Mixed Convection Nanofluid Flow Past An Exponential Permeability Plate With Radiation Effect

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Abstract:

This study examines the characteristics of the boundary layer and mixed convection phenomena in the flow of nanofluids across a vertical flat plate. Specifically, it focuses on nanofluids consisting of copper (Cu) particles dispersed in water and investigates how different fluid variables, such as porosity and permeability, affect the flow's behaviour. This work utilizes a shooting method to solve the governing nonlinear differential equations. Similarity transformations convert the partial differential equations into higher-order ordinary differential equations. The fluctuations in fluid properties have a notable impact on heat transmission and flow characteristics, making this research relevant for improving the efficiency of thermal systems. The study gives useful insights for combining mixed convection and nanofluids to improve heating and cooling operations in various technical fields. The numerical findings demonstrate how changing porosity, permeability, and nanoparticle concentration affect the velocity and temperature profiles inside the boundary layer. The results highlight the need to consider the varying properties of fluids when building and analyzing thermal management systems based on nanofluids.

***Key Words*:** Boundary layer, mixed convection, nanofluids, variable fluid properties, Radiation, vertical flat plate.

1. Introduction.

In many engineering contexts, convective heat transfer in a fluid-saturated porous medium occurs often. The subject of convective flows has drawn more attention recently because they are widespread in porous media over many engineering disciplines such as civil, chemical and mechanical engineering. They include, for example, electrochemical outgassing, subterranean energy storage systems, food processing and storage, insulation of fibrous materials, nuclear reactors that run on solar power, geothermal applications (heating or cooling a space with natural resources), thermal insulation of buildings, design for pebble bed nuclear reactors mounted on enormous natural or man-made porous bodies, heat-treating metal work-pieces mixed below or beside layers of heat transfer media to effect a given temperature gradient across surface, mechanical engineering related to the underground disposal of nuclear waste. The flow of mixed convection over a flat surface within a saturated porous medium or through a channel filled with a porous medium is a fundamental problem in heat transfer. Many works have been studied on practical problems occurring simultaneously in mixed convective heat and mass transfer fluids with different geometry surfaces by numerous authors [1]-[3]. Nield and Bejan [4] have given an excellent summary of free convection flow in porous media. Merkin [5] considered mixed convection boundary layer flow on an impermeable vertical surface embedded in a saturated porous medium. Mohammadein and El-Shaer [6] have studied combined free and forced convective flow past a semi-infinite vertical plate embedded in a porous medium incorporating the variable permeability.

Nanofluids have emerged as a groundbreaking advancement in the realm of heat transfer and thermal regulation within fluid dynamics. These specially formulated colloidal suspensions exhibit remarkable enhancements in thermal properties compared to conventional fluids, consisting of nanoparticles dispersed in a primary fluid medium. The incorporation of nanoparticles into basic fluids like water, ethylene glycol or different oils results in notable modifications in the thermal and physical properties of the base fluids.

The utilization of nanofluids is crucial in the realm of thermal management due to their ability to enhance the efficiency of heat exchange mechanisms. Their improved ability to conduct heat and facilitate heat transfer through movement make them perfect for various uses such as cooling electronic gadgets, exchanging heat in systems and storing thermal energy by effectively dissipating heat, nanofluids possess the capacity to substantially enhance the performance and longevity of electronic components. Choi [7] proposed the notion of nanofluids and showcased how integrating nanoparticles into base fluids can greatly enhance their thermophysical properties, which include viscosity, density, specific heat, and thermal conductivity. Buongiorno [8] delved into the exploration of convective transport within Nanofluids. The study of mixed convection in Cu-H_2O nanofluids on a vertical plate has been extensively conducted due to its great use in improving heat transfer in different engineering systems, including cooling systems, electronic devices, and energy systems. The analysis of boundary layer flow for Natural convection of a nanofluid passing a vertical plate was conducted by Kuznetsov [9].

The incorporation of Cu-H_2O nanofluids in mixed convection scenarios, such as the flow around vertical plates, has exhibited superior thermal efficiency in comparison to conventional fluids. Specifically, the Cu-H_2O nanofluid showcases heightened skin friction coefficient and heat transfer rate when juxtaposed with alternative nanofluids like Al_2O_3-H_2O, rendering it more efficient for thermal purposes. To explore the thermal characteristics and heat exchange qualities of the Cu-H_2O nanofluid in conditions of combined convection, conducted experiments within a vertical passage by Amiri et al. [10]. Sheikholeslami et al. [11] utilize simulations to investigate the fluid dynamics and heat transfer processes involved in mixed convection heat transfer within a vertical tube. Many computational techniques have been used to analyse in great detail the study of mixed convection from a vertically heated plate immersed in a porous material with varying porosity and permeability parameters. The investigation of convection boundary layer flow over an isothermal vertical plate involving an incompressible and viscous fluid with variable thermal conductivity is conducted by Nalinakshi et al. [12]. The intricate scenario in the realm of heat transfer theory, with significant theoretical and practical consequences, is scrutinized by Mohd Hafizi et al. [13], this entails a steady mixed convective boundary layer motion over a vertical interface immersed in a thermally stratified porous medium filled with a nanofluid. Cho et al. [14] conducted a computational investigation on the heat transfer characteristics of nanofluids comprised of water in a lid-driven enclosure under mixed convection conditions for three distinct nanophotonics, their findings revealed the superior performance of Cu–water nanofluid, along with the observation that the Nusselt number escalates with the increase in nanoparticle volume fraction. Subsequently, Cho [15] explored the impact of an inclined magnetic field on heat transfer and entropy generation within a cavity filled with nanofluid, this investigation scrutinized parameters such as nanoparticle volume fraction, amplitude of the wavy surface, ratio of irreversibility distribution, as well as the Richardson, Reynolds and Hartmann numbers. Nishanthan and Soniya [16] investigated temperature-dependent thermophysical features of a two-dimensional flow of nano-liquid Cu-H_2O over a linearly elongated sheet in a magnetic field. El-Dawy et al. [17] examine the convective heat and mass transfer phenomena occurring in a vertical plate embedded in a porous medium saturated with nanofluids, Their study specifically delves into the influence of variations in permeability and thermal diffusivity on these transport processes. Furthermore, [18-23] studied the concept of the variable viscosity property of the fluid. In this direction, to investigate the flow patterns, several works have been carried out based on constant and variable viscosity models.

The contemporary period is defined by a growing curiosity in studying fluid dynamics and thermal conduction in porous materials. This heightened interest can be attributed to the wide-ranging applications of flow through a porous medium, for instance, in the realms of geothermal energy recovery, crude oil extraction, storage of radioactive nuclear waste, and fibre insulation. In their research, Syakila and Pop [24] delved into an examination of the steady mixed convection boundary layer flow around a vertical flat plate that is immersed in a porous medium containing nanofluids. A study of the convection boundary layer flow over an isothermal vertical plate is conducted by Singh [25], focusing on an incompressible and viscous fluid with variable thermal conductivity. Khaled Djeffal's [26] investigation is centred on the presence and attributes of solutions to a third-order non-linear differential equation that pertains to mixed convection boundary layer flow over a permeable vertical surface within a porous medium. This study underscores the significance of factors including the mixed convection parameter and temperature parameter. The existence of a porous substance modifies the flow characteristics and enhances the system's overall thermal conductivity [27-29].

Furthermore, heat transfer through thermal radiation plays a significant role in the field of fluid dynamics for equipment design, among other engineering processes. It is particularly significant in various engineering branches, including mechanical, aerospace, chemical, environmental, solar power, and space technology applications, where higher operating temperatures are required. Nuclear power plants, gas turbines, glass manufacturing, furnace design, propulsion systems for missiles, satellites, aircraft, plasma physics and spacecraft re-entry aero thermodynamics with high temperatures and thermal radiation effects are a few examples of industrial applications. There is very little literature on the influence of radiation in interior flows since it is exceedingly difficult to develop solutions for both convection and radiation. The analysis of thermal radiation transmission across a vertical surface has been extensively documented in multiple research investigations carried out by different researchers as referenced in the literature [30-34]. Soundalgekar and Takhar [35] investigated the impact of radiation on the free convection phenomena occurring in the flow of a gas over a semi-infinite flat plate, employing the Cogley-Vincentine-Giles equilibrium model. Takhar et al. [36] investigated the MHD influences on the free convective flow of a radiating gas over a semi-infinite plate, employing the Rosseland approximation method for modelling radiative heat transfer. Grosan and Pop [37] conducted a numerical examination of the impact of radiation on the stable mixed convection heat transfer within a vertical channel. The utilization of the Rosseland approximation was implemented in the representation of the conduction radiation heat transfer, with the assumption of constant temperatures at the walls. The findings indicated a reduction in the occurrence of reverse flow with a rise in the radiation parameters. The synergistic impact of laminar flow, mixed convection, and surface radiation heat transfer on thermally developing airflow in a vertically oriented channel subjected to side heating has been investigated experimentally. This investigation involved varying thermal and geometric parameters and was conducted by Rajamohan Ganesan et al. [38].

The investigation of thermal energy transfer involving radiation phenomena during mixed convection over a vertically heated surface characterized by varying levels of porosity and permeability, employing nanofluids composed of Cu-H_2O, uncovers a multitude of crucial findings. The incorporation of copper particles into aqueous solutions leads to a notable improvement in heat transfer effectiveness because of the excellent thermal conductivity demonstrated by copper in contrast to pure water. In a vertical channel, Surender et al. [39] examined the constant laminar flow and mixed convection heat transfer of a Cu-H_2O nanofluid under the effects of thermal radiation and Navier slip. The influence of radiation is evident in studies conducted on MHD radiative flow of hybrid alumina-copper/water nanofluids, researchers [40-41] have observed that heightened radiation parameters hinder boundary layer separation and enhance heat transfer, especially under conflicting flow conditions. Lin Zhang et al. [42] investigated the impact of particle size of Cu-water nanofluid on the efficacy of innovative heat exchangers. Through numerical simulation, enhancements were made to the design of a conventional fixed tube plate heat exchanger, wherein the straight tube was replaced with a corrugated tube.

A literature review carried out by numerous researchers has examined the analysis of heat transfer utilizing base fluids, porous medium, and magnetic influences. Initially, the constancy of fluid properties was maintained, however, subsequent exploration involved diverse geometries and fluctuating fluid characteristics. Many scholars have investigated heat transfer by introducing nanoparticles into base fluids, forming nanofluids, revealing a notable enhancement in heat transfer efficiency compared to the base fluid. Nevertheless, limited research has been conducted on the alteration of fluid properties of heat energy transport.

The primary objective of this research is to scrutinise the mixed convective heat transfer in a saturated porous medium for a viscous, incompressible fluid. The investigation will concentrate on a semi-infinite vertical plate, considering the impact of thermal radiation and nanofluids with varying fluid properties.

Mathematical Formulation And Solution:

We consider the steady mixed convection boundary layer flow past a vertical semi-infinite flat plate embedded in a nanofluid-filled porous medium. The x-coordinate is measured along the plate from its leading edge, and the y-coordinate is normal to it. Let U_o be the velocity of the fluid in the upward direction and the gravitational field is acting in the downward direction as shown in Figure 1. The plate is maintained at a uniform temperature T_w which is always greater than the free stream values existing far from the plate (i.e., $T_w > T_\infty$).

Fig. 1: Physical configuration of the system

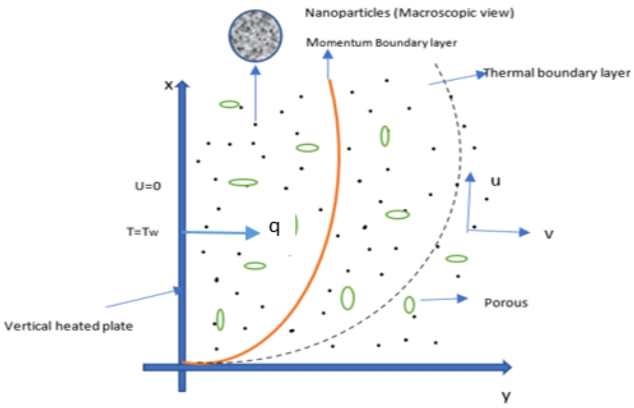

Table 1. Thermophysical properties of water and nanoparticles[43]

Physical Properties	Water/base fluid	Cu(Copper)
$\rho\ (kg/m^3)$	997.1	8933
$c_p\ \ (J/kg.K)$	4179	385
$k\ (W/m.K)$	0.613	401
$\sigma\ \ (S/m)$	0.05	59.6×10^6

Assume that there exists thermal equilibrium between the base fluid, Copper, and the nanoparticles, with no occurrence of slip between them. The medium is considered non-scattering, absorbing, and emitting radiation, where the Rosseland approximation is applied to elucidate the radiative flux within the energy equation. By the nanofluid model, the equations governing the flow problem are established under these aforementioned assumptions.

Conservation of Mass,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

Momentum Equation,

$$\rho_{nf}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=g\left(\rho g\right)_{nf}\left(T-T_{\infty}\right)+\mu_{nf}\frac{\partial^2 u}{\partial y^2}+\mu_{nf}\frac{\varepsilon(y)}{k(y)}\left(U_0-u\right) \quad (2)$$

Energy Equation,

$$\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\kappa_{nf}}{\left(\rho c_p\right)_{nf}}\frac{\partial^2 T}{\partial y^2}-\frac{\partial q_r}{\partial y}+\frac{\mu_{nf}}{\left(\rho c_p\right)_{nf}}\left(\frac{\partial u}{\partial y}\right)^2 \quad (3)$$

The above governing equations need to be solved subject to the following boundary

$$u=0, v=0, T=T_w \text{ at } y=0 \quad (4)$$

$$u=U_0, v=0, T=T_{\infty} \text{ as } y\to\infty \quad (5)$$

Where u and v are the velocity components along the x and y direction, g is the acceleration due to gravity, T is the Temperature of the nanofluid, $(\rho\beta)_{nf}$ the thermal expansion of the nanofluid, ρ_{nf} the density of the nanofluid, μ_{nf} the dynamic viscosity of the nanofluid, $k(y)$ the variable permeability of the nanofluid, $\varepsilon(y)$ the porosity of the saturated of the porous medium, U_0 be the velocity of the fluid in the upward direction, $(\rho c_p)_{nf}$ the heat capacitance of the nanofluid, κ_{nf} the thermal conductivity of the nanofluid, q_r the radiative heat flux.

The radiative heat flux can be written as (using the Rosseland approximation[44])

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \quad (6)$$

Where, σ^* the Stefan Boltzmann constant and the Rosseland mean observation co-efficient is k^*. Now by assuming a modest temperature difference with the flow and by using the Taylor series to expand the T^4 and T_{∞} where the temperature may be stated as a linear function of temperature as shown in the following equation,

$$T^4=T_{\infty}^4+4T_{\infty}^3\left(T-T_{\infty}\right)+6T_{\infty}^2\left(T-T_{\infty}\right)^2+\cdots$$

Ignoring higher-order terms in the above equation beyond the first-order $\left(T-T_{\infty}\right)$, we get

$$T^4\cong 4TT_{\infty}^3-3T_{\infty}^4 \quad (7)$$

On Substituting Eq.(8) into Eq.(7) we get

$$\frac{\partial q_r}{\partial y}=-\frac{16\sigma T_{\infty}^3}{3k^*}\frac{\partial^2 T}{\partial y^2} \quad (8)$$

The Eq. (3) will be reduced to the form:

$$\left(\rho c_p\right)_{nf}\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\kappa_{nf}\frac{\partial^2 T}{\partial y^2}+\frac{16\sigma T_{\infty}^3}{3k^*}\frac{\partial^2 T}{\partial y^2}+\mu_{nf}\left(\frac{\partial u}{\partial y}\right)^2 \quad (9)$$

We now introduce the following dimensionless variables f and θ as well as similarity variable η .

$$\eta = \left(\frac{y}{x}\right)\left(\frac{U_0 x}{\gamma}\right)^{\frac{1}{2}}, \ \psi = \sqrt{\gamma U_0 x}\, f(\eta), \ \theta = \frac{T - T_\infty}{T_w - T_\infty} \tag{10}$$

Where a prime represents differentiation concerning η and $T = T_w$ is the plate temperature.

In equation (6) the stream function $\psi(x, y)$ is defined by $u = \frac{\partial \psi}{\partial y}, \ v = -\frac{\partial \psi}{\partial x}$, such that the continuity equation (1) is satisfied automatically and the velocity components are given by,

$$u = U_0 f'(\eta), \ v = -\frac{1}{2}\sqrt{\frac{\upsilon U_0}{x}}\left(f(\eta) - \eta f'(\eta)\right) \tag{11}$$

The properties of the nanofluid are described as follows and given in the Reference [Choi [45]]:(12-15)

The density $\rho_{nf} = (1-\varphi)\rho_f + \varphi \rho_s$ (12)

viscosity of the nanofluid $\mu_{nf} = \frac{\mu_f}{(1-\varphi)^{2.5}}$ (13)

Heat Capacitance $(\rho c_p)_{nf} = (1-\varphi)(\rho c_p)_f + \varphi(\rho c_p)_s$ (14)

Thermal conductivity $\kappa_{nf} = \kappa_f \left(\frac{(\kappa_s + 2\kappa_f) - 2\varphi(\kappa_f - \kappa_s)}{(\kappa_s + 2\kappa_f) + \varphi(\kappa_f - \kappa_s)}\right)$ (15)

φ is the nanoparticle volume fraction, ρ_f and ρ_s are the density of the base fluid and nano-particles respectively, c_{pf} and c_{ps} - specific heat of the base fluid and nanoparticles respectively and κ_f and κ_s are thermal conductivity of base fluid and nanoparticles respectively.

Following Chandrashekhar and Namboodiri [13], the variable permeability $k(\eta)$, the variable porosity $\varepsilon(\eta)$ are given by,

$$k(\eta) = k_0\left(1 + d e^{-\eta}\right)$$

(16) $\varepsilon(\eta) = \varepsilon_0\left(1 + d^* e^{-\eta}\right)$ (17)

Were k_0, ε_0 and are the permeability, and porosity at the edge of the boundary layer respectively, σ^* is the ratio of the thermal conductivity of a solid to the conductivity of the fluid, d and d^* are treated as constants having values of 3.0 and 1.5 respectively.

transformed equations are:

$$f''' + \frac{A_1A_2}{2} ff'' + \frac{Gr}{R_e^{\,2}}\theta A_1A_2 + \frac{(1-f')\left(1+d^*e^{-\eta}\right)}{R_eK\left(1+de^{-\eta}\right)} = 0 \tag{18}$$

$$2A_1\left(A_4+R\right)\theta'' + Ec\Pr f''^2 + A_1A_3\Pr f\theta' = 0 \tag{19}$$

Were,

$$A_1 = \frac{1}{(1-\phi)^{2.5}},\ A_2 = (1-\phi)+\phi\frac{\rho_s}{\rho_f},\ A_3 = (1-\phi)+\phi\frac{\rho_s\left(c_p\right)_s}{\rho_f\left(c_p\right)_f},\ A_4 = \kappa_f\left[\frac{\kappa_s+2\kappa_f-2\phi\left(\kappa_f-\kappa_s\right)}{\kappa_s+2\kappa_f+\phi\left(\kappa_f-\kappa_s\right)}\right]$$

$\Pr = \frac{\mu_f\left(c_p\right)_f}{k_f}$ is the Prandtl number, $\alpha^* = \mu/\bar{\mu}$ is the ratio of viscosities, $E = \frac{U_0^{\,2}}{(c_p)_f\left(T_w-T_\infty\right)}$ is the Eckert number, $K = \frac{k_0}{x^2\varepsilon_0}$ is the local permeability parameter,

$\text{Re} = \frac{U_0x}{\nu_f}$ is the local Reynolds number and $E = \frac{U_0^{\,2}}{(c_p)_f\left(T_w-T_\infty\right)}$ is the local Gasthof number.

The transformed boundary conditions are:

$$f = 0,\ f' = 0, \theta = 1 \quad \text{at} \quad \eta = 0 \tag{20}$$

$$f' = 1, \theta = 0 \ \text{as} \ \eta \to \infty \tag{21}$$

The skin friction and the rate of heat transfer can be calculated by

$$\tau = -\frac{1}{(1-\varphi)^{2.5}}\, f''(0)/\sqrt{\text{Re}} \qquad \text{and} \qquad Nu = -\frac{k_{nf}}{k_f}\sqrt{\text{Re}}\,\theta'(0)$$

Where τ is the skin friction and Nu is the Nusselt number.

Numerical Method:

The boundary value problems described by equations (18) and (19) are highly nonlinear, with the third and second orders respectively. The non-linear boundary value problem (BVP) is addressed using the Shooting technique. The complex non-linear boundary value problem, consisting of a third-order equation in and a second-order equation in, has been simplified into a system of five simultaneous first-order equations with five unknowns.

$$f_1' = f_2,\ f_2' = f_3$$

$$f_3' = -\left(\frac{A_1A_2}{2} f_1 f_3 + \frac{Gr}{R_e^{\,2}}\theta A_1A_2 + \frac{(1-f_1)\left(1+d^*e^{-\eta}\right)}{R_eK\left(1+de^{-\eta}\right)}\right) \tag{22}$$

$$f_4' = f_5$$

$$f_5' = -\left(\frac{1}{2}\Pr f_1 f_5 \frac{A_3}{(A_4+R)} + Ec\Pr f_3^{\,2}\frac{1}{(A_4+R)A_1}\right) \tag{23}$$

Where $f_1 = f,\ f_2 = f',\ f_3 = f'',\ f_4 = \theta,\ f_5 = \theta'$ and a prime denotes differentiation concerning η .

The boundary conditions now become

$$f_1 = 0,\ f_2 = 0,\ f_4 = 1 \quad \text{at} \quad \eta = 0 \tag{24}$$

$$f_2 = 1,\ f_4 = 0 \quad \text{as} \quad \eta \to \infty \tag{25}$$

Results And Discussions:

The present work employs varying fluid parameters and nanofluids to examine the phenomenon of mixed convection over a vertically heated plate. The complicated system of governing equations is composed of strong interdependencies and non-linear partial differential equations. Similarity transformations are used to convert the equations into higher-order ordinary differential equations. We employ the Runge-Kutta method to transform higher-order ordinary differential equations (ODEs) into first-order ODEs and subsequently utilize the shooting process.

The delineation of the boundary conditions as stipulated in equations (24) and (25) holds significant importance for the research. The characteristics of the nanofluid can be found in Table 1. The outcomes of the computations are depicted in Figures 2-9, along with Tables 1. The relevant parameters and thermophysical properties of the base fluid (water) and the nanoparticle (copper) are taken into consideration. The selection of volume fraction falls within a specified range. The velocity and temperature profiles are presented for both uniform permeability (UP) and variable permeability (VP) scenarios. The values of d and d^* are assumed as 0 for uniform permeability and 3.0 and 1.5 correspondingly for variable permeability. Upon combining the base fluid with a nanofluid containing copper nanoparticles at different volume fractions, distinctive changes are observed in the velocity and temperature plots.

Fig. 2 displays the velocity profiles for various values of the volumetric fraction parameter. As the volumetric fraction increases, the velocity decreases. This is because a rise in the volume percentage causes the Nano fluid’s density to increase, slowing the fluid’s velocity. Furthermore, the interactions occurring among the nanoparticles in suspension and the molecules of the base fluid may result in the generation of supplementary frictional forces. These forces function as a resistance to the flow of the fluid, consequently diminishing its total velocity, this higher viscosity which is evident in both the UP and VP cases. However, a different pattern may be seen in the temperature profiles. The reason for this is that copper nanofluids become thicker thermal boundary layers when they are mixed with nanoparticles, which increases their level of thermal conductivity. As such, the fluid's capacity to diffuse heat is improved when the temperature gradient decreases. Heat transport is more efficient in cases with variable permeability than in cases with uniform permeability as in Fig. 3.

The mixed parameter, which quantifies the ratio of buoyant forces to viscous forces, is a key factor in defining the cooling qualities of mixed convection flow over a vertical plate containing nanofluid. The mixed convection parameter of the heated plate is <<0 and the cooled plate has >>0. An additional cooling effect can be produced by adding a nanofluid, which is a suspension of nanoparticles in a base fluid. Nanoparticles often boost heat transfer by increasing the fluid's effective thermal conductivity.
The greater cooling of the vertical plate in Fig. 4 is attributed to both enhanced thermal conductivities of the nanofluid and buoyancy-driven natural convection.
In both the UP and VP scenarios, the temperature is found to be lower in VP than in UP due to the mixed convection parameter, which may result in a larger temperature differential between the fluid and the plate. As shown in Fig. 5, it is also noted that the effect of VP on the temperature distribution is more substantial for higher values.

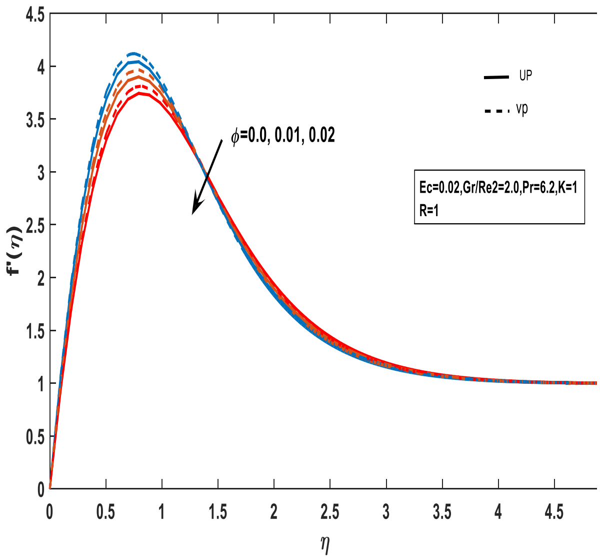

Fig 2: Velocity Profile for various values of ϕ

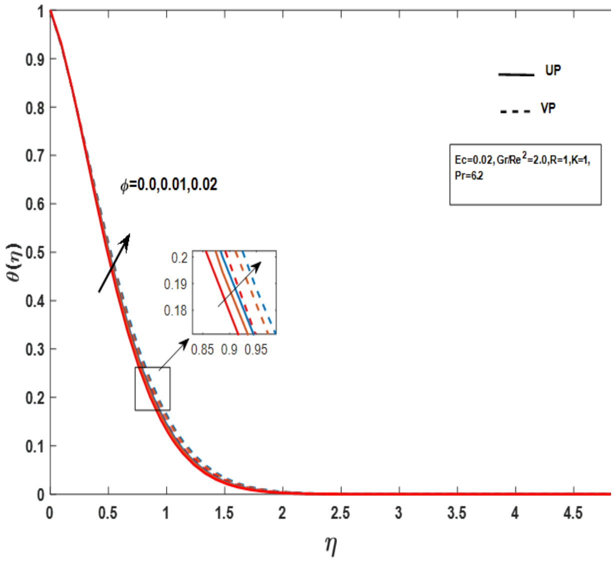

Fig 3: Temperature Profile for various values of ϕ

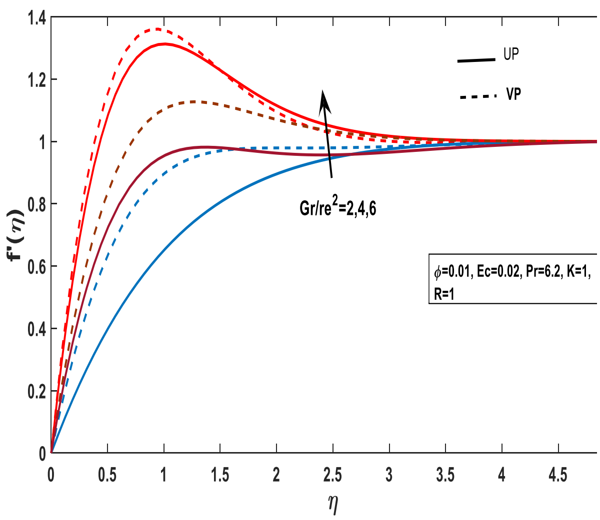

Fig 4: Velocity Profile for various values of Gr/Re^2

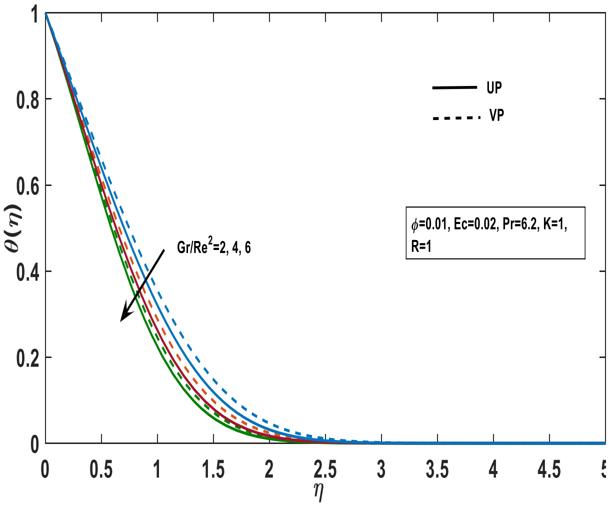

Fig 5: Temperature Profile for various values of Gr/Re^2

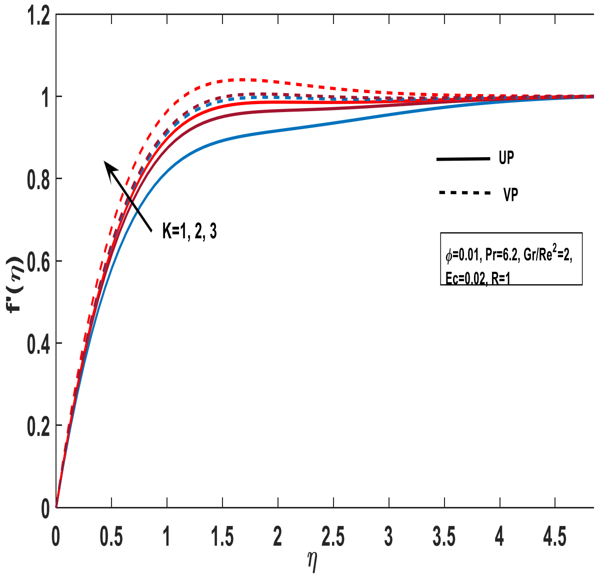

Fig 6: Velocity Profile for various values of K

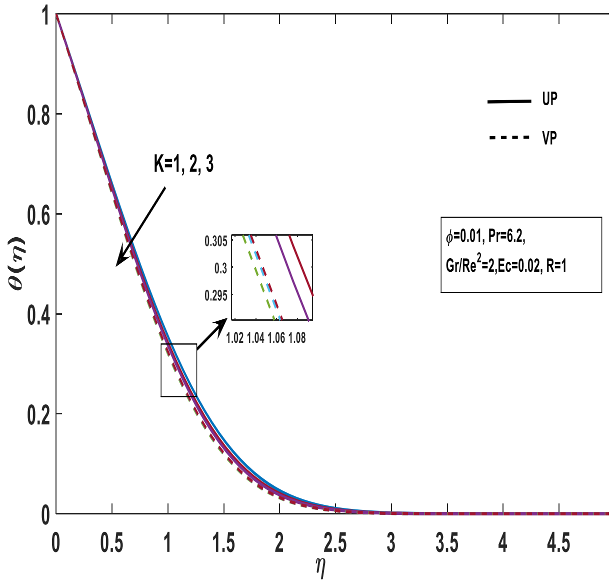

Fig 7: Temperature Profile for various values of *K*

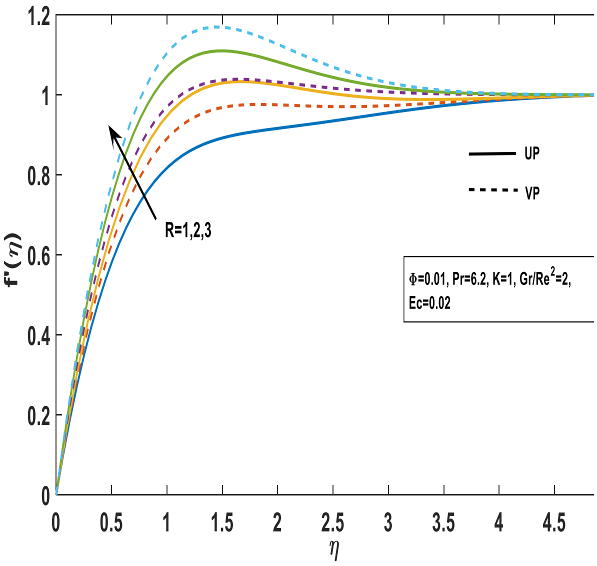

Fig 8: Velocity Profile for various values of *R*

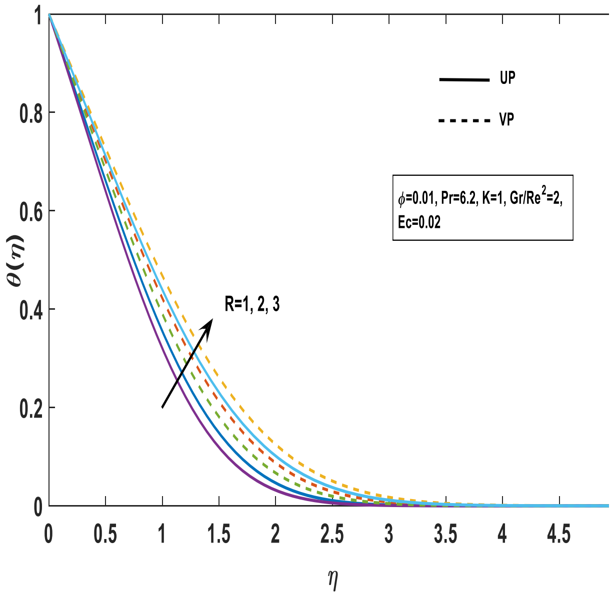

Fig 9: Temperature Profile for various values of *R*

The impact of the porosity parameter on the velocity and temperature profile is shown in Fig. 6 and 7 for both UP and VP cases. Permeability refers to the ability of a porous media to allow liquids to pass through it. Due to the increase of porosity, the velocity field increases. Also, the temperature profiles reduce with increasing porosity parameters. The Cu-water nanofluid velocity patterns at different radiation parameter values are shown in Fig. 8. The velocity profile rises with an increasing radiation parameter value, and then gradually falls to reach the free stream velocity, it is observed that velocity increases in VP then UP case. The temperature profiles at different radiation parameter values are shown in Fig. 9. The temperature profile increases as the radiation impact grows because the increasing radiation creates a greater heat flow, leading to a rise in the boundary layer's temperature. The temperature falls steadily until it reaches the temperature of the free stream. At lower radiation parameter values, the fluctuation in temperature profiles becomes more noticeable.

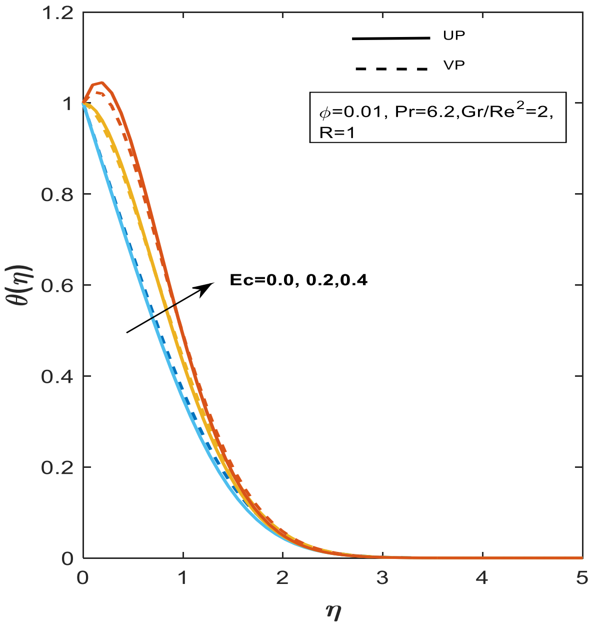

Fig 10: Temperature Profile for various values of *Ec*

Table 2: Comparative analysis for $f''(0)$ and $-\theta'(0)$ when Ec=0.1, Pr=7.0,R=0 for UP case :							
El-Dawy et al. [17]				Present work			
Gr/Re^2	ϕ	$f''(0)$	$-\theta'(0)$	Gr/Re^2	ϕ	$f''(0)$	$-\theta'(0)$
0.2	0	1.1799	0.3355	0.2	0	1.1782	0.3354
0.5		1.3222	0.3351	0.5		1.3211	0.3199
0.8		0.5927	0.2769	0.8		0.5826	0.3001
0.2	0.01	1.3359	0.3361	0.2	0.01	1.3357	0.3212
0.5		0.5869	0.2122	0.5		0.5870	0.2222
0.8		1.2271	-1.0927	0.8		1.2272	-1.0865
0	0.02	1.3509	0.3350	0	0.02	1.3502	0.3347
0.5		1.0187	-0.6405	0.5		1.0185	-0.6305
0.8		0.7003	-0.0536	0.8		0.7001	-0.0446

Ec increases because of the relationship between kinetic energy and enthalpy difference, which raises the temperature, which can be seen in both UP cases, fig.10 . Fig. 11 shows the increase in skin friction with an increase in K In Fig. 12, it is observed that the heat gets transferred from the sheet to the nanofluid, which increases the temperature of the fluid. The comparison is depicted in Table 2, the agreement of the results of the model up to three decimal places validates in absence of radiation.

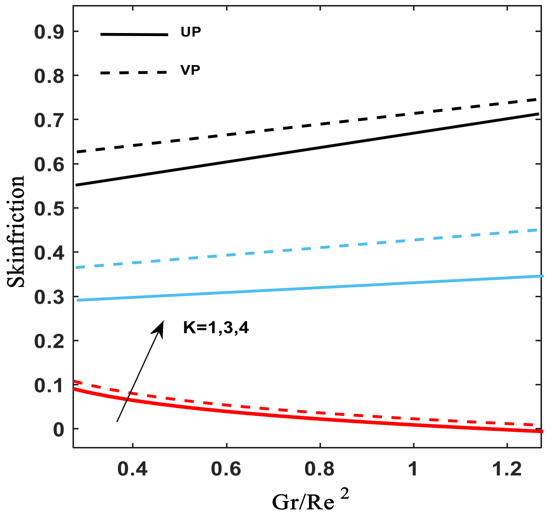

Fig.11: Skin friction for various values of K

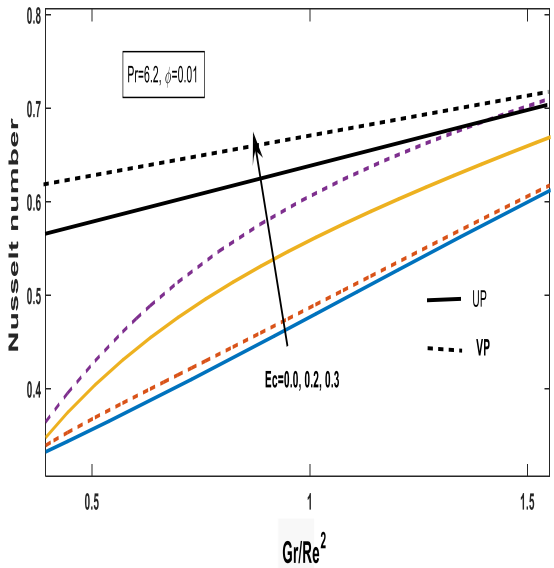

Fig.12: Nusselt number for various values of Ec

Conclusion:

The following conclusions are drawn for the physical interest of a vertical plate in a sparsely packed porous medium in the existence of thermal radiation, the permeability and porosity are assumed to be variable.

1. The velocity profile reduces with increasing volume fraction where a reverse trend can be observed in the temperature profile for both UP and VP cases.
2. The vertical plate's cooling is enhanced by nanofluid thermal conductivity and buoyancy-driven natural convection. VP temperatures are lower than UP due to mixed convection, causing a larger temperature differential. VP's effect is more significant for higher values.

3. The velocity profile increases with the radiation parameter value, reaching free stream velocity in the VP case. Temperature profiles show increased radiation impact due to greater heat flow and boundary layer temperature.
4. As the porosity parameter increases, the velocity field also increases, while the temperature profiles decrease for both UP and VP cases.
5. The temperature of the nanofluid increases with increasing of Eckert number.
6. The Russell number leads to increases with an increase in the Eckert number.
7. Skin-friction coefficient increases with the increase of porous parameter K.

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