

# A Study of Fuzzy Generalized #Rg-Closed Sets and their Role in Fuzzy Topological Spaces

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## Abstract:

Fuzzy topological spaces offer a natural extension of classical topological spaces by incorporating degrees of membership for points. In this study, we explore the concept of fuzzy generalized #rg-closed sets, a fusion of fuzzy sets, generalized closure operators, and the #rg-closed property. The primary objective of this investigation is to analyse the characteristics, properties and the functional role of fuzzy generalized #rg-closed sets within the framework of fuzzy topological spaces.

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## Introduction:

The fuzzy logic and topology introduced and developed by Zadeh[15], Wong[14], Chang[2] and many more. In recent times, Jenifer J. Karnel and HolabasayyaSankannavar[3,4] introduced generalized #rg-closed closed and open sets in topological spaces. Extending this concept to fuzzy topological spaces, we define a new class of fuzzy generalized sets namely, fuzzy generalized #rg-closed sets and investigate their properties. In this section, we list out the definitions and the results which are needed in sequel.

## Preliminaries:

The fuzzy semi-closure (resp. fuzzy  $\alpha$ -closure, fuzzy semi-pre closure) of a fuzzy subset  $A$  of  $X$ , denoted by  $scl(A)$  (resp.  $\alpha cl(A)$ ,  $spcl(A)$ ), is defined to be the intersection of all fuzzy semi-closed (resp. fuzzy  $\alpha$ -closed, fuzzy semi-pre closed) sets of  $X$  containing  $A$ . It is known that  $scl(A)$  (resp.  $\alpha cl(A)$ ,  $spcl(A)$ ) is a fuzzy semi-closed (resp. fuzzy  $\alpha$ -closed, fuzzy semi-pre closed) set[11].

**Definition 2.1** A fuzzy subset  $A$  of a fuzzy topological space  $(X, T)$  is called:

- i) fuzzy semi-openset[5] if and only if there exists a fuzzy open set  $B$  in  $X$  such that  $B \leq A \leq cl(B)$ .
- ii) fuzzy regular open set[5] of  $X$  if  $int(cl(A))=A$ .
- iii) fuzzy generalized closed (briefly, fg-closed) [12] if  $cl(A) \leq U$  whenever  $A \leq U$  and  $U$  is fuzzy open set in  $X$ .
- iv) fuzzy strongly generalized closed (briefly, fg\*-closed)[10] if  $cl(A) \leq U$  whenever  $A \leq U$  and  $U$  is fuzzy g-open set in  $X$ .
- v) fuzzy generalized semiclosed (briefly, fgs-closed)[9] if  $scl(A) \leq U$  whenever  $A \leq U$  and  $U$  is fuzzy open set in  $X$ .
- vi) fuzzy regular generalized closed (briefly, frg-closed)[8] if  $cl(A) \leq U$  whenever  $A \leq U$  and  $U$  is fuzzy regular open set in  $X$ .

- vii) fuzzy weakly closed (briefly, fw-closed)[6] if  $\text{cl}(A) \leq U$  whenever  $A \leq U$  and  $U$  is fuzzy semi-open set in  $X$ .
- viii) fuzzy weakly generalized closed (briefly, fwg-closed)[1] if  $\text{cl}(\text{int}(A)) \leq U$  whenever  $A \leq U$  and  $U$  is fuzzy open set in  $X$ .
- ix) fuzzy  $\pi$ -generalized closed (briefly,  $\pi\text{rg}$ -closed)[1] if  $\text{cl}(A) \leq U$  whenever  $A \leq U$  where  $U$  is fuzzy  $\pi$ -open set in  $X$ .
- x) fuzzy regular weakly generalized closed (briefly, frwg-closed)[7] if  $\text{cl}(\text{int}(A)) \leq U$  whenever  $A \leq U$  and  $U$  is fuzzy regular open set in  $X$ .
- xi) fuzzy  $\#$ regular generalized closed (briefly,  $\# \text{rg}$ -closed)[13] if  $\text{cl}(A) \leq U$  whenever  $A \leq U$  and  $U$  is fuzzy  $\text{rw}$ -open in  $X$ .

The complements of the above mentioned fuzzy open sets are called their respective fuzzy closed sets. From above generalized fuzzy set definitions and their fundamental results, we have the following lemma.

**Lemma 2.2** The following results are very precious to conduct the further study;

- i) Every fuzzy closed set is fuzzy  $g^*$ -closed set but not conversely[10].
- ii) Every fuzzy regular closed set is fuzzy closed set but not conversely[5].
- iii) Every fuzzy  $\pi$ -closed set is fuzzy closed set but not conversely[1].
- iv) Every fuzzy closed set is fuzzy  $\# \text{rg}$ -closed set but not conversely[13].
- v) Every fuzzy  $\# \text{rg}$ -open set is fuzzy  $g$ -open set but not conversely[13].
- vi) Every fuzzy  $g$ -closed set is fuzzy  $wg$ -closed set but not conversely[1].
- vii) Every fuzzy  $\text{rg}$ -closed set is fuzzy  $\text{rwg}$ -closed set but not conversely[7].
- viii) Every fuzzy  $g$ -closed set is fuzzy  $gs$ -closed set but not conversely[9].
- ix) Every fuzzy  $\pi$ -open set is fuzzy open set but not conversely[1].

**Fuzzy generalized  $\# \text{rg}$ -closed sets** (briefly  $\text{Fg}\# \text{rg}$ -closed sets):

**Definition 3.1** A fuzzy subset  $A$  of a fuzzy space  $X$  is said to be fuzzy generalized  $\# \text{rg}$ -closed (briefly,  $\text{Fg}\# \text{rg}$ -closed) if  $\text{cl}(A) \leq U$  whenever  $A \leq U$  and  $U$  is fuzzy  $\# \text{rg}$ -open set in  $X$ .

We denote the collection of all  $\text{FG}\# \text{rg}$ -closed sets in  $X$  by  $\text{FG}\# \text{RGC}(X)$ .

Firstly, we have to prove the class of fuzzy  $G\# \text{rg}$ -closed sets lies between classes of fuzzy  $g^*$ -closed sets and fuzzy  $\text{rg}$ -closed sets in fuzzy spaces.

**Theorem 3.2** Every strongly fuzzy generalized closed (briefly,  $\text{fg}^*$ -closed) set in fuzzy space  $X$  is fuzzy  $G\# \text{rg}$ -closed set in space  $X$ , but not conversely.

*Proof.* Let  $A$  be a fuzzy  $g^*$ -closed set in fuzzy space  $X$ . Suppose  $U$  is fuzzy  $\# \text{rg}$ -open set in fuzzy space  $X$  such that  $A \leq U$ . Since lemma 2.2(v) and by the definition of fuzzy  $g^*$ -closed set we have  $\text{cl}(A) \leq U$ . Therefore  $\text{cl}(A) \leq U$  whenever  $A \leq U$  and  $U$  is fuzzy  $\# \text{rg}$ -open set in fuzzy space  $X$ . Hence  $A$  is fuzzy  $G\# \text{rg}$ -closed set.

**Example 3.3** Let  $X = \{a, b, c, d, e\}$  and fuzzy subsets are  $0_x = \{(a, 0), (b, 0), (c, 0), (d, 0), (e, 0)\} = 0$ ,  $\alpha_1 = \{(a, 1), (b, 0), (c, 0), (d, 0), (e, 0)\}$ ,  $\alpha_2 = \{(a, 0), (b, 0), (c, 0), (d, 1), (e, 1)\}$ ,  $\alpha_3 = \{(a, 1), (b, 0), (c, 0), (d, 1), (e, 1)\}$  and  $1_x = \{(a, 1), (b, 1), (c, 1), (d, 1), (e, 1)\} = 1$ . Then fuzzy topology of  $X$  is  $T = \{0_x, \alpha_1, \alpha_2, \alpha_3, 1_x\}$ . The fuzzy subsets  $\beta_1 = \{(a, 1), (b, 1), (c, 0), (d, 1), (e, 0)\}$ ,  $\beta_2 = \{(a, 1), (b, 1), (c, 0), (d, 0), (e, 1)\}$ ,  $\beta_3 = \{(a, 1), (b, 0), (c, 1), (d, 1), (e, 0)\}$ ,  $\beta_4 = \{(a, 1), (b, 0), (c, 1), (d, 0), (e, 1)\}$ ,  $\beta_5 = \{(a, 1), (b, 1), (c, 0), (d, 1), (e, 1)\}$ ,  $\beta_5 = \{(a, 1), (b, 0), (c, 1), (d, 1), (e, 1)\}$  are fuzzy  $G\# \text{rg}$ -closed sets but not fuzzy  $g^*$ -closed sets in  $X$ .

**Theorem 3.4** Every fuzzy  $G\# \text{rg}$ -closed set is fuzzy  $\text{rg}$ -closed set in fuzzy space  $X$ , but not conversely.

*Proof.* Let  $A$  be an arbitrary fuzzy  $G\# \text{rg}$ -closed set in fuzzy space  $X$ . Suppose  $U$  is any fuzzy regular open set in fuzzy space  $X$  such that  $A \leq U$ . Since lemma 2.2(ii) and  $A$  is fuzzy  $G\# \text{rg}$ -closed set in fuzzy space  $X$ , it follows that  $\text{cl}(A) \leq U$  whenever  $A \leq U$  and  $U$  is fuzzy regular open in fuzzy space  $X$ . Hence  $A$  is fuzzy  $\text{rg}$ -closed set.

The converse of above theorem need not be true which can be shown in following example.

**Example 3.5** Let  $X = \{a, b, c, d, e\}$  and fuzzy subsets are  $0_x = \{(a, 0), (b, 0), (c, 0), (d, 0), (e, 0)\} = 0$ ,  $\alpha_1 = \{(a, 1), (b, 0), (c, 0), (d, 0), (e, 0)\}$ ,  $\alpha_2 = \{(a, 0), (b, 0), (c, 0), (d, 1), (e, 1)\}$ ,  $\alpha_3 = \{(a, 1), (b, 0), (c, 0), (d, 1), (e, 1)\}$  and  $1_x = \{(a, 1), (b, 1), (c, 1), (d, 1), (e, 1)\} = 1$ . Then fuzzy topology of  $X$  is  $T = \{0_x, \alpha_1, \alpha_2, \alpha_3, 1_x\}$ . The fuzzy subsets  $\beta_1 = \{(a, 1), (b, 0), (c, 0), (d, 1), (e, 1)\}$ ,  $\beta_2 = \{(a, 0), (b, 1), (c, 0), (d, 0), (e, 1)\}$ ,  $\beta_3 = \{(a, 0), (b, 0), (c, 1), (d, 1), (e, 1)\}$ , are fuzzy rg-closed sets but not fuzzy G#rg-closed sets in fuzzy space  $X$ .

**Remark 3.6** Lemma 2.2(i) and also from Theorem 3.2, every fuzzy  $g^*$ -closed set is fuzzy G#rg-closed set but not conversely. Hence every fuzzy closed set is fuzzy G#rg-closed set but not conversely.

**Remark 3.7** Lemma 2.2(ii). Also from Remark 3.6, every fuzzy closed set is fuzzy G#rg-closed set but not conversely. Hence every fuzzy regular closed set is fuzzy G#rg-closed set but not conversely.

**Remark 3.8** Lemma 2.2(iii). Also from Remark 3.6, every fuzzy closed set is fuzzy G#rg-closed set but not conversely. Hence every fuzzy  $\pi$ -closed set is fuzzy G#rg-closed set but not conversely.

The converse of above remarks need not be true which can be shown in following example.

**Example 3.9** Let  $X = \{a, b, c\}$  and fuzzy subsets are  $0_x = \{(a, 0), (b, 0), (c, 0)\} = 0$ ,  $\alpha_1 = \{(a, 1), (b, 0), (c, 0)\}$ ,  $\alpha_2 = \{(a, 1), (b, 1), (c, 0)\}$  and  $1_x = \{(a, 1), (b, 1), (c, 1)\} = 1$ . Then fuzzy topology of  $X$  is  $T = \{0_x, \alpha_1, \alpha_2, 1_x\}$ . The fuzzy subsets  $\beta_1 = \{(a, 1), (b, 0), (c, 1)\}$  is fuzzy G#rg-closed set but not fuzzy closed sets, fuzzy regular closed sets and fuzzy  $\pi$ -closed sets in fuzzy space  $X$ .

**Theorem 3.10** Every fuzzy G#rg-closed set in  $X$  is fuzzy  $g$ -closed set in  $X$ , but not conversely.

*Proof.* Let  $A$  be an arbitrary fuzzy G#rg-closed set in  $X$ . Suppose  $U$  is any fuzzy open set in  $X$  such that  $A \leq U$ . Since lemma 2.2(iv) and  $A$  is fuzzy G#rg-closed set in  $X$ , it follows that  $cl(A) \leq U$  whenever  $A \leq U$  and  $U$  is fuzzy open set in fuzzy space  $X$ . Hence  $A$  is fuzzy  $g$ -closed set.

**Remark 3.11** From Theorem 3.10, every fuzzy G#rg-closed set is fuzzy  $g$ -closed set but not conversely and from lemma 2.2(vi). Hence every fuzzy G#rg-closed set is fuzzy  $wg$ -closed set but not conversely.

**Remark 3.12** From Theorem 3.4, every fuzzy G#rg-closed set is fuzzy rg-closed set but not conversely and from lemma 2.2(vii). Hence every fuzzy G#rg-closed set is  $rwg$ -closed set but not conversely.

**Remark 3.13** From Theorem 3.11, every fuzzy G#rg-closed set is fuzzy  $g$ -closed set but not conversely and from lemma 2.2(viii). Hence every fuzzy G#rg-closed set is fuzzy  $gs$ -closed set but not conversely.

**Theorem 3.14** Every fuzzy G#rg-closed set in  $X$  is fuzzy  $\pi g$ -closed set in fuzzy space  $X$ , but not conversely.

*Proof.* Let  $A$  be an arbitrary fuzzy G#rg-closed set in fuzzy space  $X$ . Suppose  $U$  is any fuzzy  $\pi$ -open set in  $X$  such that  $A \leq U$ . Since every fuzzy  $\pi$ -open set is fuzzy open set and every fuzzy open set is fuzzy #rg-open set and  $A$  is fuzzy G#rg-closed set in fuzzy space  $X$ . It follows that  $cl(A) \leq U$  whenever  $A \leq U$  and  $U$  is fuzzy  $\pi$ -open in space  $X$ . Hence  $A$  is fuzzy  $\pi g$ -closed set.

The converse of above theorem need not be true which can be shown in following example.

**Example 3.15** Let  $X = \{a, b, c, d\}$  and fuzzy subsets are  $0_x = \{(a, 0), (b, 0), (c, 0), (d, 0)\} = 0$ ,  $\alpha_1 = \{(a, 1), (b, 0), (c, 0), (d, 0)\}$ ,  $\alpha_2 = \{(a, 1), (b, 1), (c, 0), (d, 0)\}$ ,  $\alpha_3 = \{(a, 1), (b, 1), (c, 1), (d, 0)\}$  and  $1_x = \{(a, 1), (b, 1), (c, 1), (d, 1)\} = 1$ . Then fuzzy topology of  $X$  is  $T = \{0_x, \alpha_1, \alpha_2, \alpha_3, 1_x\}$ . The fuzzy subsets  $\beta_1 = \{(a, 0), (b, 0), (c, 1), (d, 0)\}$ ,  $\beta_2 = \{(a, 1), (b, 0), (c, 1), (d, 0)\}$ ,  $\beta_3 = \{(a, 0), (b, 1), (c, 1), (d, 0)\}$ ,  $\beta_4 = \{(a, 1), (b, 1), (c, 1), (d, 0)\}$  are fuzzy  $\pi g$ -closed sets but not fuzzy G#rg-closed sets in  $X$ .

**Remark 3.16** Fuzzy G#rg-closed sets and fuzzy weakly closed sets are independent as shown from following examples.

**Example 3.17** Let  $X = \{a, b, c\}$  and fuzzy subsets are  $0_x = \{(a, 0), (b, 0), (c, 0)\} = 0$ ,  $\alpha_1 = \{(a, 1), (b, 0), (c, 0)\}$ ,  $\alpha_2 = \{(a, 0), (b, 1), (c, 1)\}$ ,  $\alpha_3 = \{(a, 1), (b, 0), (c, 1)\}$  and  $1_x = \{(a, 1), (b, 1), (c, 1)\} = 1$ . Then fuzzy topology of  $X$  is  $T = \{0_x, \alpha_1, \alpha_2, \alpha_3, 1_x\}$ . The fuzzy subset  $\beta_1 = \{(a, 0), (b, 0), (c, 1)\}$  is fuzzy  $wg$ -closed set but not fuzzy G#rg-

closed set in fuzzy space  $X$ , let  $U = \{0_x, \alpha_1, \alpha_3, 1_x\}$  then  $\beta_2 = \{(a, 1), (b, 1), (c, 0)\}$  is fuzzy  $G\#rg$ -closed set but not fuzzy  $w$ -closed sets in fuzzy space  $X$ .

**Remark 3.18** Fuzzy  $G\#rg$ -closed sets and fuzzy pre-closed sets are independent as shown from following examples.

**Example 3.19** Let  $X = \{a, b, c\}$  and fuzzy subsets are  $0_x = \{(a, 0), (b, 0), (c, 0)\} = 0$ ,  $\alpha_1 = \{(a, 1), (b, 0), (c, 0)\}$ ,  $\alpha_2 = \{(a, 1), (b, 0), (c, 1)\}$  and  $1_x = \{(a, 1), (b, 1), (c, 1)\} = 1$ , then fuzzy topology of  $X$  is  $T = \{0_x, \alpha_1, \alpha_2, 1_x\}$ . The fuzzy subset  $\beta_1 = \{(a, 0), (b, 0), (c, 1)\}$  is fuzzy pre-closed set but not fuzzy  $G\#rg$ -closed set in fuzzy space  $X$ , let  $U = \{0_x, \alpha_1, 1_x\}$  then  $\beta_1 = \{(a, 1), (b, 1), (c, 0)\}$ ,  $\beta_2 = \{(a, 1), (b, 0), (c, 1)\}$  is fuzzy  $G\#rg$ -closed set but not fuzzy pre-closed sets in fuzzy space  $X$ .

**Remark 3.20** Fuzzy  $G\#rg$ -closed sets and fuzzy  $\#rg$ -closed sets are independent each other.

**Theorem 3.21** The union of two fuzzy  $G\#rg$ -closed subsets of fuzzy space  $X$  is always fuzzy  $G\#rg$ -closed set in fuzzy space  $X$ .

*Proof.* Let  $A$  and  $B$  are fuzzy  $G\#rg$ -closed sets in fuzzy space  $X$ . Consider  $U$  be any fuzzy  $\#rg$ -open set in  $X$  such that  $A \vee B \leq U$  i.e.  $A \leq U$  and  $B \leq U$ . Since  $A$  and  $B$  are fuzzy  $G\#rg$ -closed sets, then  $cl(A) \leq U$  and  $cl(B) \leq U$ . Hence  $cl(A \vee B) = cl(A) \vee cl(B) \leq U$  Implies  $cl(A \vee B) \leq U$  whenever  $U$  is fuzzy  $\#rg$ -open set. Therefore  $A \vee B$  is fuzzy  $G\#rg$ -closed set in fuzzy space  $X$ .

**Remark 3.22** The intersection of two fuzzy  $G\#rg$ -closed subsets of fuzzy space  $X$  need not be fuzzy  $G\#rg$ -closed set in fuzzy space  $X$ .

**Example 3.23** Let  $X = \{a, b, c\}$  and fuzzy subsets are  $0_x = \{(a, 0), (b, 0), (c, 0)\} = 0$ ,  $\alpha_1 = \{(a, 1), (b, 0), (c, 0)\}$  and  $1_x = \{(a, 1), (b, 1), (c, 1)\} = 1$ , then fuzzy topology of  $X$  is  $T = \{0_x, \alpha_1, 1_x\}$ . The fuzzy subset  $\beta_1 = \{(a, 0), (b, 1), (c, 1)\}$  and  $\beta_2 = \{(a, 1), (b, 0), (c, 1)\}$  be two fuzzy  $G\#rg$ -closed subsets of fuzzy space  $X$ . But  $\beta_1 \wedge \beta_2 = \{(a, 0), (b, 0), (c, 1)\}$  which is not contained in fuzzy  $G\#rg$ -closed set in fuzzy space  $X$ . Hence fuzzy intersection of two fuzzy  $G\#rg$ -closed sets is not always fuzzy  $G\#rg$ -closed set in fuzzy space  $X$ .

**Theorem 3.24** If a fuzzy subset  $A$  is fuzzy  $G\#rg$ -closed in fuzzy space  $X$  and  $cl(A) \wedge (1 - cl(A)) = 0_x$ , then  $cl(A) - A$  does not contain any non-zero fuzzy  $\#rg$ -closed set in fuzzy space  $X$ .

*Proof.* On contrary, suppose that  $A$  is fuzzy  $G\#rg$ -closed set in fuzzy space  $X$ . Let  $F$  be any fuzzy  $\#rg$ -closed subset of  $cl(A) - A$ . Then  $F \leq cl(A) - A$  implies that  $F \leq cl(A) \wedge (1 - A) \leq 1 - A$  and so  $A \leq 1 - F$ . But  $1 - F$  is fuzzy  $\#rg$ -open set, then  $cl(A) \leq 1 - F$  that implies  $F \leq 1 - cl(A)$  and also  $F \leq cl(A)$ , it follows that  $F \leq cl(A) \wedge (1 - cl(A)) = 0_x$ . Thus  $F = 0_x$ . Therefore  $cl(A) - A$  does not contain a non-zero fuzzy  $\#rg$ -closed set in fuzzy space  $X$ .

**Example 3.25** If  $cl(A) - A$  contains no non-zero fuzzy  $\#rg$ -closed subset in fuzzy space  $X$ , then  $A$  need not be fuzzy  $G\#rg$ -closed set. Let  $X = \{a, b, c, d\}$  and fuzzy subsets are  $0_x = \{(a, 0), (b, 0), (c, 0), (d, 0)\} = 0$ ,  $\alpha_1 = \{(a, 1), (b, 0), (c, 0), (d, 0)\}$ ,  $\alpha_2 = \{(a, 0), (b, 1), (c, 1), (d, 0)\}$ ,  $\alpha_3 = \{(a, 1), (b, 1), (c, 1), (d, 0)\}$  and  $1_x = \{(a, 1), (b, 1), (c, 1), (d, 1)\} = 1$ . Then fuzzy topology of  $X$  is  $T = \{0_x, \alpha_1, \alpha_2, \alpha_3, 1_x\}$ . The fuzzy subset  $\beta = \{(a, 0), (b, 0), (c, 1), (d, 0)\}$  then  $cl(\beta) - \beta = \{(a, 0), (b, 1), (c, 0), (d, 1)\}$  does not contain any non-zero fuzzy  $\#rg$ -closed set, but  $A$  is not a fuzzy  $G\#rg$ -closed set in fuzzy space  $X$ .

**Corollary 3.26** If fuzzy subset  $A$  of fuzzy space  $X$  is a fuzzy  $G\#rg$ -closed set in  $X$  and  $cl(A) \wedge (1 - cl(A)) = 0_x$ , then  $cl(A) - A$  does not contain any non-zero fuzzy regular open set in fuzzy space  $X$  but not conversely.

*Proof.* Follows from Theorem 3.24 and the fact that every fuzzy regular open set is fuzzy  $\#rg$ -open in fuzzy space  $X$ .

**Theorem 3.27** Let  $A$  be fuzzy  $G\#rg$ -closed set, then  $A$  is fuzzy closed set if and only if  $cl(A) - A$  is fuzzy  $\#rg$ -closed set.

*Proof.* Let  $A$  is fuzzy  $G\#rg$ -closed set in fuzzy space  $X$ , then  $cl(A) - A = A$  and so  $cl(A) - A = 0_x$  which is fuzzy  $\#rg$ -closed set.

Conversely, suppose that  $\text{cl}(A) - A$  is fuzzy  $\#rg$ -closed set. Since  $A$  is fuzzy  $G\#rg$ -closed, then by Theorem 3.24,  $\text{cl}(A) - A = 0_x$  that is  $\text{cl}(A) = A$  and hence  $A$  is fuzzy closed set.

**Theorem 3.28** If  $A$  is fuzzy  $G\#rg$ -closed subset of fuzzy space  $X$  such that  $A \leq B \leq \text{cl}(A)$ , then  $B$  is always fuzzy  $G\#rg$ -closed set in  $X$ .

*Proof.* Let  $A$  be fuzzy  $G\#rg$ -closed set in fuzzy space  $X$  such that  $A \leq B \leq \text{cl}(A)$ . If  $A \leq B \leq \text{cl}(A)$  implies that  $\text{cl}(A) \leq \text{cl}(B) \leq \text{cl}(\text{cl}(A))$  that implies  $\text{cl}(A) \leq \text{cl}(B) \leq \text{cl}(A)$  that is  $\text{cl}(A) = \text{cl}(B)$ . Let  $U$  is fuzzy  $\#rg$ -open set in fuzzy space  $X$  such that  $B \leq U$  then  $A \leq U$  because  $A \leq B$ . Since  $A$  is fuzzy  $G\#rg$ -closed set,  $\text{cl}(A) \leq U$  that implies  $\text{cl}(B) \leq U$  where  $U$  is fuzzy  $\#rg$ -open set in fuzzy space  $X$ . Therefore,  $B$  is fuzzy  $G\#rg$ -closed set in fuzzy space  $X$ .

The converse of the above Theorem need not be true as seen from the following example.

**Example 3.29** Let  $X = \{a, b, c, d\}$  and fuzzy subsets are  $0_x = \{(a, 0), (b, 0), (c, 0), (d, 0)\} = 0$ ,  $\alpha_1 = \{(a, 1), (b, 0), (c, 0), (d, 0)\}$ ,  $\alpha_2 = \{(a, 0), (b, 1), (c, 1), (d, 0)\}$ ,  $\alpha_3 = \{(a, 1), (b, 1), (c, 1), (d, 0)\}$  and  $1_x = \{(a, 1), (b, 1), (c, 1), (d, 1)\} = 1$ . Then fuzzy topology of  $X$  is  $T = \{0_x, \alpha_1, \alpha_2, \alpha_3, 1_x\}$ . The fuzzy subset  $\beta_1 = \{(a, 0), (b, 0), (c, 0), (d, 1)\}$  and  $\beta_2 = \{(a, 0), (b, 0), (c, 1), (d, 1)\}$ , then  $\beta_1$  and  $\beta_2$  are fuzzy  $G\#rg$ -closed sets in fuzzy space  $X$ , but  $\beta_1 \leq \beta_2$  is not fuzzy subset in  $\text{cl}(\beta_1) = \{(a, 0), (b, 0), (c, 0), (d, 1)\}$ .

**Theorem 3.30** In a fuzzy topological space  $X$ , if fuzzy  $\#rg$ -open sets of  $X$  are  $\{1_x, 0_x\}$ , then every fuzzy subset of  $X$  is a fuzzy  $G\#rg$ -closed set.

*Proof.* Let  $X$  is a fuzzy space and  $F\#RGO(X) = \{1_x, 0_x\}$ . Suppose  $A$  be any arbitrary fuzzy subset of  $X$ , if  $A = 0_x$ , then  $1_x$  is fuzzy  $G\#rg$ -closed set in fuzzy space  $X$ . If  $A \neq 0_x$  then  $1_x$  is the only fuzzy  $\#rg$ -open set containing  $A$  and so  $\text{cl}(A) \leq 1_x$ . Hence by definition  $A$  is fuzzy  $G\#rg$ -closed set in fuzzy space  $X$ .

**Theorem 3.31** If a fuzzy subset  $A$  of a fuzzy space  $X$  is both fuzzy open and fuzzy  $G\#rg$ -closed set, then it is fuzzy closed set in fuzzy space  $X$ .

*Proof.* Suppose a fuzzy subset  $A$  of a fuzzy space  $X$  is fuzzy open and fuzzy  $G\#rg$ -closed set. Since every fuzzy open set is fuzzy  $\#rg$ -open set in fuzzy space  $X$ . Now  $A \leq A$  and by definition of fuzzy  $G\#rg$ -closed set,  $\text{cl}(A) \leq A$  and also  $A \leq \text{cl}(A)$  then we have  $\text{cl}(A) = A$ . Hence  $A$  is fuzzy closed in  $X$ .

**Corollary 3.32** If a fuzzy subset  $A$  of a fuzzy topological space  $X$  is both fuzzy  $\#rg$ -open and fuzzy  $G\#rg$ -closed set, then it is fuzzy closed in fuzzy space  $X$ .

**Corollary 3.33** If  $A$  be fuzzy open and fuzzy  $G\#rg$ -closed set in fuzzy space  $X$ . Suppose that  $F$  is fuzzy closed in fuzzy space  $X$ , then  $A \wedge F$  is fuzzy  $G\#rg$ -closed set.

*Proof.* Let  $A$  is fuzzy open and fuzzy  $G\#rg$ -closed set in fuzzy space  $X$ , by Theorem 3.41  $A$  is fuzzy closed set. Since  $F$  is fuzzy closed set in fuzzy space  $X$  and every fuzzy closed set is fuzzy  $G\#rg$ -closed set i.e. both  $A$  and  $F$  are fuzzy closed, then  $A \wedge F$  is also fuzzy closed set in  $X$ .

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