Modelling, 55(5–6), 1722–1733,https://doi.org/10.1016/j.mcm.2011.11.017

[32] Maihami R, Karimi B (2014) Optimizing the pricing and replenishment policy for non- instantaneous deteriorating items with stochastic demand and promotional efforts. Comput. 532Oper. Res. 51, 302-312, https://doi.org/10.1016/j.cor.2014.05.022

[33] Maihami R, Ghalehkhondabi I, Ahmadi E (2021) Pricing and inventory planning for non-instantaneous deteriorating products with greening investment: A case study in beef industry. Journal of Cleaner Production, 295, 126368, https://doi.org/10.1016/j.jclepro.2021.126368

[34] Marchi B, Zanoni S (2023) Technical note on “Inventory management in supply chains with consideration of Logistics, green investment and different carbon emissions policies”. Computers & Industrial Engineering, Volume 175, 108870, ISSN 0360-8352, https://doi.org/10.1016/j.cie.2022.108870

[35] Mashud A.H.M, Wee H.M, Huang C.V (2019) Preservation technology investment, trade credit and partial backordering model for a non-instantaneous deteriorating inventory. RAIRO-Oper. Res., 55 (2021) S51-S77, https://doi.org/10.1051/ro/2019095

[36] Mashud A. H. M, Roy D, Daryanto Y, Ali M. H (2020) A sustainable inventory model with imperfect products, deterioration, and controllable emissions. Mathematics, 8(11), 2049, https://doi.org/10.3390/math8112049

[37] Mashud A. H. M, Pervin M, Mishra U, Daryanto Y, Tseng M. L, Lim M. K (2021) A sustainable inventory model with controllable carbon emissions in green-warehouse farms. Journal of Cleaner Production, 298, 126777, https://doi.org/10.1016/j.jclepro.2021.126777

[38] Mishra U, Cárdenas-Barrón L.E, Tiwari S, Shaikh A.A, Treviño-Garza G(2017) An inventory model under-price and stock dependent demand for controllable deterioration rate with shortages and preservation technology investment. Ann. Oper. Res. 254, 1-2, 165- 190, https://doi.org/10.1007/s10479-017-2419-1

[39] Mishra U, Tijerina-Aguilera J, Tiwari S, Cardenas-Barron L.E, (2018) Retailer 's joint ordering, pricing, and preservation technology investment policies for a deteriorating item under permissible delay in payments. Math. Probl Eng. 6962417,https://doi.org/10.1155/2018/6962417

[40] Mishra U, Wu J-Z, Tsao Y-C, Tseng M-L(2019) Sustainable inventory system with controllable non-instantaneous deterioration and environmental emission rates. J. Clean. Prod, https://doi.org/10.1016/j.jclepro.2019.118807

[41] Mishra U, Wu J-Z, Sarkar B (2020) A sustainable production-inventory model for a controllable carbon emissions rate under shortages. J. Clean. Prod. 25620, 120268, https://doi.org/10.1016/j.jclepro.2020.120268

[42] Mishra U, Wu J-Z, Sarkar B (2021) Optimum sustainable inventory management with backorder and deterioration under controllable carbon emissions. Journal of Cleaner Production, 279, 123699, https://doi.org/10.1016/j.jclepro.2020.123699

[43] Palanivel M, Priyan S, Mala P (2018) Two-warehouse system for non-instantaneous deterioration products with promotional effort and inflation over a finite time horizon. J Ind Eng Int 14, 603–612, https://doi.org/10.1007/s40092-017-0250-6

[44] Paul A, Pervin M, Roy S.K, Maculan N, Weber G-H (2022) A green inventory model with the effect of carbon taxation. Ann Oper Res 309, 233–248 , https://doi.org/10.1007/s10479-021-04143-8

[45] Rapolu C.N, Kandpal, D.H (2020) Joint pricing, advertisement, preservation technology investment and inventory policies for non-instantaneous deteriorating items under trade credit. Opsearch 57, 274– 300, https://doi.org/10.1007/s12597-019-00427-7

[46] Saha S, Nielsen I, Moon I (2017) Optimal retailer investments in green operations and preservation technology for deteriorating items. Journal of Cleaner Production, 140, 1514–1527, https://doi.org/10.1016/j.jclepro.2016.09.229

[47] San-José L. A, Sicilia J, Barron L.E., Rosa M. G. (2024)A sustainable inventory model for deteriorating items with power demandand full backlogging under a carbon emission tax. Int. J. Production Economics 268 109098, https://doi.org/10.1016/j.ijpe.2023.109098

[48] Shah N.H, Shah P.H, Patel M.B (2021) Retailer's inventory decisions with promotional efforts and preservation technology investments when supplier offers quantity discounts.OPSEARCH 58, 1116–1132, https://doi.org/10.1007/s12597-021-00516-6

[49] Shah N.H, Rabari K, Patel E (2023) Greening efforts and deteriorating inventory policies for price-sensitive stock-dependent demand, International Journal of Systems Science: Operations & Logistics, Volume 10, Issue 1, https://doi.org/10.1080/23302674.2021.2022808

[50] Shaikh A.A, Mashud A.H.M, Uddin M.S, Khan M.A (2017) Non-instantaneous deterioration inventory model with price and stock dependent demand for fully backlogged shortages under inflation. Int. J. Bus. Forecast. Market. Intell. 3 (2), 152-164, https://doi.org/10.1504/IJBFMI.2017.084055

[51] Singh S. R, Khurana D, Tayal S (2016) An economic order quantity model for deteriorating products having stock dependent demand with trade credit period and preservation technology. Uncertain Supply Chain Manag 4, 29–42, https://doi.org/10.5267/j.uscm.2015.8.001

[52] Singh S.R, Rathore H (2015) Optimal payment policy with preservation technology investment and shortages under trade credit. Indian Journal of Science and Technology 8(S7), 203–212, https://doi.org/10.17485/ijst/2015/v8is7/64489

[53] Soni H.N, Chauhan A.D (2018) Joint pricing, inventory, and preservation decisions for deteriorating items with stochastic demand and promotional efforts. J. Ind. Eng. Int. 14(4), 831–843, https://doi.org/10.1007/s40092-018-0265-7

[54] Soni H. N, Suthar D. N (2019) Pricing and inventory decisions for non-instantaneous deteriorating items with price and promotional effort stochastic demand. Journal of Control and Decision, 1–25,https://doi.org/10.1080/23307706.2018.1478327

[55] Sundara Rajan R, Uthayakumar R (2017) Analysis and optimization of an EOQ inventory model with promotional efforts and back ordering under delay in payments. J. Manag Anal. 4(2), 159–181, https://doi.org/10.1080/23270012.2017.1280423

[56] Szmerekovsky J.G, Zhang J (2009)Pricing and two-tier advertising with one manufacturer and one retailer. Eur. J. Oper. Res. 192(3), 904–917,https://doi.org/10.1016/j.ejor.2007.10.005

[57] Tat R, Taleizadeh A. A, Esmaeili M (2015) Developing economic order quantity model for non-instantaneous deteriorating items in vendor-managed inventory (VMI) system.International Journal of Systems Science, 46(7), 1257–1268,https://doi.org/10.1080/00207721.2013.815827

[58] Thomas A, Mishra U (2022) A sustainable circular economic supply chain system with waste minimization using 3D printing and emissions reduction in plastic reforming industry. Journal of Cleaner Production, 345, 131128, https://doi.org/10.1016/j.jclepro.2022.131128

[59] Toptal A, Özlü H, Konur D (2014) Joint decisions on inventory replenishment and emission reduction investment under different emission regulations. International Journal of Production Research, 52, 1, 243-269, https://doi.org/10.1080/00207543.2013.836615

[60] Tsao Y.C (2014) Joint location, inventory, and preservation decisions for non-instantaneous deterioration items under delay in payments. Int J SystSci 1(1):1–14, https://doi.org/10.1080/00207721.2014.891672

[61] Tsao Y. C, Sheen G. J (2008) Dynamic pricing, promotion and replenishment policies for a deteriorating item under permissible delay in payments. Computers & Operations Research, 35(11), 3562–3580, https://doi.org/10.1016/j.cor.2007.01.024

[62] Vandana, Das A.K (2022) Two-warehouse supply chain model under preservation technology and stochastic demand with shortages. *OPSEARCH* 59, 1587–1612, https://doi.org/10.1007/s12597-022-00600-5

[63] Wu K-S, Ouyang, L-Y, Yang, C-T (2006) An optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand and partial backlogging. International Journal of Production Economics, 101(2), 369–384, https://doi.org/10.1016/j.ijpe.2005.01.010

[64] Wu K-S, Ouyang L-Y, Yang C-T (2009) Coordinating replenishment and pricing policies for non-instantaneous deteriorating items with price-sensitive demand. International Journal of Systems Science, 40(12), 1273–1281, https://doi.org/10.1080/00207720903038093

[65] Yadav S, Khanna A (2021) Sustainable Inventory Model for Perishable Products with Expiration Date and Price Reliant Demand Under Carbon Tax Policy. Process Integration and Optimization for Sustainability, 5(3), 475–486, https://doi.org/10.1007/s41660-021-00157-8

[66] Yang C.T, Dye C.Y, Ding J.F (2015) Optimal dynamic trade credit and preservation technology allocation for a deteriorating inventory model. ComputInd Eng 87, 356–369, https://doi.org/10.1016/j.cie.2015.05.027

[67] Zhang J-L, Chen J, Lee C-Y (2008) Joint optimization on pricing, promotion and inventory control with stochastic demand. International Journal of Production Economics, 116(2), 190–198, https://doi.org/10.1016/j.ijpe.2008.09.008

[68]