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Study of Core Methodologies and their Modified Versions for Image Segmentation

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Abstract: - The field of image segmentation basically concerns with the dividing image into its constituent objects. It is a widely studied field so a lot of research have been conducted and many methods are proposed to provide solution to this problem. Some of them are novel and propose a new technique and others are the modification to improve the efficiency of the methods proposed in past. This paper discusses the core methodology proposed by various researchers. The main purpose is to study the key mathematical equations and main steps that drives the different methods used in the field of image segmentation. The core algorithms their main mathematical foundation from the papers and literature of the various researchers have been put forth in this paper to look into them on a single platform. For that different papers have been surveyed in which some of them give the novel idea and other are the efficient extension of those novel ideas.

Keywords*:* Image segmentation, image object, interactive image segmentation, threshold

1. Introduction

In the field of image processing image segmentation is widely used because of its various usages and applications. Images can be thought as the collection of different objects taken together. To separate these objects from one another we require image segmentation, basically image segmentation is the process of dividing image into separate objects on the basis of some key property such that a given pixel of image can be classified to one region to which its proximity is most [1][2]. The general idea of dividing image into region follows that the particular region consist of homogeneous pixels and adjacent regions have different pixel value. One pixel must belong to a particular region such that, R_i intersection with R_j is NULL.

When solving the problem of image segmentation some question always arise, like while portioning an image what is the precise criterion for the good partition and what can be the efficient way to compute the partition .

The most generic methods [3] used for segmentation are based on threshold. The focus is to find a point in image which can divide the image into two parts mainly called as the foreground and background. Many algorithms have been proposed on this strategy would be discussed later, the main concern of most of the methods is to find an optimal threshold such that the division of the image can be done properly according to the interest. But many problems incur due to reasons like a) No optimal pixel value exist to use as threshold. b) Sometime single threshold value is not sufficient, so we require more than one threshold.

The region base segmentation is also quite popular in which we try to maximize the intra region relation of the pixels and minimize the inter region relation. The region based segmentation methods basically requires seed pixels to fetch the region of interest. These algorithm are very useful when some particular object from image is of interest and we only want that particular region to be fetched.

Most of the image segmentation methods are interaction based i.e. they require some point of interaction with the image [4]. This interaction can be in the form of seed input or the region of interest bases or ROI. The image segmentation algorithms are broadly classified as region based, contour based, Graph Cut based methods, Region growing/merging, Random walk based, super pixel based and many more. Some of these method are highly computing intensive but with the help of modern computing power and the parallel algorithm implementation the task of segmentation has become attainable at most of the point.

The content of the paper is mainly dedicated in the discussion of the image segmentation methods based on the concept of clustering, thresholding, region growing, entropy and other popular methods for handling segmentation. With these widely used methods we also review and discuss the other methods used by various researchers based on these techniques.

(a) Original Images **(b) Ground truth images**

Figure 1 To show the human annotated ground truth segmentation

The above figure 1 shows some of the images labelled with the ground truth [101] (Human annotations) to see the idea of segmentation. These can also be used as the benchmark to test the result obtained from the different segmentation algorithms.

2. Related Work

2.1 Otsu Image Segmentation (Thresholding):

Otsu algorithm is basically dividing the image into two parts ie in foreground and background on the basis of threshold [4]. The idea of otsu is to find an optimal threshold by assigning intensity level to a certain class such that the inter class variance is maximized. This method take account of two parameters: the probability density function and of each class and the probability of occurrence of each class.

According to the method the classes are distinct with their intensity value if they are properly thresholded and it uses histogram based computation which can be thought of as 1-D array.

The intensity values in images can be divided into {0,1,2…………L-1} where L is the maximum grey level. Let the size of an image be M*N.

The frequency of the grey level with value i can be n_i(number of pixels with i intensity value). There can be $n_0, n_1 \ldots\ldots\ldots n_L$ such values such that

$M*N = n_0 + n_1 + \ldots\ldots\ldots n_{L-1.}$ (1)

The probability of i value pixel can be calculated as $p_i = n_i/(M*N)$ (2)

Given that
$\sum_{i=0}^{L-1} p_i = 1, p_{i>=0}$ (3)

Let select the threshold T(k)=k 0<k <L-1 which divide the image into two classes C_1 and C_2, where C_1 have [0,k] and C_2 contains [k+1,L-1]

The cumulative probability that a pixel belong to class C_1 is $P_1(k)$ which is given by

$P1(k) = \sum_{i=0}^{k} p_i$ (4)

Similarly for class C_2

$P_2(k) = \sum_{i=k+1}^{L-1} p_i = (1 - P_1(k))$ (5)

The Mean gray value of the pixel for class C_1 is

$m_1(k)=\sum_{i=0}^{k} ip(i/C_i)$

$=\sum_{i=0}^{k} ip(C_1\ /i)p(i)/p(C_1)$ (6)

$= \frac{1}{P_1(k)} \sum_{i=0}^{k} ip(i)$ (7)

$p(C_1/i)$ is the probability of *C_1* given i, it is 1 because the concentration is on the values of I from C_1

$p(i/C_1)$ is the probability of value i that it comes from *C_1*

$p(C_1)$ is the probability of class C_1 which is $P_1(k)$

similarly

$m_2(k)= \frac{1}{P_2(k)} \sum_{i=k+1}^{L-1} ip(i)$ (8)

the overall mean of whole image

$m_g=\sum_{i=0}^{L-1} ip(i)$ (9)

the above equations can be verified by

$P_1 m_1 + P_2 m_2 = m_g$ (10)

Where $P_1 + P_2 = 1$ (11)

To evaluate the effectiveness of the threshold at level k use the dimensionless metric

$\eta=\sigma^2_B/\sigma^2_G$ (12)

where σ^2_B is the between class variance

$\sigma^2_B = P_1(m_1 - m_g)^2 + P_2(m_2 - m_g)^2$ (13)

One dimensional thresholding approach was [5][6] considered for segmentation of image into foreground and background. The extension of this approach two dimensional thresholding Gong et al. [7] introduced in various literatures helps the better segmentation.

The 2D thresholding method gives better results for the images corrupted by the noise.

the idea of this approach is to take original image *f(x,y)* and a neighbourhood averaged image *a(x,y)*.

An image can be visualize as a function of f:NXN→D.consider a threshold point T ∈ D which divides the image

$$f_t(x,y) = \begin{cases} I_0 & if\ \ f(x,y) < T \\ I_1 & if\ \ f(x,y) \geq T \end{cases} \quad (14)$$

Let a(x,y) be the averaged image defined by the function

$$g(x,y) = \frac{1}{n^2} \sum_{i=-n/2}^{n/2} \sum_{j=-n/2}^{n/2} f(x+i, y+j) \quad (15)$$

Where n≤N

Then represent these values by pair [f(x,y),g(x,y)] *the o*riginal gray level and the averaged neighbourhood pair. Lets consider a vector pair(S,T) in the 2D space defined as a two dimensional threshold divides the image as

$$f_{S,T} = \begin{cases} I_0 & if\ \ f(x,y) < S\ and\ g(x,y) < T \\ I_1 & if\ \ f(x,y) \geq S\ and\ g(x,y) \geq T \end{cases} \quad (16)$$

Where $1 \leq I_0$,S,T and $I_1 \leq L$

Let $s_{i,j}$ be the frequency pair (i,j) where f(x,y)=i and g(x,y)=j $0 \leq s_{i,j} \leq N^2$ .then the join probability can be give by

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$p_{i,j} = s_{i,j} / N^2$, where i,j=0,1........L; (17)

$$\sum_{i=1}^{L}\sum_{j=1}^{L} p_{i,j} = 1 \qquad (18)$$

Now $p_{i,j}$ be the 2D histogram of the image Shown in figure 2

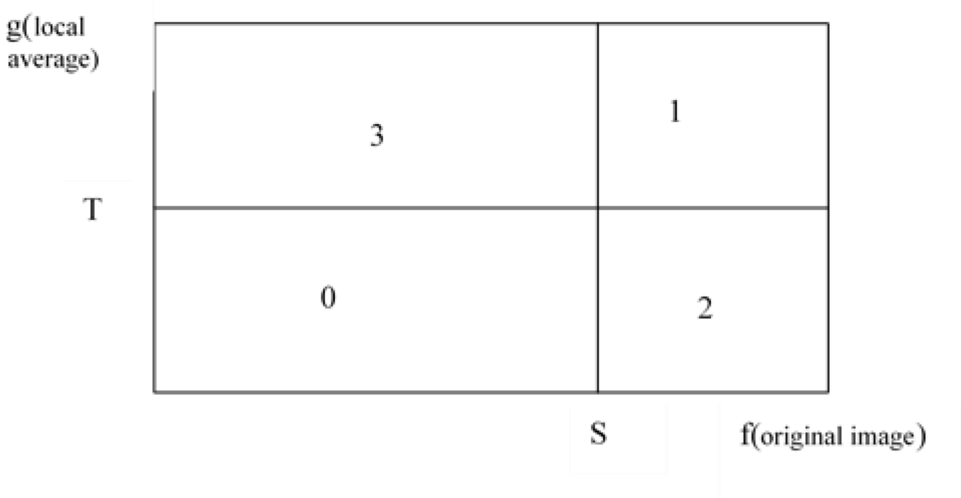


Figure 2 2D gray level histogram

From the above figure we can see that the the 2D space gets divided into four regions by S,T .The regions near the diagonal i.e. 0,1 is can be considered for background and foreground, the region away from the diagonal consist of the pixels either edges or the noise.

If L be the gray level in f and g then there will be L^2 values in the histogram. The thresholds are then optimizes to give a better result using two dimensional entropic thresholding. So the two dimensional thresholding method not only gives a better segmentation but also deals with hthe problem of noise by taking it into account. The [7] method also enhance the computation capability by using recursive structure from $O(L^4)$ [4] to $O(L^2)$.

The [8] tries to improve the 2D Otsu by using the genetic algorithm. The main aim of the paper was to optimize the 2D Otsu and enhance the computation time. This method uses the characteristics of genetic algorithms to optimize the threshold in 2D histogram .

Otsu Method [5] basically depend on between maximizing class variance but [9] uses an approach base on maximizing within class variance. The approach is to minimize wihin class variance which is somehow adopted from the statistical measure ANOVA.This method is can be considered as the variant of the otsu [2] method proposed earlier. The methodology of the algorithm consider a set of pixels $S=\{g_i$, $i=0,1,\ldots\ldots\ldots\ldots L$ $|g_0<g_1<\ldots\ldots\ldots\ldots g_L\}$, the frequency representation of the pixels is given by $\{h_i\}$ (histogram of image).

Take a pixel point g_k divides the set S into two classes C_1 and C_2 such that $C_1=(g_0\ldots\ldots\ldots\ldots g_k\}$ and $C_2=\{g_{k+1}\ldots\ldots\ldots\ldots\ldots\ldots g_L\}$ given that $C_1 \subset S$, $C_2 \subset S$,$C_1 \cup C_2= S$ and $C_1 \cap C_2 = \phi$.

For every partition

$$p_i = \begin{cases} \sum_{j=0}^{k} h_j & i=1 \\ \sum_{j=k+1}^{L} h_j & i=2 \end{cases} \qquad (19)$$

where p_i is the class probability.

The mean of each class is given by

$$\mu_i = \begin{cases} \sum_{j=0}^{k} (g_j h_i / p_1) & i = 1 \\ \sum_{j=k+1}^{L} (g_j h_i / p_2) & i = 2 \end{cases} \quad (20)$$

The class variance is defined as

$$\sigma_i = \begin{cases} \sum_{j=0}^{k} (g_j - \mu_1)^2 h_i / p_1) & i = 1 \\ \sum_{j=k+1}^{L} (g_j - \mu_2)^2 h_i / p_2) & i = 2 \end{cases} \quad (21)$$

The otsu [5][11] strategy of defining threshold is to minimize($\sigma_1 p_1 + \sigma_2 p_2$) whereas the approach used in [9] minimize the within class variance by minimizing($\sigma_1 + \sigma_2$) . The analyses presented in the paper is is done by shifting a pixel from one class to another and by evaluating how the pixel basically classified nearest to the centroid of the class from which it has low average distance.The major analysis of this research reveals that two factors class probability and class variance plays a crucial role in determining the threshold for the segmentation. The use of relative distance and average distance have given more accurate threshold than otsu as given in result.

A Novel method [10] which used two stage technique for the colour image segmentation was proposed based on coarse segmentation and delicate segmentation. Coarse segmentation as the first stage uses the refined otsu [5] algorithm. It first pre-process the image with Gaussian filter to add smoothness. The otsu [5] method use the variance as

$$\eta_o = \sigma_B^2(t) / \sigma_T^2 \quad (22)$$

Where $\sigma_B^2(t)$ is the between class variance and σ_T^2 is the total variance.

To achieve better result a new factor

$$\eta_c = (\mu_1 - \mu_0)/(\mu_h - \mu_l) \quad (23)$$

Where μ_1 is the average gray level of the high region and μ_0 is the average gray level of the low region. μ_h and μ_l defines the highest and lowest gray level in histogram. The idea is that higher the value of η_c the farther the two classes in separation. These two factors are not sufficient to take account of the overlapping of the classes for that a new parameter

$\eta_{area} = S_1 /(S_1 + S_2 + S_3)$ was introduced where η_{area} is the normalize classify area where S_1 is the area of P_1VP_2 S_2 and S_3 represent the area of region V_0VP_2 and V_0VP_1 respectively as shown in figure 2.

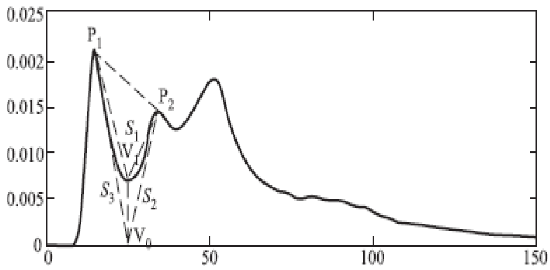


Figure 3: P_1,P_2 represents the peak and V represents the valley V_0 is the projection of V

If V_0 and V coincide then η_{area} is one which shows very low overlapping, if this value is high then classes are almost inseparable.

η_o and η_c represents the distance of two classes where η_{area} is the degree of separation.. The effective threshold is given by

$$\eta = 0.5 \times (\eta_o + \eta_c) \log_2 (1 + \eta_{area}) \qquad (24)$$

If η is closer or equal to one then the background and foreground are completely separable and if it is tending to zero the two classes are inseparable. By using the above threshold we can obtain the coarse segmentation after which we can obtain the contour of the object.

Now after the coarse segmentation is done after that the delicate segmentation is applied. The delicate segmentation is applied through narrow band techniques.first a mask of nxn is applied to dilate the contour obtained in the process of coarse segmentation, the value of n can be from 3 to 7.

After this procedure the image gets divided into three parts namely foreground, background and To-be-determined (TBD) region. Then a window of size w x w is used to scan the TBD region such that w > n so that it can comprise all the three classes. For every pixel calculate the average of foreground and background AF and AB respectively. Then calculate the Euler distance between the narrowband pixel and AF lets assume AIF and with AB let us assume AIB . for the better results a laxative parameter l_r is defined between 0 and 1. Now the status of the pixel is updated as

p= "object" if AIF < l_r X AIB

p= "background" if AIB < l_r X AIF

p= " uncertain" otherwise

in the above proposed method there is no need to define initial energy function, contours or labels for foreground or background. Evey time the second stage i.e. delicate segmentation is not required , only the coarse segmentation produces the efficient result.

2.2 Region Growing:

Region growing [4] plays a very important and key role in the field of image segmentation. The basic idea of region growing is to merge the pixels of the image together on the basis of some given criteria. While segmenting the image we require the homogeneous pixels to be together to form a connected region. The region grow does the same by choosing a seed pixel from an area and the start growing in the neighbourhood region.

The generic algorithm for region growing is

1. *Choose a seed pixel from the image*
2. *While the stopping criteria is not met*

a. *Check whether there exists the pixel which can merged with the existing region on the basis of some criteria(eg. Threshold)*

b. *Grow the region*

3. *End*

The neighbourhood condition while growing region is usually the eight-connectedness or the four connectedness of the current pixel.

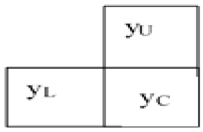
Some measures to be considered while using region growing

If we group the pixels into the region without taking care of the connectivity , it may lead to the wrong or misleading results

The stopping rule plays an important role in region growing, while growing the region by adding the neighbouring pixels on should clearly specify the stopping criteria of the algorithm.

A very famous algorithm [4] was proposed to find the different connected region in the image by providing different colour to every region considered as blob

The following algorithm considers three pixel point arranged in angular window



Where y_C is the current pixel, y_U is the upper pixel of current pixel, y_L is the left pixel of the current pixel.
The algorithm proceeds as follows

Let the initial color C=1, scan the image from left to right and top to bottom

1. *If f(y_C)=0 the continue*

a. *Else*

i.*if(f(y_U)=1 and f(y_L)=0)*

1. *colour (y_C)= colour(y_U)*

ii.*if (f(y_L)=1 and f(y_U)=0)*

1. *colour (y_C)= colour(y_L)*

iii.*if (f(y_L)=1 and f(y_U)=1)*

iv.*begin*

1. *colour (y_C)= colour(y_L)*
2. *colour(y_L) is equivalent to colour(y_U)*

v.*end*

vi.*if (f(y_L)=0 and f(y_U)=0)*

1. *colour y_L=C ; C=C+1*

b. *end*

A new approach for region growing[14] was proposed to find the boundary of the blobs in the image. This method uses a novel approach to define the discontinuity measures, average contrast and peripheral contrast .The process mentioned in the paper starts like any other process by choosing a starting point or seed pixel. Lets choose an arbitrary seed point, a boundary pixel is combined with current region if it has the highest grey level among the neighbouring region. By doing this the pixels with high gray levels will be absorbed first. After absorbing the all high gray level values the absorption of monotonically low and low grey level value pixels starts.

For the stopping condition of the algorithm the criteria size of region N is used which ca be simply measured by counting the number of pixels in the mapping. According to the condition the index i of the pixel must be less than N.

For defining average contrast and peripheral contrast some terminologies are defined. Current boundary(CB) is the set of pixels adjacent to the current region. Internal Boundary (IB) is the set of all pixels which forms the outermost part of the current region.

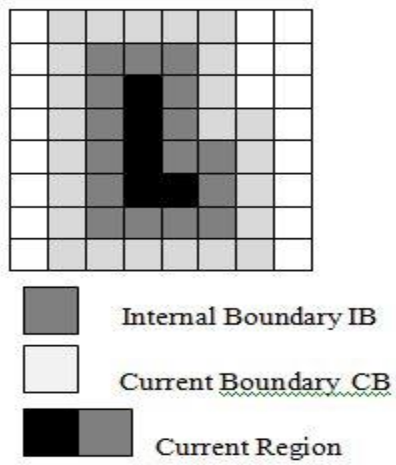


Figure 4: Figure to show the CB,IB, current region.

With these terminologies the average contrast is defined as for a region *i* it is the difference between the average of the region pixel and the CB.

$$c(i)=\frac{1}{i}\sum_{t=1}^{i}y_t-\frac{1}{k-i}\sum_{t=i+1}^{k}y_t \qquad (25)$$

Where y_1..........................y_i are the pixels from current region and y_{i+1}.......................y_k are the pixels from CB.

The region growing will give increasing average contrast value as long as the region gets high intensity values, when region starts growing in the background i.e. towards low intensity values the average contrast value starts decreasing. The result of this segmentation gives the average contrast boundary (ACB).

The peripheral contrast is defined as the difference between the average of IB and the average of CB which gives the average magnitude of the rate of change(gradient) of the pixels in CB for the growing region. This measure is less susceptible to the noise than using simple gradient function to find the difference between the two points. For a homogenous regions the maximal point can be obtained easily but for the noisy region there are more than one maxima. To overcome this problem last local maxima is considered before the ACB.
A new method for region growing [16] was proposed which was closed to the use of morphological watershed. The mechanism starts with the picking seed points which are grouped into n sets.The procedure starts with the seeds and form the sets A_1, A_2...........................A_n.. After allocating the pixels to one of the set some of the pixels remains unallocated.
Let T be the collection of all unallocated pixels.

$$T=\{x\notin\bigcup_{i=1}^{n}A_i \mid N(x)\cap A_i \qquad (26)$$

Where $N(x)\rightarrow\{$*Set of immediate neighbour of pixel x*$\}$.
The immediate neighbours are defined in 8-connectedness of the pixel x.
For x $\in$ T , $N(x)$ should meet with any one of the A_i. Then the pixel belongs to any index i , $i(x)\in\{1,2,3$.....................$n\}$ such that

$$N(x)\cap A_i\neq\phi \qquad (27)$$

A factor $\partial(x)$ was introduced to see how much x varies from its neighbours

$$\partial(x)=\left|\, g(x)-\underset{g\in A_i(x)}{means}[g(y)]\right| \qquad (28)$$

Where $g(x)$ is the gray level value of the image point x. If $n(x)$ meets with more than one region i.e. two or more A_i ,$i(x)$ is considered in such a way that $N(x)$ *meets* A_i and $\partial(x)$ is minimized. In these circumstances x can be added to the set of boundary pixels.
A method based use the colour space Y,C_b, C_r [17] was proposed to segment the images using region growing. This is done by first changing the R,G,B colour space to the Y,C_b, C_r and the seeds are selected to perform the segmentation using region growing.
Calculate the standard deviation in 3 x 3 region of Y,C_b, C_r

$$\sigma_x=\sqrt{\frac{1}{9}\sum_{i=1}^{9}(x_i-\bar{x})^2} \qquad (29)$$

Where x_i is the current pixel to consider
Total standard deviation is given by

$$\sigma=\sigma_x+\sigma_{Cb}+\sigma_{Cr} \qquad (30)$$

And the normalized standard deviation

$$\sigma_N=\sigma/\sigma_{\max} \qquad (31)$$

Where $\sigma_{\max}$ is the maximum standard deviation.
It defined a similarity factor of pixel with its neighbourhood

$$H=1-\sigma_N \qquad (31)$$

Condition1 :By using similarity first condition for a pixel to be seed pixel candidate
The H value should be greater than threshold
Then calculate the Euclidian distance in Y,C_b, C_r space of pixel to its eight connected pixels

$$d_i = \frac{\sqrt{(Y - Y_i)^2 + (C_b - C_{bi})^2 + (C_r - C_{ri})^2}}{\sqrt{Y^2 + C_b^2 + C_r^2}} \quad (32)$$

For $i=1,2..............8$,

$$d_{\max} = \max_{i=1}^{8}(d_i) \quad (33)$$

By using this the second condition for the see pixel is defined as
Condition 2: the Euclidian distance of seed pixel should be highest in neighbourhood less than threshold
Condition 1 checks the highest similarity to its neighbourhood and condition 2 checks that the pixel is not on the boundary .
For the selection of the optimal threshold otsu [5] method have been used.
The main steps of the algorithm are as follows

1) Convert the image from RGB to Y, C_b,C_r
2) Seeds ae selected using automatic seed selection method
3) Apply Region growing for the segmentation
4) Apply region merging wherever required, mainly to overcome over segmentation.

The idea of distributed region growing is proposed [50] for the image segmentation for the very large size images .This method is most suitable for the large size images captured through satellites through remote sensing. The main strategy to solve the problem is to divide the image into tiles and then apply the segmentation algorithm separately.

Working with the distributed region growing algorithm requires considering a lot of thins and not an easy implementation. When the image is divided into tiles then some mechanism is required to deal with the border of the tiles.

The problem is dealt by

1) Split the image into tiles and generate dataset which are independent

2) Segmentation algorithm is executed on each tile.

3) The neighbouring segment those touches the border are stitched efficiently

The strategy assumes that the pixels which are internal to the segments are less disturbed in comparison to those are present on borders.

The procedure of the image division is starts with tiling the images which is based on coordinate system independent of image boundaries. Two important benefit of this scheme is that firstly the same division method can be applied for different images covering the same area. Secondly it provides the reproducibility.

The division of image depend on the geographical grids known as geo cells. The topmost geo cell layer comprise of single cell i.e. whole image. The cell is the divided into four cells and each individual cell a again gets divided into four cells. This is done recursively unless desired size is not achieved. The label of each geo cell is unique which provide information about its position in hierarchical structure. Labelling is basically done on the basis of z-curve ZXWY. The W again gets divided into four quadrant and labelled as WW,WX,WY,WZ and so on.

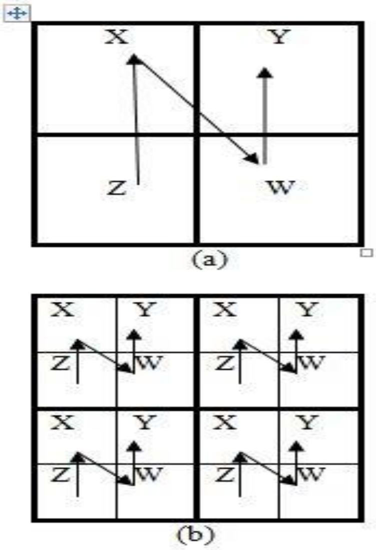


Figure 5 (a) z curve (b) Furthur partiotion of image for next level

After the division of the image ,the execution of segmentation method on each tile is possible which is also known as independent image segmentation. After this segmentation some post processing is required for the efficient results. Inner segments are assumed to be properly generated, so they can be easily filtered and can be stored in cloud repository. Te segments those touches the tile borders are required to post process to remove the artefacts.The basic strategy of post processing used is SPP(simple post processing) which allows the segments from adjacent tiles to merge together on the fulfill of some condition. This is simple merging process is not much efficient For the better efficiency the hierarchical post processing technique is used. In this technique the hierarchical level in internal segments are processed and stored. The segments those touches the tiles boundaries are grouped together for the processing. The grouping is performed on the basis of the geo cell position with respect to the upper level. The new label can be generated by removing the last letter for the label WXY→WX. This procedure reduce the artefact to a large extent. To improve the results more the Hierarchical Post processing with re-segmentation is used. The results on very large size image are shown .

In the paper [19] flooded region growing method have been emphasised. Region growing shows two few basics problems 1) high computational complexity 2) the algorithms are highly sequential which make them difficult to run on parallel system because the pixels are added sequentially 3) the performance of the algorithm degrades with the wear and blurry edges. The algorithm presents in the paper is based on the tissue like P system [20]. In the flooding region growing method all the pixel regions grows simultaneously like flood on the basis of feature like color, texture etc. The first stage of the algorithm started with assigning unique value to every pixel based on their index as every pixel is treated as separate membrane. After the similarity of the pixels are determined and the flooding of the region starts. The approach in this aper is to run the region growing method on the parallel system for that CUDA is used.

This paper [21] presents the variants of the seeded region growing. The approach of Linear seeded Region Growing is initially explained which talks about treating image as the combination of linear planes. After that the concept of Quadratic seeded region growing is introduces which consider the concept of quadratic surfaces. Original Seeded region growing methods do not take account of shape and boundary of the region which is handled in the Stabilised seeded region growing. Many improvements for the seeded region growing have been proposed over time [25-28] to improve the quality and the efficiency of this method. The method based on complex texture [105] appears to be suitable for dynamic scenes. The iterative boundary based system [106] computer to decide and limit the user input.

2.3 K-means Algorithm:

The k means algorithm [29][30] is used to divide the set of data into k clusters
Algorithm
Input:

- The initial k points chosen randomly to define initial clusters

* The set of all data points D

Output: Set of clusters which divided into k parts.

Method:

1) *Randomly choose k initial points from D as the initial centres for clusters*
2) *Repeat*

a. *Allocate each point to the cluster to which the point have maximum similarity on the basis of the mean value of points in cluster*

b. *The mean value of the cluster is updated every time*

3) *Until no change*

The square error criteria is given by

$$E = \sum_{i=1}^{k} \sum_{p \in C_i} | p - m_i |^2 \qquad (34)$$

Where E denotes the sum of square error, p represent the point in the space, m_i is the mean of i^{th} cluster C_i .

This paper [31] combines the matting with K means to improve the segmentation process. The basic idea of matting [32-34] is to combine foreground f_k and background b_k in some proportion

$$I_k = \alpha_k f_k + (1-\alpha_k) b_k \qquad (35)$$

α_k is the opacity of pixel range in *[0,1]* also known as alpha-matte.

The algorithm initially used two approach ,first use the k means segmentation and then apply closed form. The final result is produced by combining the both approach

Methodology

Apply the k means algorithm

a. *Take original image*

b. *K means segmentation*

c. *RGB to L* a* b**

d. *N-colour compositing*

e. *Generate n clusters*

2) *Closed form*

a. *Detect parameter(th_alpha,windowsize,level)*

b. *Segmentation from scribble*

c. *Background removal*

The segmentation approach proposed is the combination of various methods to produce an efficient segmentation results [35].

The local histogram is re quantized in $N_b= 5*5*5=125$ Descriptors bins, basically $N_b=q^3$ dividing the RGB colour cube into *125* small boxes.

N_x be the set of location x in the $N_x * N_x$ window centered at x.

h[ ] be an array of bin descriptor *[h[0],h[1]……………..h[N*$_b$*-1]]*

for every pixel belongs to N_x *with* R_x, G_x, B_x

$$k \leftarrow q^2 \lfloor q.R_x / 256 \rfloor + q.\lfloor q.G_x / 256 \rfloor + \lfloor q.B_x / 256 \rfloor \qquad (36)$$

$h[k]=h[k]+1/[N_w]^2$ (37)

where *h[ ] be the bin descriptor ranges from h[0]………h[*N_b*-1]*

After calculating the 125 bins K means algorithm [36]is applied to group them together

Segmentation map fusion.The color spaces {RGB,HSV,YIQ,LUV } are taken into account [37]

The segmentation map fusion is the procedure considered for combining the local histograms.

For the purpose of finding the similarity bhattacharya similarity coefficient has been considered.

A normalized histogram of an image is given by

$h\{(n,x)\}_{n=0\ldots\ldots\ldots\ldots Nb-1}$ at pixel location x

Refrence histogram $\{ h^*(n)\}_{n=0\ldots\ldots\ldots\ldots Nb-1}$ *repesent the other cluster to be compared*

Distance measure is given by

$$D_B(h^*, h(x)) = (1 - \sum_{n=0}^{N_b-1} \sqrt{h^*(n)h(n;x)})^{1/2} \quad (38)$$

To avoid the over segmentation the merging process is governed by

$$D_{merge} = \min_{x \in R}\{\sum_c D_B[h^0(n), h^t(n;x)]\} \quad (39)$$

Where $h^0(n)$ represents the normalized nonparametric histogram of pixels of the region which is to be merged $h^t(n;x)$ is the normalized histogram.

Fuzzy clustering method (FCM) based on multiple spatial information have been proposed to improve the efficiency of already used FCM [112].the paper [113] propose algorithm NCB_FCM based on FCM and make use of comentropy mainly for the suppression of speckle noise and edge preservation. The SLIC algorithm with the information of colour and mean distance information is used for the effective segmentation [114]. To improve the efficiency of traditional multiphase image segmentation method it is combined with the clustering for the optimization [115]. Fuzzy c means clustering combined with morphological reconstruction to handle noise problems [116]. The swarm optimization method combined with the clustering [117] as swarm optimization provides the capability of global optimization and power of parallel processing. The wavelet transform which provide the multi resolution aspect of the image combined with the clustering helps the speed up of the segmentation process and reduce complexity [118]. Mediodshift based clustering method have been used to find the fault area in the infrared images [119].

2.4 Snakes(Active Contour):

Active contour model [38]is defined as the energy minimization function driven by some external forces which causes it to moves towards the lines or edges. Snakes can be used to detect edges ,lines, contours and tracking motion.

Basic snake model is basically controlled by the image forces and external constrained forces [38][40]

Let the parameter of the snake *v(s)be (x(s),y(s))*

$$E^*_{snake} = \int_0^1 E_{snake}(v(s))ds \quad (40)$$

$$= \int_0^1 E_{\text{int}}(v(s))ds + E_{image}(v(s))ds + E_{con}(v(s))ds \quad (41)$$

E_{int} *is the Internal Energy of curve due to bending*

E_{image} *are the image forces*

E_{con} *be the External constrained force*

$$E_{\text{int}} = (\alpha(s)|v_s(s)|^2 + \beta(s)|v_{ss}(s)|^2)/2 \quad (42)$$

The first order term of the curve is controlled by $\alpha(s)$.

The second order makes the snake act like thin plate and then two can be adjusted using $\alpha(s)$ and $\beta(s)$.

For a particular image the task of the user is to put the snake somewhere near the feature, After getting close enough ,the energy minimization function will drive the snake all the way.

Image forces are the energy functions that drives the snake towards the features. This can be defined as the combination of three energy functions

$E_{image}=W_{line} E_{line} + W_{edge}E_{edge} + W_{term} E_{term}$ (43)

E_{line} is defined in terms of $I(x,y)$

$E_{line} = I(x,y)$

The E_{line} depends on the sign of the sign of W_{line} , the snake will mov either toward the light line or dark line

$E_{edge} = -|\nabla I(x,y)|^2$ then the snake will move towards the large image gradient.

For the smooth movement of the snake the extra smoothness is added to the image by applying gaussian filter

$$E_{line} = -(G_\sigma * \nabla^2 I)^2 \quad (44)$$

Where G_σ is the gaussian filter with standard deviation σ ,[2] the maximum of this lies in the zero crossing of $G_\sigma * \nabla^2 I$.

This energy is added to ensure that the snake moves towards the zero crossing .

$C(x,y) = G_\sigma(x*y)*I(x,y)$ be the smoothed image with gaussian filter.

$$E_{term} = \frac{C_{yy}C_x^2 - 2C_{xy}C_xC_y + C_{xx}C_y}{(C_x^2 + C_y^2)^{3/2}} \qquad (45)$$

By the combination of E_{edge} and E_{term} the snake cabe moved towards the edges or where the terminators lies. The main problem with the snake method is that it depends on the initial contour. In this paper [41] Hough Transform in the target area is applied for edge localization. Along with the Markov Random field the concept of snake is used in this paper [42].the concept of game theory is also applied with the above two models.

2.5 Geodesic distance:

The concept of Geodesic distance for the segmentation [41] labels the input data to forground *F* and background *B.* The distance is given by

$$D_l = \min_{s\epsilon\omega_l} d(s,x), l\epsilon\{F,B\} \qquad (46)$$

$$d(s_1,s_2) = \min_{C_{s_1,s_2}} \sum_{x,y} W_{xy} \qquad (47)$$

C_{s_1,s_2} is the cost of path from s_1 to s_2 . The weight W depends on the local gradient in neighborhood.

Then the concept of matting is applied for the soft segmentation. The paper[42] combines the concept of Geodesic distance with the edge information from the graph cut. The problem with only using Geodesic distance concept is that they are bias towards the seed which minimize the path. The method of seed growing incorporated with geodesic distance [44] helps in the better interactive image segmentation. The geodesic metric used with the Eikonel partial differential equation framework [45] is proposed for better edge feature and curvature regularization.

2.6 Entropy:

For the gray scale image segmentation thresholding based methods are widely used in various places. To find a optimal threshold has been always a challenging task. Many methods have been proposed by various researchers to solve the problem. Entropy bases methods based [46] on the randomness of the input have proven their effectiveness in deciding the threshold for the segmentation.

For an input image with n distinct gray levels , $p_1, p_2 \ldots\ldots p_n$ can be defined as the probability of each gray level. Now all these probabilities for the gray levels can be divided into two probabilities distribution $1\ldots\ldots s$ and $s+1\ldots\ldots n$.

$$A \rightarrow \frac{p_1}{P_s}, \frac{p_2}{P_s} \ldots\ldots.. \frac{p_s}{P_s}$$

$$B \rightarrow \frac{p_{s+1}}{1-P_s}, \frac{p_{s+2}}{1-P_s} \ldots\ldots\ldots \frac{p_n}{1-P_s} \qquad (48)$$

where

$$P_s = \sum_{i=1}^{s} p_i\,,\ \ 1 - P_s = \sum_{i=s+1}^{n} p_i \quad (49)$$

The entropies associated with the given distribution A,B

$$H(A) = -\sum_{i=1}^{s} \frac{p_i}{P_s} \ln \frac{p_i}{P_s} \qquad (50)$$

$$= -\frac{1}{P_s}\left[\sum_{i=1}^{s} p_i \ln p_i - \ln P_s\left(\sum_{i=1}^{s} p_i\right)\right] \qquad (51)$$

$$= -\frac{1}{P_s}[-H_s - P_s \ln P_s]\,, where\ H_s = -\sum_{i=1}^{s} p_i \qquad (52)$$

$$= \ln P_s + \frac{H_s}{P_s} \qquad (53)$$

Similarly for another distribution B,

$$H(B) = -\sum_{i=s+1}^{n} \frac{p_i}{1-P_s} \ln \frac{p_i}{1-P_s} \qquad (53)$$

Which can be evaluated same as above to

$= ln(1-P_s) + \frac{H_n - H_s}{1-P_s}, where\ H_n = -\sum_{i=s+1}^{n} p_i lnp_i$ (54)

Now consider $\varphi(s)$ be the sum of *H(A)* and *H(B)*

$\varphi(s) = \frac{H_s}{P_s} + lnP_s + ln(1-P_s) + \frac{H_n - H_s}{1-P_s}$ (55)

$\varphi(s) = \frac{H_s}{P_s} + lnP_s(1-P_s) + \frac{H_n - H_s}{1-P_s}$ (56)

In order to maximize the function for discrete for discrete value *s* considering it a uniform distribution,

Then, $P_s = \frac{s}{n}$ (57)

$\varphi(s) = ln\frac{s}{n}\left(1-\frac{s}{n}\right) + \frac{-s}{n}ln\frac{1}{n}\Big/\frac{s}{n} + \frac{-(n-s)}{n}ln\frac{1}{n}\Big/\frac{(n-s)}{n}$ (58)

Which evaluates to

$\varphi(s) = \ln(n-s)\, s$ (59)

This can be maximized or attain a maximum value at *s=(1/2)n* and distribution become symmetric i.e. *H(A)=H(B).*

Another Entropy based segmentation [47] derived from shanon entropy [46] which is defined for some discrete distribution p_i, $i=1\ldots\ldots N$ the entropy becomes

$H = -\sum_{i=1}^{N} p_i logp_i \ ,\sum_{i=1}^{N} p_i = 1$ (60)

According to the Boltzmann law with the given gray level g_i of pixel i the number of ways a image *I* can be created

$I(g_1, g_2 \ldots\ldots, g_N) = \frac{G!}{g_1!\ldots\ldots g_N!}$ (61)

Where *G* is the total pixels in image

The problem with this law is that the pixel of image can be arranged in any order which is not possible for a given image because the order of pixel in mage produce relevancy to image. To introduce this relevancy measure using variance in the neighborhood of N_3(3 X 3).

The entropy measures become

$H = -\sum_{i=1}^{N} p_i log\frac{p_i}{m_i}$ (62)

Where

$m_i = 1 + \sigma_i^2$ (63)

$\sigma_i^2 = \sum_{i\epsilon N_3} \frac{(g_i - \mu_{N_3})^2}{9}$ (64)

$p_i = \frac{g_i}{G}$ (65)

For the threshold selection by considering the image relevancy mentioned above

$H = -\sum_{g=0}^{n-1} f_g \frac{g}{G} log\frac{g}{G}$ (66)

Where

$G = \sum_{i=1}^{N} g_i = \sum_{g=0}^{n-1} g f_g$ (67)

G is the total illumination in the image, n is the number of gray-levels of given image and f_g provides the count of pixels with grey-level g.

Now to divide the image into two classes the entropy of the corresponding class

$$H_0(C) = -\sum_{i=1}^{N} \frac{g_i}{G_0(T)} log \frac{g_i}{G_0(T)} \qquad (68)$$

$$= -\sum_{g=0}^{T} f_g \frac{g}{G_0(T)} log \frac{g}{G_0(T)} \qquad (69)$$

$$H_1(T) = \sum_{i=1}^{N} \frac{g_i}{G_1(T)} log \frac{g_i}{G_1(T)} \qquad (70)$$

$$= -\sum_{g=T+1}^{n-1} f_g \frac{g}{G_1(T)} log \frac{g}{G_1(T)} \qquad (71)$$

Where

$$G_0(T) = \sum_{g=0}^{T} g f_g \;\; and \;\; G_1(T) = \sum_{g=T+1}^{n-1} g f_g \qquad (73)$$

By including the spatial relevance information defined above in terms of m_i the entropy is not defined in terms of histogram information

$$H_0(T) = -\sum_{i=1}^{N} g_i/G_0(T) log \frac{g_i/G_0(T)}{m_i}$$

(74)

$$H_1(T) = -\sum_{i=1}^{N} g_i/G_i(T) log \frac{\frac{g_i}{G_1(T)}}{m_i} \qquad (75)$$

For obtaining the optimum threshold we need to maximize this total segmentation entropy τ

$$\tau = arg \left\{ \begin{matrix} max \\ 0 \le T \le n-1 \end{matrix} \{H_0(T) + H_1(T)\} \right. \qquad (76)$$

According to the bring instead of maximizing the sum of entropy the individual class entropy can be maximize as much as possible. In order to do that the smaller class entropy is maximized.

$$\tau = arg \left\{ \begin{matrix} max \\ 0 \le T \le n-1 \end{matrix} \{min(H_0(T), H_1(T))\} \right. \qquad (77)$$

The above statement will try to provide the point where the entropy of foreground and background is nearly same.

2.7 Scissors:

This paper [53] presents an interactive tool known as intelligent scissors. Fully automating the task of segmentation is a very hard process. Even the tracing the boundary of an object in image is not an easy process. Live wire boundary [54] can create a boundary using the traced point on the object of interest. For the working of the intelligent scissors Let p and q be the two adjacent or neighboring pixel points. The local cost can be represented by $l(p,q)$ and the various other image features f_z (Laplacian zero crossing) , f_g (Gradient magnitude) , f_d (Gradient direction), f_p (Edge pixel value) , f_i (Inside pixel value), f_o (Outside pixel value) can be taken into account. Now the local cost function can be given as,

$$l(p,q) = \omega_z . f_z(q) + \omega_g . f_g(q) + \omega_d . f_d(p,q) + \omega_p . f_p(q) + \omega_i . f_i(q) + \omega_o . f_o(q)$$

(78)

ω can be defined as the corresponding weights for the different factors. The zero crossing is basically used for the edge localization. f_z is assigned zero value if the Laplacian is zero otherwise assign it the value 1.

f_g can be calculated using the derivative in x and y direction i.e I_x and I_y

$G = \sqrt{I_x^2 + I_y^2}$ (79)

So the static cost function becomes

$f_c = \frac{\max(G')-G'}{\max(G')} = 1 - \frac{G'}{\max(G')}$ (80)

Where $G' = G - \min(G)$ (81)

Gradient direction f_d is to determine the change in the boundary whether it is sharp or smooth. Pixel value features (f_p , f_i , f_o), f_p is the simple pixel scaling feature and can be calculated as

$f_p(p) = \frac{1}{255} I(p)$ (82)

As the range of a typical image ranges from 0 to 255.

$f_i(p) = \frac{1}{255} I(p + k.D(p))$ (83)

$f_i(p) = \frac{1}{255} I(p - k.D(p))$ (84)

D(p) is the unit vector of gradient direction. k can be any chosen constant.

As this paper presents an interactive tool for the image segmentation. The key addition is Laplacian-crossing which helps the better localization of edges. The nodes can be added and removed from list in active time makes it less expensive for computation results the improvement in the speed of interaction for the optimal path generation.

2.8 Graphcut:

A new technique to segment the images using graphs properties is introduced [56] for the interactive segmentation .The main task is to divide the image into background and foreground. The main advantage of the scheme is that it provide global optimal solution using interactive segmentation.

Let *P* be the set of data element and *N* represents all the neighbourhood represented by unordered pair *{p,q}*.

Let $A = (A_1 .... A_p .... A_{|P|})$.be a binary vector ,where A_p specifies the *p* assigns to *P*. A_p is either an object point short "obj" or a background point "bkg".

The cost function $E(A) = \lambda .R(A)+B(A)$ (85)

$$R(A) = \sum_{p \in P} R_p(A_p) \qquad (86)$$

$$B(A) = \sum_{\{p,q\} \in N} (B_{\{p,q\}} . \partial(A_p, A_q)) \qquad (87)$$

$$\partial(A_p, A_q) = \begin{cases} 1 & if\ A_p \neq A_q \\ 0 & otherwise \end{cases} \qquad (88)$$

$\lambda \geq 0$ defines the *R(A)* region property versus Boundary property *B(A).*

The assumption for *R(A)* is such that penalty to assigning the pixel *p* to background to foreground is given.

$R_p(.)$ can show how the *p* can fit into foreground or background.

B(A) tells the boundary property of segment *A.*

$B_{\{p,q\}} \geq 0$ is the penalty for a discontinuity between *p* and *q*.

$B_{\{p,q\}}$ is large when *p,q* are similar(intensity perspective).

$B_{\{p,q\}}$ is close to zero if *p,q* is very dissimilar.

The graph $G(V,E)$ consist of V as the set of all vertices and E is the set of all edges.

A cut C is the subset of E ,such that new graph $G(C)=\langle V,E\backslash C\rangle$

Total cost

$$|C| = \sum_{e \in C} w_e \qquad (89)$$

W_e is the non negative weight cost.The method assume that the cut of the graph can be calculated in polynomial time.

Assume that O is the subset of all object pixel marked as foreground and K be the subset of all background pixel marked as Background.

$$O \subset P \ and \ \ K \subset P, \ O \cap K = \phi$$

$$\forall p \in O \quad A_p = obj$$
$$\forall p \in K \quad A_p = bkg$$

To segment an image into foreground or Background two additional nodes are used S is a source node for object and T is the sink node for background.

Now set of vertices $V = P \cup \{S,T\}$.

The set of edges contains

The set o edges now contains two more type of undirected n-links (neighbourhood links) and t-links (terminal links)

$\forall p$ have two links $\{p,S\}$ and $\{p,T\}$. For each neighbouring pixel N, p and q will be denoted as $\{p,q\}$

$$E = N \cup \{\{p,S\},\{p,T\}\} \qquad (90)$$

Now the weights of the edges

Edge	*Weight*	*for*
$\{p,q\}$	$B_{\{p.q\}}$	$\{p,q\} \in N$
$\{p,S\}$	$\lambda.R_p(bkg)$	$p \in P, p \notin O \cup K$
	M	$p \in O$
	0	$p \in K$
$\{p,T\}$	$\lambda.R_p(obj)$	$p \in P, p \notin O \cup K$
	0	$p \in O$
	M	$p \in K$

$$M = 1 + \max_{p \in P} \sum_{q:\{p,q\} \in N} B_{\{p,q\}} \qquad (91)$$

Let see how $\hat{C}$ segment the image and check where $\hat{C}$ is optimal.

F be set all fesiable cuts

C cuts exactly on t-link at each P

$\{p,q\} \in C$ p,q are t-linked to different terminals

---

if $p \in O$ then $\{p,T\} \in C$

if $p \in K$ then $\{p,S\} \in C$

Max- flow algorithm is used for determining the minimum cut [57]in the graph.

If the minimum cut is already calculated and the new seeds are again added the new seeds are again added the cost of two t-links are updated.

$\{p,S\}$	$\lambda.R_p(bkg)$	M
$\{p,T\}$	$\lambda.R_p(obj)$	0

and then compute the maximum flow to find the moment of the graph.

The problem occur here is that assigning a new weight can reduce the capacity of the link. If there is flow through these edges then the link may break

For a new object the t-link is updated as

t-link	initial cost	Add	New cost
$\{p,S\}$	$\lambda.R_p(bkg)$	$M+\lambda.R_p(obj)$	$M+C_p$
$\{p,T\}$	$\lambda.R_p(obj)$	$M+\lambda.R_p(bkg)$	C_p

$R_p(obj)= -lnP_n(I_p|O)$

$R_p(bkg)= -lnP_n(I_p|K)$

$$B_{\{p,q\}} \alpha \ e^{\exp((-\frac{(I_p-I_q)^2}{2\sigma^2}))} \cdot \frac{1}{dist(p,q)} \qquad (92)$$

$$|I_p - I_q| < \sigma$$

The method proposed in this paper [58] uses the graph cut but with the slight modification. Instead of using some random markings for foreground and background this method uses the polygon to make the same. After creating the polygons the polygons are then used for foreground boundary and background boundary.

The points inside the foreground boundary are marked as F_G and the points between foreground boundary and background boundary are marked as B_G.

To speed up the process first watershed algorithm is applied to pre segment the image. This creates the small regions in the image and these regions are then treated as the node for the graph for the flow network.

As foreground and background consist of many colours so there can be many classes those can be produced

$FG=\{FGC_1, FGC_2, FGC_3 \ldots\ldots\ldots\ldots\ldots\ldots FGC_K\}$

$BG=\{BGC_1, BGC_2, BGC_3 \ldots\ldots\ldots\ldots\ldots\ldots BGC_K\}$

t-link

$$R_p(foreground) = \min\left(||p - FGC_k||^2\right)/[(\min\left(||p - FGC_k||^2\right) + \min\left(||p - BGC_k||^2\right)] \qquad (93)$$

$$R_p(background) = \min\left(||p - BGC_k||^2\right)/[(\min\left(||p - FGC_k||^2\right) + \min\left(||p - BGC_k||^2\right)] \qquad (94)$$

n-link

$$B_{\{p,q\}} = 1/(1+\| p-q \|^2) \qquad (95)$$

The idea in this paper [59] is to define a new energy term which measures the L1 distance between the object and the background . The L1 can be maximized globally so that object and background can be separated in one cut. The proposed method try to minimize the energy function.

$$E(S) = |S|.H(\theta^S) + |\bar{S}|.H\left(\theta^{\bar{S}}\right) + |\partial S| \qquad (96)$$

θ^S are the histogram inside the object S

$\theta^{\bar{S}}$ are the histogram inside background $\bar{S} = \Omega / S$

Ω is the set of all image pixel

H(.) is the entropy for probability distribution.

The image segmentation based on bounded box is very popular. Most of the existing techniques uses tight bounding box for the implementation. This paper [66] introduces a new strategy Loose Cut algorithm which is based on loosely coupled bounding box to handle the problem. The idea proposed here uses the min-max flow algorithm for the optimization.

Graph cut basically tries to minimize the function

$$E_{GC}(X,\theta) = \sum_i D(x_i,\theta) + \sum_{i,j\epsilon N} V(x_i,x_j) \qquad (97)$$

N denotes the pixels in neighboring system

$D(x_i,\theta)$ defines the labelling cost of pixel i as foreground or background on the model θ

$V(x_i,x_j)$ penalized the discontinuity for the smoothness of label.

$\theta = (M_f, M_b)$

M_f be the foreground gaussian mixture model,

M_b be the background gaussian mixture model,

Loose Cut on the other hand uses the given energy function

$$E(X,\theta) = E_{GC}(X,\theta) + \beta E_{LC}(X) \qquad (98)$$

E_{GC} is the Grabcut Energy

E_{LC} is the energy term for increasing label consistency by keeping $\beta > 0$.

Now the proposed global similarity constraint assume M_f have K_f Gaussian components then M_f^i contains means μ_f^i $i = 1,2,\dots\dots, K_f$ and

M_b have K_b Gaussian components contain means μ_b^i $i = 1,2,\dots\dots, K_b$

For every M_f^i the foreground components its nearest $M_b^{j(i)}$ background component can be determined as

$$j(i) = arg_{j\epsilon 1\dots k_b}^{min} |\mu_f^i - \mu_b^j|$$

So the similarity of M_f^i Gaussian component and entire background M_b can be determined as

$$S(M_f^i, M_b) = \frac{1}{|\mu_f^i - \mu_b^{j(i)}|} \qquad (99)$$

So the global global similarity function can be defined as

$$Sim(M_f, M_b) = \sum_{i=1}^{k_f} S(M_f^i, M_b) \qquad (100)$$

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The similarity index needs to be less than some threshold for estimating θ.

Other Graph-cut based methods based on diffusion likelihood [102] , on the approach of two phase pixel level segmentation [103] and GC based on Geodesic [104].

3 Evaluation Metric:

3.1 Hamming distance:

The idea [71] is based on Hamming distance of two segmentation S and R

Let D_H be the directional Hamming distance

$$D_H(S \to R) = \sum_{r_i \in R} \sum_{S_k \neq S_j, S_k \cap r_i \neq 0} |r_i \cap S_k| \qquad (101)$$

| . | is the size of the set. Hamming distance for region based evaluation

$$p = 1 - \frac{D_H(S \to R) + D_H(R \to S)}{2 * |S|}, p \in [0,1] \qquad (102)$$

3.2 Local Consistency Error:

It allows refine labelling between segmentation and ground truth [72].

$$LCE(S, R, p_i) = \frac{1}{N} \sum_i \min \{E(S, R, p_i), E(R, S, p_i)\} \qquad (103)$$

$E(S, R, p_i)$ gives the level at which two segmentations agree at pixel *p*.

3.3 Partition distance measure:

It gives a new measure [73] given the two partition P and Q form region , the partition distance is the number of pixels that need to be deleted from S such that P and Q become identical. $d_{sym}(Q,P)=0$ gives that no pixel need to be deleted.

3.4 Distance Distributed Signature [71]:

$$b = 1 - \frac{D_B(B_s, B_R) + D_B(B_R, B_s)}{C(|R| + |S|)} \qquad (104)$$

Where |R| and |S| are the number of boundary points. $D_B(B_S+B_R)$ represents the discrete function which measures the discrepancy form B_S to B_R. B_S is the boundary point and B_R is the boundary ground truth.

Intersection over union: Checks for the labelled pixel correctly as forground or background compared to ground truth

$$IoU = |S \cap G| / |S \cup G| \qquad (105)$$

Where *S* is the segmented image and *G* is the ground truth image.

3.5 Probabilistic Random Index(PRI): This method PRI [74] is evaluated as

$$PRI(S_{test}, \{S_k\}) = \frac{1}{\binom{N}{2}} \sum_{\substack{i,j \\ i<j}} [c_{ij} p_{ij} + (1 - c_{ij})(1 - p_{ij})] \qquad (105)$$

S_{test} → The segmented image to be compared.

S_k → The ground truth image.

N is the total pixels.

p_{ij} → be the random number chosen from Bernoulli Distribution.

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c_{ij} → pair of pixels I and j with the same label.

3.6 Variation of Information:

It is basically used for clustering application [77] but can be extended for the image segmentation.

$$VI(S_{seg}, S_{hum}) = H(S_{seg}) + H(S_{hum}) - 2I(S_{seg}, S_{hum}) \qquad (106)$$

H and *I* represents the Entropy of the mutual information.

Other evaluation metrics like Modified Hausdroff Distance (MHD) [78] which calculate the displacement between segmented and ground truth results. The Number of clicks [79] and F-Boundary Score (FB) [80] are also widely used evaluation methods

4. Conclusion and Future Work

In this paper the ideas of segmentation from the point of view of various researchers presented by them in different papers have been put together. By analyzing the different research paper this can be figure out that there are various core or novel methods for the image segmentation and lot of the recent research is based on these methods. We provide the main algorithms used by different papers and present them at a single platform. The basic key concepts used in the algorithm proposed has been provided in simple form. This paper is basically concerned with the state of art methods for image segmentation. Now the methods based on deep learning are widely proposed which is the future work of this research survey.

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