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Two-Wheel Robot Swing-Up Using Scissor Pair Control Moment Gyro

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Abstract:- In this study, two wheel inverted pendulum robot is equipped with a scissor pair control moment gyro (CMG) for controlling its tilt angle. The objective is utilizing the produced torque from CMG to swing up the robot from ground to the upright position. The physical system is designed and its mathematical model is derived using Lagrange method, capturing the complex dynamics and constraints, including locked drive wheels. To achieve the challenging task of swinging up the robot from extreme initial conditions (90-degree body tilt angles on either side) to a stable zero-state upright position, a full-state feedback Linear Quadratic Regulator (LQR) controller is employed. The nonlinear equations of the system and the controller are solved numerically by MATLAB software. Furthermore, to validate the derived model and controller, the system is implemented on computer-aided engineering software (Simulink Multibody), and the results are meticulously compared with mathematical outcomes.

Keywords*:* Control Moment Gyro, Inverted Pendulum, Swing-Up, LQR, Two Wheel Robot.

1. Introduction

The landscape of mobile robotics is experiencing significant advancements due to development of electronics, sensors and actuators, with these versatile machines finding application across a multitude of environments[1],[2]. These encompass air-based, water-based, and land-based settings, where they play pivotal roles in diverse sectors, including households, agriculture[3], industry[4], military operations[5], and even the frontier of space exploration[6].

One standout among these mobile robots is the Two-Wheel Inverted Pendulum Robot, affectionately known as "TWIP." Within the commercial sector[7], TWIP has emerged as a subject of intense investigation due to its inherent attributes, notably its simplicity, remarkable maneuverability, and capacity to operate effectively in confined spaces.

In the academic sphere, TWIP presents a captivating challenge to researchers[8],[9], given its intrinsic instability as an underactuated system[10]. The cart pole, regarded as a simplified counterpart to the two-wheel robot, has been a focal point for extensive research efforts[10]. This includes endeavors related to both swinging it up[11] from challenging initial conditions and maintaining a balanced equilibrium upright state [12].

Recent developments have introduced mechanical devices capable of generating torque to control the body tilt angle within the inverted pendulum structure. Among these devices are Reaction Wheels (RWs) [13] and Control Moment Gyros (CMGs)[14],[15]. While reaction wheels have their merits, they are hampered by their dependence only on mass moment of inertia, resulting in limited torque capacity that varies with the specific flywheel and motor used. In contrast, CMGs offer distinctive advantages. They derive their torque not only from the flywheel's mass moment of inertia but also from its fixed angular velocity.

CMGs introduce a unique torque generation mechanism. The direction of torque produced by CMGs is orthogonal to both the flywheel's angular momentum and the gimbal rotation. This results in a torque component that varies according to the cosine of the gimbal angle. Although CMGs inherently produce torque in multiple directions as the gimbal angle changes, the scissor pair control moment gyro (SPCMG) configuration has been devised to address this complexity. SPCMG enable the duplication of desired torque components while effectively nullifying undesired ones.

In this paper, the primary objective is to explore the efficacy of employing the SPCMG to accomplish the formidable task of swinging up the robot body from a 90-degree tilt angle. This endeavor is undertaken while treating the relative motion between the drive wheels and the robot body as a unified entity. In this configuration, the SPCMG assumes sole responsibility for manipulating the swing-up and balancing of the system. The derivation of the mathematical model for this inherently nonlinear system begins by utilizing Lagrange's equations to elegantly capture its dynamic intricacies. Subsequently, linearization around the zero states position is undertaken to facilitate the exploration of control strategy.

A closed-loop system is formed by applying full-state feedback control to the system. Optimal gains are acquired by applying the linear quadratic regulator (LQR) to the linearized system. The nonlinear system is subsequently regulated using the optimal gains obtained, and the closed-loop nonlinear system is solved using MATLAB software, resulting in the acquisition of response data and control signals, enabling us to visualize and analyze them.

To bolster the credibility of mathematical model and controller, a robust MATLAB/Simulink Multibody model is constructed, enabling a meticulous comparison of simulation results with theoretical outcomes.

In summation, this paper constitutes a comprehensive exploration of the complexities and challenges inherent to TWIP. It delves deep into the intricacies of control, leveraging the SPCMG to achieve the task of swinging up the robot body from extreme initial conditions. This work seamlessly integrates mathematical modeling, control strategy, and simulation tools to shed light on this intricate facet of mobile robotics.

2. System Modeling And Control

A. System Setup

Illustrated in Fig. 1, the conceptual robot design comprises a body configured as an inverted pendulum, actuated by a pair of wheels. During the swinging-up phase, the drive wheels are intentionally immobilized, resulting in zero relative motion between the body and the wheels. Upon achieving an upright position, the balance phase starts as discussed in [14]. In this phase the braked drive wheels are released allowing the robot's base, centrally situated between the drive wheels, commences a translation along the z-axis, represented by the displacement z(t). The pivotal component of this setup is the SPCMG, strategically integrated within the robot's base. This integration ensures that the torque generated by the SPCMG is perpetually aligned with the direction of the body's tilting, denoted as φ(t), as the precision angle of the gimbal, θ(t), attains a certain velocity.

The flywheels exhibit counter-rotating behavior, spinning in opposing directions at an identical fixed angular velocity denoted as Ω. Simultaneously, the gimbal angle, θ(t), is rigorously constrained to follow corresponding motions in opposite directions, as visually depicted in Fig. 2.

The resulting unidirectional torque is generated as a cosine function of the instantaneous value of the gimbal angle. This torque will have the dual role of swinging up the robot in swing-up phase and subsequently stabilizing it in the upright position in balance phase.

In swing-up phase, the control block diagram illustrated in Fig. 3 shows that gimbal speed is utilized as control signal obtained from full-state feedback controller. Controller gains are calculated by linearizing the system and applying LQR to obtain optimal gains.

B. Mathematical Modeling

In this section, the mathematical model that describes the dynamics of the robot during swinging-up phase is derived. The system under investigation possesses two degrees of freedom: the body tilt angle, denoted as φ, and the gimbal precision angle, represented as θ.

The analysis begins by determining the kinetic and potential energy associated with each constituent part of the system with robot base level as zero energy level. This step allows to encapsulate the system's energy state and serves as a foundational element for the subsequent mathematical lagrangian derivations for robot of parameters presented in *Table I*. The kinetic energies of flywheel T_f, gimbal T_g, wheel T_w and body T_b can be defined as follows, respectively

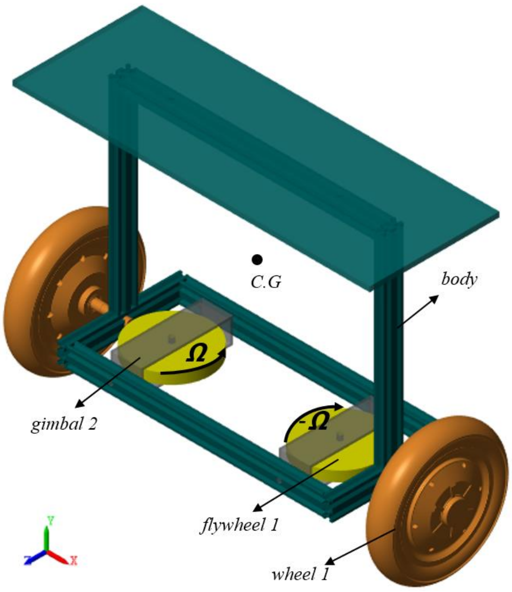

Fig. 1. Two wheel robot equipped with scissor pair control moment gyro

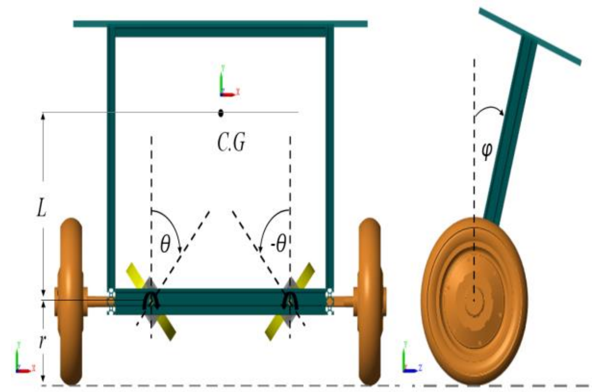

Fig. 2. Gyro-stabilized two wheel robot degrees of freedom

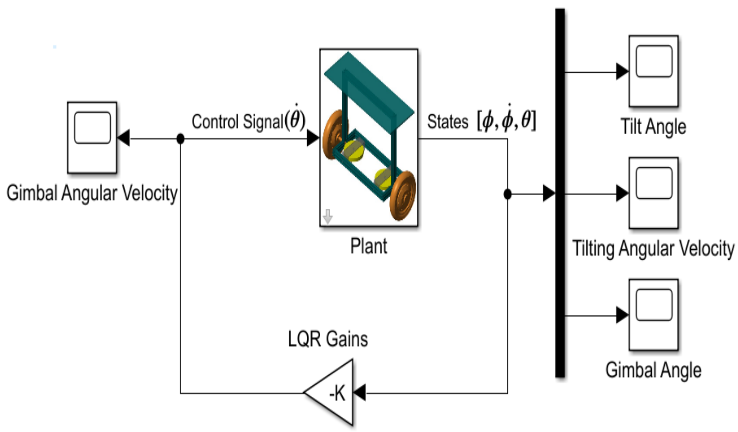

Fig. 3. Control Block Diagram of Two Wheel Robot Swing-Up

$$T_f = \frac{1}{2}\left[m_f\left(\dot{\phi}r\right)^2 + I_{f_x}\left(\dot{\phi}\cos(\theta)\right)^2 + I_{f_y}\left(-\Omega - \dot{\phi}\sin(\theta)\right)^2 + I_{f_z}\left(\dot{\theta}\right)^2\right] \quad (1)$$

$$T_g = \frac{1}{2}\left[m_g\left(\dot{\phi}r\right)^2 + I_{g_x}\left(\dot{\phi}\cos(\theta)\right)^2 + I_{g_y}\left(-\dot{\phi}\sin(\theta)\right)^2 + I_{g_z}\left(\dot{\theta}\right)^2\right] \quad (2)$$

$$T_w = \frac{1}{2}\left[m_w\left(\dot{\phi}r\right)^2 + I_{w_x}\left(\dot{\phi}\right)^2\right] \quad (3)$$

$$T_b = \frac{1}{2}\left[m_b\left(\left(L\dot{\phi}\cos(\phi) + \dot{\phi}r\right)^2 + \left(-L\dot{\phi}\sin(\phi)\right)^2\right) + I_{b_x}\left(\dot{\phi}\right)^2\right] \quad (4)$$

So the total kinetic energy is

$$T = T_b + 2T_f + 2T_g + 2T_w \quad (5)$$

During operation, only the center of mass of body changes with time and other parts remains at zero level so

$$V = m_b gL\cos(\phi) \quad (6)$$

Since only two degrees of freedom existed, Lagrange's equations of the system can be written as

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\phi}}\right) - \frac{\partial T}{\partial \phi} + \frac{\partial V}{\partial \phi} = 0 \quad (7)$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) - \frac{\partial T}{\partial \theta} + \frac{\partial V}{\partial \theta} = 2\tau_c \quad (8)$$

Where τ_c is the control torque applied to each gimbal. So, the nonlinear equations are obtained and rearranged in terms of $\ddot{\phi}, \ddot{\theta}$ as follow

Table I. Robot Parameters

Parameter	Symbol	Value
Length between base and body center of gravity	L	0.137545 m
Radius of each wheel	r	0.127 m
Gravitational constant	g	9.80665 m/s^2
Mass of each flywheel	m_f	2.46278 kg
Mass of each gimbal	m_g	0.978686 kg
Mass of each wheel	m_w	2.34 kg
Mass of Body	m_b	2.29855 kg
Flywheel mass moment inertia in x. z directions	I_{f_x}, I_{f_z}	0.00356039 kg/m^2
Flywheel mass moment of inertia in y direction	I_{f_y}	0.0069566 kg/m^2
Gimbal mass moment of inertia in x direction	I_{g_x}	0.00410189 kg/m^2
Gimbal mass moment of inertia in y direction	I_{g_y}	0.00396961 kg/m^2
Gimbal mass moment of inertia in z direction	I_{g_z}	0.00068736 kg/m^2

Parameter	Symbol	Value
Wheel mass moment of inertia in x direction	I_{w_x}	0.0208127 kg/m^2
Mass moment of inertia of body in x direction	I_{b_x}	0.0795059 kg/m^2
Flywheel speed	Ω	1000 rad/s

$$\ddot{\phi} = \frac{m_b g L \sin(\phi) - 2I_{f_y}\Omega\dot{\theta}\cos(\theta) - 2\dot{\phi}\dot{\theta}\sin(2\theta)\left(I_{f_y} + I_{g_y} - I_{f_x} - I_{g_x}\right) + m_b L r \dot{\phi}^2 \sin(\phi)}{I_{b_x} + 2I_{w_x} + r^2 m_T + 2\left(I_{f_x} + I_{g_x}\right)\cos^2(\theta) + 2\left(I_{f_y} + I_{g_y}\right)\sin^2(\theta) + m_b L^2 + 2m_b L r\cos(\phi)} \quad (9)$$

$$\ddot{\theta} = \frac{2\tau_c + \dot{\phi}^2 \sin(2\theta)\left(I_{f_y} + I_{g_y} - I_{f_x} - I_{g_x}\right) + 2I_{f_y}\Omega\dot{\phi}\cos(\theta)}{2\left(I_{f_z} + I_{g_z}\right)} \quad (10)$$

C. State Space Model and Control

The robot is considered to be controlled by signal that determine gimbal speed control directly rather than gimbal torque. This choice simplifies the problem making system to be reduced to only three states and allows to represent the phase portraits of system in 3D graph.

Define $x = [x_1, x_2, x_3]^T$ and $u = \dot{\theta}$, where $x_1 = \phi$, $x_2 = \dot{\phi}$ and $x_3 = \theta$

Then the system dynamics can be rewritten as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} x_2 \\ f(x,u) \\ u \end{bmatrix} \quad (11)$$

Since $f(x,u) = \ddot{\phi}$ mentioned in 13

Linearization is performed to system 11 for equilibrium states $x_{eq} = [0,0,0]^T$ to represent the system in form of

$$\dot{x} = Ax + Bu \quad (12)$$

as follow:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ A_{21} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \\ 1 \end{bmatrix} u \quad (13)$$

Where $A_{21} = \frac{m_b g L}{C}$, $B_2 = \frac{-2I_{f_y}\Omega}{C}$ as $C = I_{b_x} + 2\left(I_{f_x} + I_{g_x} + I_{w_x}\right) + m_b L^2 + m_T r^2 + 2m_b L r$ The cost function of linear quadratic regulator of state space model is:

$$J = \int_0^\infty \left(x^T(t) Q x(t) + u^T(t) R u(t)\right) dt \quad (14)$$

Weighting matrices Q and R are chosen as :

$$Q = \begin{bmatrix} 1000 & 0 & 0 \\ 0 & 10000 & 0 \\ 0 & 0 & 2500 \end{bmatrix} \text{ and } R = 2500$$

And control input that achieve desired states while minimizing cost function is

$$u = -k\left(x - x_{desired}\right) \quad (15)$$

In swing-up phase, the target is to put the robot in upright position while resetting gimbals to its zero position.

Substituting $x_{desired} = x_{eq} = [0,0,0]^T$, so

$$u = -kx \quad (16)$$

The solution of gain K is standard and it is given as:

$$K = R^{-1}B^T P \quad (17)$$

The gain matrix K is used to control the gimbals angular velocity of non-linear system, where P is determined by solving the following Algebraic Riccati Equation (ARE):

$$PA + A^T P - PBR^{-1}B^T P + Q = 0 \quad (18)$$

Optimum gains (17) , deduced from robot parameters of **Error! Reference source not found.***Table I* and system (13) can be calculated as follow K= [-1.0879,-1.0594,-0.6325].

3. Analysis

A. Experiment Criteria

The supposed experiment is initiated from distinct initial conditions deemed critical for investigation. These conditions entail the robot being positioned at either a positive or negative 90-degree tilt angle, representing a prone orientation on the ground. Additionally, the system commences with a state characterized by zero tilt angular velocity and zero gimbal angle, hence, $x_0 = \left[\pm\frac{\pi}{2}, 0, 0\right]^T$.

The core objective of the controller is to achieve the precise swing-up of the robot's body from these initial conditions to attain a perfectly upright position, defined as a zero-degree tilt angle. To accomplish this, the gimbal angular velocity is strategically modulated, while resetting the gimbal angle to zero $x_{desired} = [0,0,0]^T$.

B. Results

The nonlinear system of equations is solved numerically using MATLAB software and response was simulated with 0.001 second time step over 20 second time span.

Fig. 4 displays the response of three key states within the nonlinear system: the body tilt angle, body angular velocity, and gimbal precision angle. The body tilt angle undergoes a notable ascent, transitioning from a starting point of -π/2 radians to a stabilized zero position in an upright configuration. Concurrently, the gimbal precision angle exhibits dynamic changes until all states ultimately converge to a state of equilibrium at zero radians. Furthermore, in Fig. 5 we observe the temporal profile of the control signal governing the gimbal angular velocity. The signal initiates with a pronounced peak value and subsequently undergoes a gradual decay, reaching a final steady-state value of zero. This control signal is instrumental in manipulating the system's response, facilitating the transition to a stable equilibrium state observed in Fig. 5.

Also in Fig. 4 and Fig. 5, we engage in a comparative analysis between the theoretical outcomes derived from solving the nonlinear system equations and the simulated results obtained through the utilization of Simulink Multibody software. It is evident that the three fundamental states as well as the control signal exhibit a remarkable degree of concurrence between the two sets of data. This pronounced alignment serves as compelling empirical evidence, attesting to the accuracy and reliability of our derived mathematical model for the robot. Moreover, it unequivocally underscores the efficacy of the applied controller, affirming its capacity to drive the robot's behavior in accordance with mathematical results.

4. Conclusion

The two-wheel robot can achieve a swing-up motion, transitioning from a grounded position to an upright stance, through the application of torque generated by a control moment gyro. During the swing-up phase, a crucial feature involves synchronizing the movement of the drive wheels and the robot body, effectively treating them as a unified entity. This synchronization eliminates concerns of instability or potential falls, as the robot can autonomously regain its upright position.

Upon attaining the upright stance, a seamless shift to the balance phase controller occurs, and the locking mechanism between the drive wheels and the robot body is released. This transition empowers the robot with the capability to move freely and maintain its equilibrium during subsequent motion.

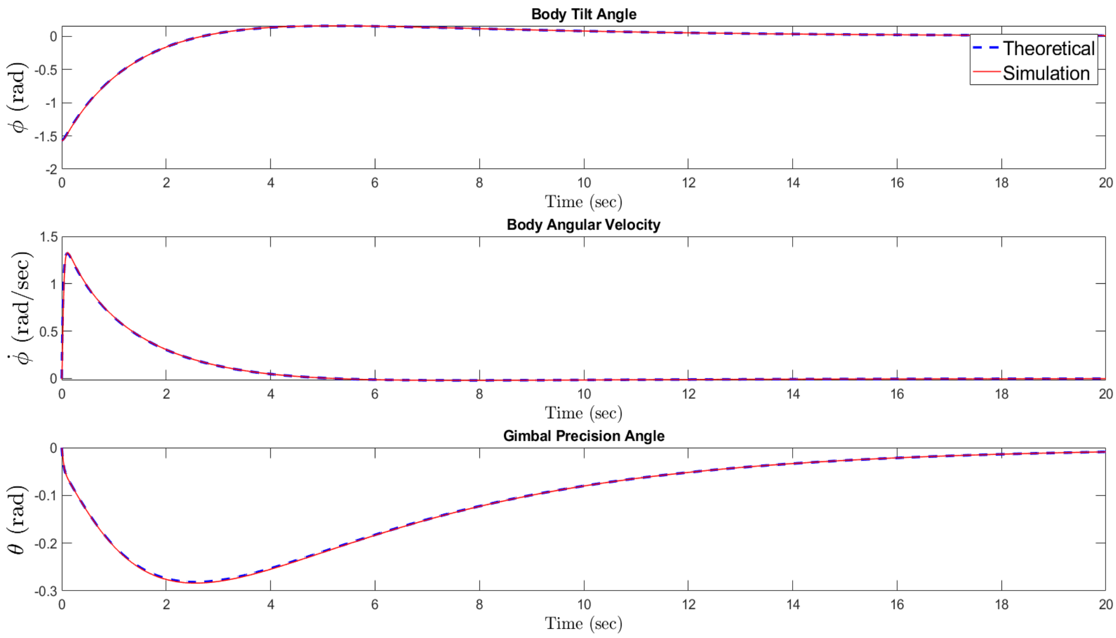

Fig. 4. Response of two wheel robot equipped with scissor pair control moment gyro in swing-up phase

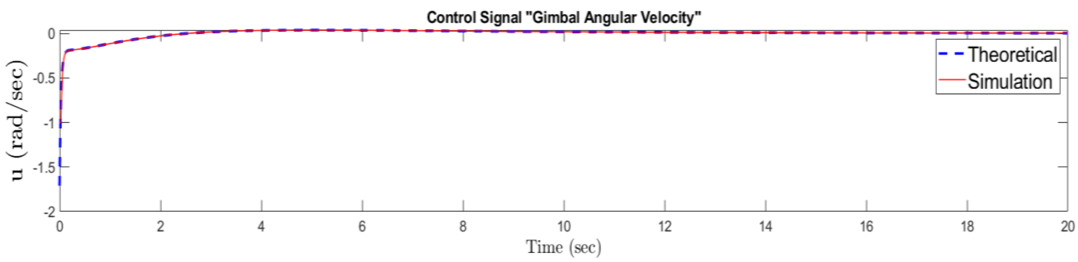

Fig. 5. Control signal for gimbal speed of scissor pair control moment gyro in swing-up phase

Refrences

[1] F. Rubio, F. Valero, and C. Llopis-Albert, "A review of mobile robots: Concepts, methods, theoretical framework, and applications," *International Journal of Advanced Robotic Systems*, vol. 16, no. 2. SAGE Publications Inc., Mar. 01, 2019. doi: 10.1177/1729881419839596.

[2] M. B. Alatise and G. P. Hancke, "A Review on Challenges of Autonomous Mobile Robot and Sensor Fusion Methods," *IEEE Access*, vol. 8. Institute of Electrical and Electronics Engineers Inc., pp. 39830–39846, 2020. doi: 10.1109/ACCESS.2020.2975643.

[3] V. Moysiadis, N. Tsolakis, D. Katikaridis, C. G. Sørensen, S. Pearson, and D. Bochtis, "Mobile robotics in agricultural operations: A narrative review on planning aspects," *Appl. Sci.*, vol. 10, no. 10, May 2020, doi: 10.3390/app10103453.

[4] A. Markis, M. Papa, D. Kaselautzke, M. Rathmair, V. Sattinger, and M. Brandstötter, "Safety of Mobile Robot Systems in Industrial Applications", doi: 10.3217/978-3-85125-663-5-04.

[5] D. Patil, M. Ansari, D. Tendulkar, R. Bhatlekar, V. N. Pawar, and S. Aswale, "A Survey on Autonomous Military Service Robot," in *International Conference on Emerging Trends in Information Technology and Engineering, ic-ETITE 2020*, Feb. 2020. doi: 10.1109/ic-ETITE47903.2020.78.

[6] Y. Wang, "Space Robotics Space Science and Technologies Series Editor: Peijian Ye." [Online]. Available: http://www.springer.com/series/16385

[7] "segway-i2-se @ eu-en.segway.com." https://eu-en.segway.com/products/segway-i2-se

[8] R. P. M. Chan, K. A. Stol, and C. R. Halkyard, "Review of modelling and control of two-wheeled robots," *Annual Reviews in Control*, vol. 37, no. 1. pp. 89–103, Apr. 2013. doi: 10.1016/j.arcontrol.2013.03.004.

[9] P. T. Thanh, D. P. Nam, V. Van Tu, T. Q. Huy, and N. Van Huong, "Robust control law using H-infinity for wheeled inverted pendulum systems," *Int. J. Mech. Eng. Robot. Res.*, vol. 8, no. 3, pp. 483–487, 2019, doi: 10.18178/ijmerr.8.3.483-487.

[10] C. C. Yih, "Sliding mode control for swing-up and stabilization of the cart-pole underactuated system," *Asian J. Control*, vol. 15, no. 4, pp. 1201–1214, 2013, doi: 10.1002/asjc.577.

[11] K. J. Åström and K. Furuta, "Swinging Up a Pendulum by Energy Control *," *IFAC Proc. Vol.*, vol. 29, no. 1, pp. 1919–1924, 1996, doi: 10.1016/s1474-6670(17)57951-3.

[12] C. Aguilar-Ibáñez, J. Mendoza-Mendoza, and J. Dávila, "Stabilization of the cart pole system: by sliding mode control," *Nonlinear Dyn.*, vol. 78, no. 4, pp. 2769–2777, 2014, doi: 10.1007/s11071-014-1624-6.

[13] S. R. Larimi, P. Zarafshan, and S. A. A. Moosavian, "A new stabilization algorithm for a two-wheeled mobile robot aided by reaction wheel," *J. Dyn. Syst. Meas. Control. Trans. ASME*, vol. 137, no. 1, 2015, doi: 10.1115/1.4027852.

[14] A. Amin, M. A. Fouz, and A. Elsawaf, "LQR Stability Control of Scissor Pair Gyro-Stabilized Two Wheel Robot," *Proc. 2023 5th Int. Youth Conf. Radio Electron. Electr. Power Eng. REEPE 2023*, 2023, doi: 10.1109/REEPE57272.2023.10086769.

[15] S. Aranovskiy, I. Ryadchikov, N. Mikhalkov, D. Kazakov, A. Simulin, and D. Sokolov, "Scissored pair control moment gyroscope inverted pendulum," in *Procedia Computer Science*, 2021, vol. 186, pp. 761–768. doi: 10.1016/j.procs.2021.04.198.