Tuijin Jishu/Journal of Propulsion Technology
ISSN: 1001-4055
Vol. 45 No. 2 (2024)

Carbon Constrained Integrated Inventory System with Linear and Quadratic Production and Compound Poisson Demand.

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Abstract

This paper develops an two-stage supply chain model with imperfect manufacturing process under variable lead time. The cost of producing a unit of product is calculated as a function of production rate. In addition, linear and quadratic production functions are used to relate the process quality and production rate. This work relaxes the assumption that lead time demand is deterministic and treats it as a compound Poisson process. Also, we characterize the effects of carbon taxation, cap-and-trade and limited carbon emission policies to reduce greenhouse gas emissions. Although the basic purpose cap-and-trade is a reduction in carbon emissions that a well-structured emissions trading system can deliver significantly environmental, economic and social benefits. The energy usage is categorized according to the manufacturing and reworking processes. Under various carbon emission standards, the suggested model determines the appropriate order quantity. A numerical example has been considered to evaluate this model using MATLAB 2014a . Sensitivity analysis section is decorated for the optimal solution of the model with respect to major cost parameters of the system are carried out, and the implications of the analysis are discussed.The results suggest that limited carbon emission policies have a lower total cost than carbon tax policy and cap-and-trade policy.

Keywords: Integrated vendor-buyer, quality management, production rate, energy consumption , different carbon policies ,compound Poisson process.

1. Introduction

Nowadays, the role of marketing is more difficult than ever, and attaining a successful conclusion in a competitive and complicated market environment. In this case, cooperation between the seller and the buyer is more vital, and this type of collaboration allows for the total cost of the entire system to be minimized. During the manufacturing process, the producer will produce their things and transfer them to the buyer. Many experts believe that the production rate is constant, however this may alter. The machine initially runs flawlessly while an item's production rate gradually increases, but it may periodically transition from an under control condition to an out of control state. Until the machine hits the out-of-control stage, the elapsed time is treated as a negative exponentially distributed random variable with a mean of $\frac{1}{\mu}$. The mean of a random variable is then considered to be a quality function.Sarkar et al. [19] expanded on this approach by introducing additional quality function as a cubic polynomial with controlled lead time.

Huang et al.[7] evaluated inventory management using three different carbon policy models, namely one with carbon taxation (CT), its goal is to reduce the negative or dangerous amounts of carbon emission (CE). The manufacturer is taxed by the government for each tonne of carbon produced in their factory. It is transformed into an electricity, natural gas, or oil tax. Because carbon tax regulation prohibits the excessive use of fossil fuels, industries are encouraged to use pollution-free and sustainable energy sources in their manufacturing. And second one is Cap-and-trade (CAP) it is a method of regulating and lowering the amount of carbon emission. The government limits a company's carbon emission under this regulation. This is known as the company's cap or initial allowance of carbon emission. And last one is Limited carbon emission (LCE) is both the vendor and the buyer will need to modify their business operations to conform to the limited carbon emission requirements. This article looks at products with variable production rates, defective rework, adjustable lead times, compound Poisson process, energy consumption and sustainable investment under the formulation of a normal distribution model. The basic goal of carbon policy is to lower the annual total cost of carbon emission. We also add process integration into the production and reworking schedules and examine its effects on shortening the rework schedule. The improvement of the defective manufacturing process into a flawless production process is suggested by the energy consumption and sustainability investment. Using the MATLAB 2014a software, we solve this mathematical formulation using the traditional optimization method. The goal of this study is to examine how the reworking process affect a alternative scenario of an imperfect production model with normal distribution.

2. Literature review

Karthick and Uthayakumar [8] conducted a thorough examination of the imperfect production model with variable setup cost under free distribution. Khara et al. [9] investigated a two-tier supply chain model with an unsatisfactory manufacturing process. Adak and Mahapatra [3] investigated the incorrect manufacturing supply chain model with probabilistic deterioration under uncertainty. Dey and colleagues[5] investigated the effects of adjustable lead time on the supply chain model. Malleswaran and Udhayakumar[12] demonstrated a vendor managed inventory model for imperfect production process using sustainability investment and energy consumption under different carbon policies. Annadurai and Udhayakumar[1] categorized controlling setup cost in (Q,r,L) inventory model with defective items.

Energy consumption (EC) refers to the energy consumed by industry to make a product or by a residential building to maintain its routines. Bhuniya et al.[4] investigated the production process in an supply chain management to determine the best energy consumption and maintenance policy.To function, production (equipment) requires both constant and variable power components. Ganesh Kumar and Uthayakumar [6] investigated the green house gases trading scheme when equal and unequal shipments were used. Marchi et al. [15] categorized a SC model with green house gases, energy consumption, and an incomplete manufacturing process under various coordination scenarios.Priyan et al. [16] developed a long-term dual supply chain model for energy consumption.

Sustainability Investment (SI) is a type of economic activity aimed at conserving and developing renewable energy sources. Kishore [10] and [11] implemented a green production system for quality control and trade credit finance. Annadurai and Udhayakumar [2] developed reducing lost-sales rate in (T,R,L) inventory model with controllable lead time.

3 Research gap and novelty

Malleeswaran and Uthayakumar [13] addressed the carbon sustainability management on reducing green house gases and waste items through emission penalties. Mishra et al. [14] presented a manufacturing inventory research for a controllable CE rate in the presence of shortages. In the green supply chain model, Paul et al. [17] created a carbon tax policy.

In the existing literature , the authors considered the classical model without carbon policies and energy consumption. But here, we are going to develop models by considering the classical model with different types of carbon policies. The use of carbon policies are mainly focused to reduce the annual total carbon emission cost. The energy consumption and sustainability investment are proposed to improve the imperfect production process into a perfect production process.We solve this mathematical formulation by using the classical optimization method with MATLAB 2014a software.

Table 1: Research gap between previous and this study

Reference	Imperfect production	Normal distribution	Compound Poisson	Sustainability investment	GHG EC policies	CE
Chung and Hou (2003)		x	x	x	x	x
Eroglu and Ozdemir (2007)		x	x	x	x	x
Khouja and Mehrez (1994)		x	x	x	x	x
Karthick and Uthayakumar (2021a)		x	x	x	x	x
Mukherjee et al. (2019)		x	x	x	x	x
Marchi et.al (2019)		x	x	x		
Ouyang et al. (2004)		x	x	x		x
Ouyang et al. (2006)		x	x	x		x
Rosenblatt and Lee (1986)		x	x	x		x
Sana (2011)		x	x	x		x
Sarkar, Gupta et al. (2014)	x		x	x	x	x
Sarkar and Chung (2020)		x	x	x	x	x
Wang and Song (2020)	x	x	x		x	
present work						

Notations and assumptions

This model is built using the notations and assumptions listed below.

4.1 Notations

K - Factor of safety

n - Quantity of shipments

q - Order amount

r - Rate of production

l_b - Duration of buyer's lead time

D - Rate of demand

S_v - The vendor's setup cost

H_v -Vendor's holding expense

H_b -Buyer's holding expense

C_1 - Rework expenses

C_0 - Cost of unit inspection

σ - Departure from the mean

π -Back-order cost per unit

y -When the process becomes "out of control," the percentage of defective items increases.

t - Production run time

r_c - Cost of manufacturing per unit

e_{Nd} - A production cycle's projected amount of faulty goods.

$\delta(P)$ - The amount of time that the system has been out of control (exponential random variable)

$b(l)$ - Cost of lead time crashing

x - Supply during the lead time

d - Delivery range

m -The multiplier of Lagrange

t_e - Carbon tax for each carbon emission unit

i - The number of trucks used on each journey

e_{ps} - Carbon emission from the manufacturing setup

k_i - Carbon emission upper limit (k = 1, 2)

$e_s - CO_2$ emissions from the storage of a unit product

e_{tp} - Carbon trading price per unit of CO_2 emitted

e_t - Carbon footprint of transporting a single product

e_p - CO_2 emissions from the production of a single unit product

e_T - Cost of one gallon of diesel-truck fuel in terms of carbon emissions (ton/gallon)

v_c - Cost of carbon emissions created by transportation over time

T_G - The number of gallons per truck per mile traveled (gallon/truck).

m_c - The cost of repairing a machine once it has broken down.

F_{pc} - The cost of fuel per unit transported ($/unit) (Marchi [15])

r_p - Time spent on production

R_w - Time spent on Reworking

sec_{RP} - Specific EC to produce a unit for reworking \ (kWh/unit)

sec_{PP}- Specific EC to produce a unit for production (kWh/unit)

θ- The relationship between production rates and reworking

$w_{PP,}w_{RP,}$- Total energy utilized by the production and reworking (kW)

$i_{PP,}i_{RP}$ - Reactive power for production and rework process-support related readiness equipment Features (kW)

$\tau_{1,}\tau_{2}$- Components of energy used in production and rework (. 0) (kWh/unit)

t_{RAN} - Total transportation cost

4.2 Assumption

1. A single-seller and a single-buyer model is considered.
2. Buyer's inventory is reviewed regularly. Filling is done whenever the inventory level falls to the reorder point.
3. The buyer orders quantity nq of goods to the seller and to the manufacturer those items were replaced by the seller in the same system n items in shipment to buyer, where n is a positive integer.
4. The cost per unit of production depends on the production rate r and the rate of production increases the quality of the products decreases.
5. Deficits are allowed and it is fully withdrawn.
6. Seller is responsible for inspection cost per unit of item.
7. Elapsed time is an exponentially distributed random variable the production system is out of control and the mean of exponential distribution is a decreasing function of production rate.
8. For the buyer, the lead time l_b has mutually exclusive components n_i independent. The i^{th} component has minimum duration M_i, the normal duration N_i and a downtime cost per unit time are E_i and $E_1 \leq E_2 \leq \cdots \leq E_n$. The lead time components must cancel each other out time starting from the minimum element of E_i and so on.
9. Let $l_{b_0} = \sum_{j=1}^{N_i} N_j$ and l_{bi} is the length of the lead time components $1,2,3, \ldots i$ crashed to their minimum duration, the expression of $l_{b_i} = \sum_{j=1}^{N_j} N_j - \sum_{j=1}^{i} \left(N_j - M_j\right)$, where $i = 1,2, \ldots N_j$ and crashing cost for the lead time per cycle is given by

$$b(l) = E_i\left(l_{b(i-1)-l_b}\right) + \sum_{j=1}^{i-1} E_j\left(N_j - M_j\right), l_b \epsilon\left[l_{bi} - l_{b(i-1)}\right]$$

10. Let $N(l_b)$ represent the total number of customers during the lead period l_b, and $N(l_b)$ has a Poisson distribution with mean λl_b. If the quantity purchased by the i^{th} demand, $i = 1,2, \ldots$, is independent and identically normal distributed with mean μ and standard deviation σ, then $\left(x(L) = \sum_{i=1}^{N(l_b)} y_i, L \geq 0\right)$ is a compound Poisson process, where $x(l_b)$ denotes the overall amount purchased by time l_b.
11. The reorder point is equivalent to the sum of expected demand during the lead period and safety stock.

5. Mathematical Model Development

The following sections describe some of the essential factors of energy consumption that are used to develop our model for both traditional, sustainability investment and various carbon policies.

5.1 Mathematical Model Development

The suggested model is based on a two-level supply chain (vendor - buyer), and the expected cost function of both the buyer and the vendor is developed in the following subsections.

5.1.1 Vendor Model

The supplier makes enough nq to fulfil the orders. The vendor must invest setup expenses while producing. The vendor's startup price is $S_v\frac{D}{nq}$ per unit of time. After manufacturing, the produced quantities should be kept as inventory before delivering the orders to the buyer. As a result, the seller must pay the holding expenses. The vendor's average inventory is

$$\frac{D}{nq}\left[\left\{nq\left(\frac{q}{r}+(n-1)\frac{q}{D}-\frac{n^2q^2}{2r}\right)\right\}-\left\{\frac{q^2}{D}(1+2+3+\cdots+(n-1))\right\}\right]$$

$$=\frac{q}{2}\left[\left(1-\frac{D}{r}\right)n-1+\frac{2D}{r}\right] \qquad (1)$$

The vendor's expected holding expenses per unit time are $H_v\frac{q}{2}\left[\left(1-\frac{D}{r}\right)n-1+\frac{2D}{r}\right]$. The quality of the produced products is then examined during the inspection process. The testing session will cost C_0D. Reworking will be done if it is determined that the made goods are flawed. The number of defective items in each manufacturing cycle is provided by Rosenblatt & Lee[Rosenblatt(1986)],

$$N_d=\begin{cases}0, & \text{if } \delta\geq t\\ yr\left(t-\delta(p)\right) & \text{if } \delta<t\end{cases}$$

The expected amount of defective items during a production cycle is provided by

$$E_{n_d}=yr\left[\frac{q}{r}+\frac{1}{F(r)}\left(e^{\frac{qF(r)}{r}}\right)-\frac{1}{F(r)}\right]. \qquad (2)$$

If $e^{\frac{F(r)q}{r}}$ is an effective estimate of the Maclaurin series and $F(r)$ is small, (Sarkar et al.,[Rosenblatt[18]]),the expected repair cost is

$$C_1\frac{D}{q}E_{n_d}=C_1DyF(r)\frac{q}{2r}.$$

The following observations lead to the unit output cost function being obtained, first one is labor and energy expenses rise as production rates do, reaching higher levels. And second on is at this level of production, the cost per instrument is very low. Costs for instruments will rise if production rates progressively increase. A particular kind of production cost function that we take into consideration is $r(c)=\left(\frac{A_1}{r}+A_2r\right)$, where A_1 and A_2 are non-negative real values. The total expected cost $ETC_v(q,r,n)$ of the vendor is defined as the sum of the setup cost, holding cost, rework cost, inspection cost, and production cost and is expressed by

$$ETC_v(q,r,n)=S_v\frac{D}{nq}+H_v\frac{q}{2}\left[\left(1-\frac{D}{r}\right)n-1+\frac{2D}{r}\right]+C_0D+C_1DyF(r)\frac{q}{2r}+Dr(C) \qquad (3)$$

5.1.2Buyer Model

The vendor receives a large order from the buyer for size R, manufactures R at a finite production rate P (P>D) in a single setup, and then ships Q to the buyer over m periods. Therefore, the seller minimizes its setup cost, and the inventory price is paid to the buyer once the buyer's lot quantity q is produced. By assumption (10), expected lead time requirement is

$$E\big(x(l_b)\big) = E\big(x_1 + x_2 + \cdots x_{N(l_b)}\big) = E\big(N(l_b)\big)E(x_i) = \lambda l_b u$$

and the variance is

$$var(x) = [E(x_i)]^2 var(N(l_b)) + E(N(l_b))var(x_i) = \lambda l_b(u^2 + \sigma^2).$$

Therefore, the reorder point is $R = \lambda u l_b + K\sum\sqrt{\lambda l_b}$, where $\sum 2 = u^2 + \sigma^2$ and K is known as the safety factor. The expected shortage per replenishment cycle is $\sum\sqrt{\lambda l_b}\alpha(K)$, where

$\alpha(K) = \phi(K) - K[1 - \Phi K]$, and ϕ, Φ denote the standard normal p.d.f and c.d.f respectively. The average on hand inventory for the buyer is given by $H_b\left(\frac{q}{2} + K\sum\sqrt{\lambda l_b}\right)$. Finally, $\frac{D}{q}b(l)$ provides the projected lead time crashing cost. $ETC_b(q, R, l_b)$ is the buyer's total expected cost determined by adding the ordering cost, holding cost, shortfall cost, and lead time crashing cost and is expressed by

$$ETC_b(q, R, l_b) = O\frac{D}{q} + H_b\left(\frac{q}{2} + K\sum\sqrt{\lambda l_b}\right) + \pi\frac{D}{q}\sum\sqrt{\lambda l_b}\alpha(K) + \frac{D}{q}b(l). \quad (4)$$

It calls for a new way of looking at supply chain's transportation system, which also encompasses supply chain management, logistics, and procurement. A significant amount of a company's supply chain expenses are related to transportation. For instance, US transportation costs account for around 6% of GDP. A component of travel expenses is transportation expenses, which also includes prices for things like taxi fares, gas, parking, meals, tips, laundry, deliveries, and phone calls. Calculating the economic total cost of the transportation involves adding the cost of gasoline and the CE tax. (Marchi et al. [15]). It can be written as

$$t_{RAN} = F_{pc}\frac{D}{T_c}r + t_e v_c \quad (5)$$

where the annual carbon emissions from truck movements are

$$v_c = \frac{niDT_G e_T}{nq}, \text{where} \quad i = \frac{q}{n}.$$

5.2 Investment in sustainability and carbon policies

If the carbon emission amount is greater than the cap set by the rules in supply chain, we consider a sustainability investment to lower the carbon emission. The cost of sustainable investment for reducing carbon emission is expressed as,

$$\Gamma(g) = \mu_1 G - \mu_2 G^2 \quad (6)$$

where μ_1 and μ_2 stand for the offset and efficiency variables for the reduction of carbon emission. For lowering the carbon emission, the supply chain manager invests the SI sum of G in a sustainable investment of $\mu_1 G$. Here, the symbol $\mu_2 g^2$ stands for a further carbon emission lowering factor. The annual carbon emission from the production procedure is $De_p + \frac{D}{nq}e_p s$.

The yearly carbon emission generated by product storage is

$$YCE(n,q,G) = De_p + \frac{D}{nq}\left(e_{ps} + ne_t d\right)\frac{q}{2}$$

$$\left(2 + n\left(1 - \frac{D}{r}\right)\right)e_s - G(\mu_1 - \mu_2 G). \qquad (7)$$

5.3 Cost-integrated with various carbon policies in the traditional model

In this section, a normal distribution model is developed, linear production and process quality are discussed. Defective products are not produced if machine operation is stopped. If the machine works, there is a chance that the manufactured item will appear defective.Therefore, the mean time to failure (MTTF) assumed to be independent of the production rate is irrelevant in the practical problem. The phenomena arise due to the above reasons.

Given that F(r) is a strictly rising function of r, it is absolutely reasonable to conclude that the MTTF function is a strictly decreasing production rate. As a result, the MTTF $\frac{1}{F(r)}$ is a strictly decreasing function of r. Thus we have

Case 1.

$$\frac{1}{F(r)} = \frac{1}{B_1 r} \times (\text{F(r) is a linear quality function in r.})$$

Case 2.

$$\frac{1}{F(r)} = \frac{1}{B_2 r + C_2 r^2} \times (\text{F(r) is a quadratic quality function in r.})$$

5.4 Case 1:$F(r)$ is linear in r

5.4.1 Carbon Taxtation

In this instance, C_t is the per unit carbon emission t_e. To cut down on carbon emission, the vendor and buyer invest sustainably in green technologies. The cost function is expressed a

$$JTC(q,K,l_b,r,n,G)_{tax} = O\frac{D}{q} + H_b(\frac{q}{2} + K\sum\sqrt{\lambda l_b})$$

$$+\pi\frac{D}{q}\sum\sqrt{\lambda l_b}\alpha(K) + \frac{D}{q}b(l) + S_v\frac{D}{nq}$$

$$+H_v\frac{q}{2}\left[\left(1 - \frac{D}{r}\right)n - 1 + \frac{2D}{r}\right] + C_0 D + C_1 D y B_1\frac{q}{2} + Dr(C) + F_{pc}\frac{D}{T_c}r + t_e v_c + t_e(De_p +$$
$$\frac{D}{nq}(e_{ps} + ne_t d) + \frac{q}{2}\left(2 + n\left(1 - \frac{D}{r}\right)\right)e_s - G(\mu_1 - \mu_2 G)).$$
(8)

Taking the first order partial derivatives of $JTC(q,K,l_b,r,n)_{tax}$ with respect to q,K,r,G and $l_b \in [l_i, l_{i-1}]$, respectively, we obtain

$$\frac{\partial JTC(q,K,l_b,r,n)_{tax}}{\partial q} = -\frac{D}{q^2}\begin{bmatrix} O + b(l) + \pi\sum\sqrt{l_b\lambda}\alpha(K) \\ +\frac{S_v}{n} + \frac{t_e}{n}\left(e_{ps} + ne_t d\right) \end{bmatrix} + \frac{1}{2}H(m)\,, \qquad (9)$$

where,

$$H(m) = H_b + H_v\left[\left(1-\frac{D}{r}\right)n - 1 + \frac{2D}{r}\right] + \frac{C_1 DyB_1}{2} + \left[2 + n\left(1-\frac{D}{r}\right)\right]E_s,$$

$$\frac{\partial JTC(q,K,l_b,r,n)_{tax}}{\partial K} = H_b \sum \sqrt{\lambda l_b} + \pi\frac{D}{q}\sum \sqrt{\lambda l_b}(\Phi(K)-1), \quad (10)$$

$$\frac{\partial JTC(q,K,l_b,r,n)_{tax}}{\partial G} = 1 - t_e\mu_1 + t_e\mu_2(2G), \quad (11)$$

$$\frac{\partial JTC(q,K,l_b,r,n)_{tax}}{\partial r} = H_v\frac{q}{2}\frac{nD}{r^2} - H_v\frac{q}{2}\frac{2D}{r^2} - \frac{DA_1}{r^2} - A_2 D, \quad (12)$$

$$\frac{\partial JTC(q,K,l_b,r,n)_{tax}}{\partial l_b} = \frac{1}{2\sqrt{\lambda l_b}}H_b K\sum + \frac{1}{2q\sqrt{\lambda l}}\pi D\sum \alpha(K) - \frac{De_i}{q}. \quad (13)$$

For fixed $JTC(q,K,l_b,r,n)_{tax}$, q,K,r,G is a concave function of $l_b \in [l_i, l_{i-1}]$, because

$$\frac{\partial^2 JTC(q,K,l_b,r,n)_{tax}}{\partial l_b^2} = \frac{-1}{4}H_b K\sum l^{\frac{-3}{2}} - \frac{1}{4}\pi D\sum l^{\frac{-3}{2}}\alpha(K) < 0. \quad (14)$$

Hence, for fixed q,K,r,G the minimum joint total cost will occur at the end points of the interval

$l_b \in [l_i, l_{i-1}]$. It can be easily shown that for a given value of $l_b \in [l_i, l_{i-1}]$, $JTC(q,K,l_b,r,n)_{tax}$, is a convex function of q,K,r,G (See Appendix for detailed proof). Therefore, for fixed $l_b \in [l_i, l_{i-1}]$ the minimum value of $JTC(q,K,l_b,r,n)_{tax}$ will occur at the point q,K,r,G which satisfies

$$\frac{\partial JTC(q,K,l_b,r,n)_{tax}}{\partial q} = 0, \frac{\partial JTC(q,K,l_b,r,n)_{tax}}{\partial K} = 0, \frac{\partial JTC(q,K,l_b,r,n)_{tax}}{\partial r} = 0 \text{ and } \frac{\partial JTC(q,K,l_b,r,n)_{tax}}{\partial G} = 0.$$

Solving the above equations for $q, \Phi(K), G$ and r, respectively, we get

$$q = \sqrt{\frac{2D\left[O + b(l) + \pi\sum\sqrt{l_b\lambda}\alpha(K) + \frac{S_v}{n} + \frac{t_e}{n}(e_{ps} + ne_t d)\right]}{dH(m)}}, \quad (15)$$

$$\Phi(K) = 1 - \frac{H_b q}{\pi D}, \quad (16)$$

$$G = \frac{t_e\mu_1 - 1}{2t_e\mu_2} \quad (17)$$

and

$$r = \sqrt{\frac{2A_1 - H_v q(n-2)}{C_1 y C_2 q + 2A_2}} \quad (18)$$

where $H_b q(n-2) < 2A_1$ and $H_b q < \pi D$.

5.4.2 Carbon Cap-and-trade

In this situation, the excess X can be sold at e_{tp} per unit to make up the difference carbon emission does not go over K_1. If carbon emission is higher than K_1, the business must purchase or invest in green expenses in order to agree to the limited carbon emission requirements. The carbon trading price e_{tp}, is used here as the average price. The cost function is expressed as

$$JTC(q,K,l_b,r,n,G)_{cap} = O\frac{D}{q} + H_b(\frac{q}{2} + K\sum\sqrt{\lambda l_b})$$

$$+\pi\frac{D}{q}\sum\sqrt{\lambda l_b}\alpha(K) + \frac{D}{q}b(l) + S_v\frac{D}{nq}$$

$$+H_v\frac{q}{2}\left[\left(1-\frac{D}{r}\right)n - 1 + \frac{2D}{r}\right] + C_0 D + C_1 D y B_1\frac{q}{2} + Dr(C) + F_{pc}\frac{D}{T_c}r + t_e v_c + e_{tp}(De_p + \frac{D}{nq}\left(e_{ps} + ne_t d\right) + \frac{q}{2}\left(2 + n\left(1-\frac{D}{r}\right)\right)e_s - G(\mu_1 - \mu_2 G) - K_1). \quad (19)$$

Hence, for fixed q, K, r, G the minimum joint total cost will occur at the end points of the interval $l_b \in [l_i, l_{i-1}]$. It can be easily shown that for a given value of $l_b \in [l_i, l_{i-1}]$, $JTC(q,K,l_b,r,n)_{cap}$, is a convex function of q,K,r,G . Therefore, for fixed $l_b \in [l_i, l_{i-1}]$ the minimum value of $JTC(q,K,l_b,r,n)_{cap}$ will occur at the point q,K,r,G which satisfies $\frac{\partial JTC(q,K,l_b,r,n)_{cap}}{\partial q} = 0, \frac{\partial JTC(q,K,l_b,r,n)_{cap}}{\partial K} = 0, \frac{\partial JTC(q,K,l_b,r,n)_{cap}}{\partial r} = 0$ and $\frac{\partial JTC(q,K,l_b,r,n)_{cap}}{\partial G} = 0$.

Solving the above equations for $q, \Phi(K), G$ and r, respectively, we get

$$G = \frac{e_{tp}\mu_1 - 1}{2e_{tp}\mu_2}, \quad (20)$$

$$q = \sqrt{\frac{2D\left[O + b(l) + \pi\sum\sqrt{l_b\lambda}\alpha(K) + \frac{S_v}{n} + \frac{e_{tp}}{n}(e_{ps} + ne_t d)\right]}{dH(m)}}, \quad (21)$$

$$\Phi(K) = 1 - \frac{H_b q}{\pi D} \quad (22)$$

and

$$r = \sqrt{\frac{2A_1 - H_v q(n-2)}{C_1 y C_2 q + 2A_2}} \quad (23)$$

where $H_b q(n-2) < 2A_1$ and $H_b q < \pi D$.

5.4.3 Limited Carbon Emission

The Lagrange multiplier is a method for determining a function's local maximum and lowest values under the restrictions. It is a useful instrument for solving nonlinear problems with equality and inequality constraints. In this instance, both the vendor and the buyer must modify their business processes in order to comply with limited carbon emission K_2. Both stakeholders could engage in sustainability technologies to reduce carbon emission. The cost function is expressed as

$$JTC(q,K,l_b,r,n,G)_{LCE} = O\frac{D}{q} + H_b(\frac{q}{2} + K\sum\sqrt{\lambda l_b})$$

$$+\pi\frac{D}{q}\sum\sqrt{\lambda l_b}\alpha(K)+\frac{D}{q}b(l)+S_v\frac{D}{nq}$$

$$+H_v\frac{q}{2}\left[\left(1-\frac{D}{r}\right)n-1+\frac{2D}{r}\right]+C_0D+C_1DyB_1\frac{q}{2}+Dr(C)+F_{pc}\frac{D}{T_c}r+t_ev_c+\mathrm{m}(De_p+\frac{D}{nq}(e_{ps}+ne_td)+\frac{q}{2}\left(2+n\left(1-\frac{D}{r}\right)\right)e_s-G(\mu_1-\mu_2G)-K_2). \quad (24)$$

Hence, for fixed q,K,r,G the minimum joint total cost will occur at the end points of the interval $l_b \in [l_i,l_{i-1}]$. It can be easily shown that for a given value of $l_b \in [l_i,l_{i-1}]$, $JTC(q,K,l_b,r,n)_{LCE}$, is a convex function of q,K,r,G . Therefore, for fixed $l_b \in [l_i,l_{i-1}]$ the minimum value of $JTC(q,K,l_b,r,n)_{LCE}$ will occur at the point q,K,r,G which satisfies $\frac{\partial JTC(q,K,l_b,r,n)_{LCE}}{\partial q}=0$, $\frac{\partial JTC(q,K,l_b,r,n)_{LCE}}{\partial K}=0$,

$\frac{\partial JTC(q,K,l_b,r,n)_{LCE}}{\partial r}=0$ and $\frac{\partial JTC(q,K,l_b,r,n)_{LCE}}{\partial G}=0$.

Solving the above equations for $q,\Phi(K),G$ and r respectively, we get

$$G=\frac{m\mu_1-1}{2m\mu_2}, \quad (25)$$

$$q=\sqrt{\frac{2D\left[O+b(l)+\pi\sum\sqrt{l_b\lambda}\alpha(K)+\frac{S_v}{n}+\frac{m}{n}(e_{ps}+ne_td)\right]}{dH(m)}}, \quad (26)$$

$$\Phi(K)=1-\frac{H_bq}{\pi D} \quad (27)$$

and

$$r=\sqrt{\frac{2A_1-H_vq(n-2)}{C_1yC_2q+2A_2}} \quad (28)$$

where $H_bq(n-2)<2A_1$ and $H_bq<\pi D$.

Since it is difficult to find an explicit general solution for $q,\Phi(K),G$ and r, we establish the following iterative algorithm to find the optimal $q,\Phi(K),G$ and r.

Algorithm

Step 1. Set $n=1$

Step 2. For every $l_g, g=0,1,2,\dots n$, perform step (a) to (e).

(a) Let K_0 this implies that $\Phi(K_0)=0.3989,(i=1,2)$.

(b) Substitute K_{i0} , $\Phi(K_{i0})$ into Eqs.16,22 and 27.

(c) Use q^{0i}, to determine $\Phi(K_{i1})$, (i=1,2) from Eqs.15, 21 and 26.

(d) Use $\Phi(K_{i1})$, to find the value of K_{i1}, $(i=1,2)$ from the normal table, and hence find $\alpha(K_{i1})$.

(e) Repeat steps from (b) to (d) until no change occurs in the values of q^{ig} and K_{ig}.

Step 3. Using the values of q^{ig*} and K_{ig}^{*} determine r^{*}, (i=1,2) from Eqs. 18, 23 and 28.

Step 4. Compute $JTC(q^{ig*}, K_{ig}^{*}, l_g, r^{*}, n)$ using the values of q^{ig*}, r^{*} and K_{ig}^{*}.

Step 5. Set $n = n + 1$, repeat the Steps from 2 to 4.

Step 6. Find the value of $min_{g=0,1,2...,n} JTC(q^{ig*}, K_{ig}^{*}, l_g, r^{*}, n)$ and set

$JTC(q^{iS*}, K_{iS}^{*}, l_S, r^{*}, n)$=$min_{g=0,1,2...,n} JTC(q^{ig*}, K_{ig}^{*}, l_g, r^{*}, n)$then $(q^{iS*}, K_{iS}^{*}, l_S, r^{*}, n)$, is the optimal solution.

5.5 Case 2: $F(r)$ is quadratic in r.

5.5.1 Carbon Taxtaion

In this instance, C_t is the per unit carbon emission t_e. To cut down on carbon emission, the vendor and buyer invest sustainably in green technologies. The cost function is expressed as

$$JTC(q,K,l_b,r,n,G)_{tax} = O\frac{D}{q} + H_b(\frac{q}{2} + K\sum\sqrt{\lambda l_b})$$

$$+\pi\frac{D}{q}\sum\sqrt{\lambda l_b}\alpha(K) + \frac{D}{q}b(l) + S_v\frac{D}{nq}$$

$$+H_v\frac{q}{2}\left[\left(1-\frac{D}{r}\right)n - 1 + \frac{2D}{r}\right] + C_0D + C_1Dy(B_2r + C_2r^2)\frac{q}{2r} + Dr(C) + F_{pc}\frac{D}{T_c}r + t_ev_c + t_e(De_p + \frac{D}{nq}(e_{ps} + ne_td) + \frac{q}{2}\left(2 + n\left(1-\frac{D}{r}\right)\right)e_s - G(\mu_1 - \mu_2G)).$$

(29)

Taking the first order partial derivatives of $JTC(q,K,l_b,r,n)_{tax}$ with respect to q, K, r, G and $l_b \in [l_i, l_{i-1}]$,respectively,

Weobtain $$\frac{\partial JTC(q,K,l_b,r,n)_{tax}}{\partial q} = -\frac{D}{q^2}\begin{bmatrix} O + b(l) + \pi\sum\sqrt{l_b\lambda}\alpha(K) \\ +\frac{S_v}{n} + \frac{t_e}{n}(e_{ps} + ne_td) \end{bmatrix}$$

$$+\frac{1}{2}H(\mathrm{m}) \qquad (30)$$

where,

$$H(m) = H_b + H_v\left[\left(1-\frac{D}{r}\right)n - 1 + \frac{2D}{r}\right] + \frac{C_1Dy(B_2r + C_2r^2)}{2r} + \left[2 + n\left(1-\frac{D}{r}\right)\right]E_s,$$

$$\frac{\partial JTC(q,K,l_b,r,n)_{tax}}{\partial K} = H_b\sum\sqrt{\lambda l_b} + \pi\frac{D}{q}\sum\sqrt{\lambda l_b}(\Phi(K) - 1), \qquad (31)$$

$$\frac{\partial JTC(q,K,l_b,r,n)_{tax}}{\partial G} = 1 - t_e\mu_1 + t_e\mu_2(2G), \qquad (32)$$

$$\frac{\partial JTC(q,K,l_b,r,n)_{tax}}{\partial r} = H_v\frac{q}{2}\frac{nD}{r^2} - H_v\frac{q}{2}\frac{2D}{r^2} - \frac{DA_1}{r^2} + \frac{C_1DyC_2rq}{2} - \frac{DA_1}{r^2} - A_2D, \quad (33)$$

$$\frac{\partial JTC(q,K,l_b,r,n)_{tax}}{\partial l_b} = \frac{1}{2\sqrt{\lambda l_b}}H_bK\sum + \frac{1}{2q\sqrt{\lambda l}}\pi D\sum \alpha(K) - \frac{De_i}{q}. \quad (34)$$

For fixed $JTC(q,K,l_b,r,n)_{tax}$, q,K,r,G is a concave function of $l_b \in [l_i, l_{i-1}]$, because

$$\frac{\partial^2 JTC(q,K,l_b,r,n)_{tax}}{\partial l_b^2} = \frac{-1}{4}H_bK\sum l^{\frac{-3}{2}} - \frac{1}{4}\pi D\sum l^{\frac{-3}{2}}\alpha(K) < 0. \quad (35)$$

Hence, for fixed q,K,r,G the minimum joint total cost will occur at the end points of the interval $l_b \in [l_i, l_{i-1}]$. It can be easily shown that for a given value of $l_b \in [l_i, l_{i-1}]$, $JTC(q,K,l_b,r,n)_{tax}$ is a convex function of q,K,r,G (See Appendix for detailed proof). Therefore, for fixed $l_b \in [l_i, l_{i-1}]$ the minimum value of $JTC(q,K,l_b,r,n)_{tax}$ will occur at the point q,K,r,G which satisfies

$$\frac{\partial JTC(q,K,l_b,r,n)_{tax}}{\partial q} = 0, \frac{\partial JTC(q,K,l_b,r,n)_{tax}}{\partial K} = 0, \frac{\partial JTC(q,K,l_b,r,n)_{tax}}{\partial r} = 0 \text{ and } \frac{\partial JTC(q,K,l_b,r,n)_{tax}}{\partial G} = 0.$$

Solving the above equations for $q, \Phi(K), G$ and r, respectively, we get

$$q = \sqrt{\frac{2D\left[O+b(l)+\pi\sum\sqrt{l_b\lambda}\alpha(K)+\frac{S_v}{n}+\frac{t_e}{n}(e_{ps}+ne_td)\right]}{dH(m)}}, \quad (36)$$

$$\Phi(K) = 1 - \frac{H_bq}{\pi D}, \quad (37)$$

$$G = \frac{t_e\mu_1 - 1}{2t_e\mu_2} \quad (38)$$

and

$$r = \sqrt{\frac{2A_1 - H_vq(n-2)}{C_1yC_2q + 2A_2}} \quad (39)$$

where $H_bq(n-2) < 2A_1$ and $H_bq < \pi D$.

5.5.2 Carbon Cap-and-trade

In this situation, the excess X can be sold at e_{tp} per unit to make up the difference carbon emission does not go over K_1. If carbon emission is higher than K_1, the business must purchase or invest in green expenses in order to agree to the limited carbon emission requirements. The carbon trading price e_{tp}, is used here as the average price. The cost function is expressed as

$$JTC(q,K,l_b,r,n,G)_{cap} = O\frac{D}{q} + H_b(\frac{q}{2} + K\sum\sqrt{\lambda l_b}) + \pi\frac{D}{q}\sum\sqrt{\lambda l_b}\alpha(K) + \frac{D}{q}b(l) + S_v\frac{D}{nq}$$

$$+H_v\frac{q}{2}\left[\left(1-\frac{D}{r}\right)n-1+\frac{2D}{r}\right]+C_0D+C_1Dy(B_2r+C_2r^2)\frac{q}{2r}+Dr(C)+F_{pc}\frac{D}{T_c}r+t_ev_c+$$
$$e_{tp}(De_p+\frac{D}{nq}(e_{ps}+ne_td)+\frac{q}{2}\left(2+n\left(1-\frac{D}{r}\right)\right)e_s-G(\mu_1-\mu_2G)-K_1).$$
(40)

Hence, for fixed q,K,r,G the minimum joint total cost will occur at the end points of the interval $l_b\in[l_i,l_{i-1}]$. It can be easily shown that for a given value of $l_b\in[l_i,l_{i-1}]$, $JTC(q,K,l_b,r,n)_{cap}$, is a convex function of q,K,r,G . Therefore, for fixed $l_b\in[l_i,l_{i-1}]$ the minimum value of $JTC(q,K,l_b,r,n)_{cap}$ will occur at the point q,K,r,G which satisfies $\frac{\partial JTC(q,K,l_b,r,n)_{cap}}{\partial q}=0$, $\frac{\partial JTC(q,K,l_b,r,n)_{cap}}{\partial K}=0$, $\frac{\partial JTC(q,K,l_b,r,n)_{cap}}{\partial r}=0$ and $\frac{\partial JTC(q,K,l_b,r,n)_{cap}}{\partial G}=0$.

Solving the above equations for $q,\Phi(K),G$ and r, respectively, we get

$$G=\frac{e_{tp}\mu_1-1}{2e_{tp}\mu_2}, \qquad (41)$$

$$q=\sqrt{\frac{2D\left[O+b(l)+\pi\sum\sqrt{l_b\lambda}\alpha(K)+\frac{S_v}{n}+\frac{e_{tp}}{n}(e_{ps}+ne_td)\right]}{dH(m)}}, \qquad (42)$$

$$\Phi(K)=1-\frac{H_bq}{\pi D} \qquad (43)$$

and

$$r=\sqrt{\frac{2A_1-H_vq(n-2)}{C_1yC_2q+2A_2}} \qquad (44)$$

where $H_bq(n-2)<2A_1$ and $H_bq<\pi D$.

5.5.3 Limited Carbon Emission

The Lagrange multiplier is a method for determining a function's local maximum and lowest values under the restrictions. It is a useful instrument for solving nonlinear problems with equality and inequality constraints. In this instance, both the vendor and the buyer must modify their business processes in order to comply with limited carbon emission K_2. Both stakeholders could engage in sustainability technologies to reduce carbon emission. The cost function is expressed as

$$JTC(q,K,l_b,r,n,G)_{LCE}=O\frac{D}{q}+H_b(\frac{q}{2}+K\sum\sqrt{\lambda l_b})$$
$$+\pi\frac{D}{q}\sum\sqrt{\lambda l_b}\alpha(K)+\frac{D}{q}b(l)+S_v\frac{D}{nq}$$
$$+H_v\frac{q}{2}\left[\left(1-\frac{D}{r}\right)n-1+\frac{2D}{r}\right]+C_0D+C_1Dy(B_2r+C_2r^2)\frac{q}{2r}+Dr(C)+F_{pc}\frac{D}{T_c}r+t_ev_c+$$
$$\mathrm{m}(De_p+\frac{D}{nq}(e_{ps}+ne_td)+\frac{q}{2}\left(2+n\left(1-\frac{D}{r}\right)\right)e_s-G(\mu_1-\mu_2G)-K_2). \qquad (45)$$

Hence, for fixed q,K,r,G the minimum joint total cost will occur at the end points of the interval $l_b\in[l_i,l_{i-1}]$. It can be easily shown that for a given value of $l_b\in[l_i,l_{i-1}]$, $JTC(q,K,l_b,r,n)_{LCE}$, is a convex function of q,K,r,G . Therefore, for fixed $l_b\in[l_i,l_{i-1}]$ the minimum value of

$JTC(q,K,l_b,r,n)_{LCE}$ will occur at the point q,K,r,G which satisfies $\frac{\partial JTC(q,K,l_b,r,n)_{LCE}}{\partial q} = 0, \frac{\partial JTC(q,K,l_b,r,n)_{LCE}}{\partial K} = 0, \frac{\partial JTC(q,K,l_b,r,n)_{LCE}}{\partial r} = 0$ and $\frac{\partial JTC(q,K,l_b,r,n)_{LCE}}{\partial G} = 0$.

Solving the above equations for $q, \Phi(K), G$ and r respectively, we get

$$G = \frac{m\mu_1 - 1}{2m\mu_2}, \quad (46)$$

$$q = \sqrt{\frac{2D\left[O + b(l) + \pi \sum \sqrt{l_b\lambda}\alpha(K) + \frac{S_v}{n} + \frac{m}{n}(e_{ps} + ne_t d)\right]}{dH(m)}}, \quad (47)$$

$$\Phi(K) = 1 - \frac{H_b q}{\pi D} \quad (48)$$

and

$$r = \sqrt{\frac{2A_1 - H_v q(n-2)}{C_1 y C_2 q + 2A_2}} \quad (49)$$

where $H_b q(n-2) < 2A_1$ and $H_b q < \pi D$.

Since it is difficult to find an explicit general solution for $q, \Phi(K), G$ and r, we establish the following iterative algorithm to find the optimal $q, \Phi(K), G$ and r.

Algorithm

Step 1. Set $n = 1$

Step 2. For every $l_g, g = 0,1,2, \ldots n$, perform step (a) to (e).

(a) Let K_0 this implies that $\Phi(K_0) = 0.3989, (i = 1,2)$.

(b) Substitute K_{i0} , $\Phi(K_{i0})$ into Eqs.37,43 and 48.

(c) Use q^{0i}, to determine $\Phi(K_{i1})$, (i=1,2) from Eqs.36, 42 and 47.

(d) Use $\Phi(K_{i1})$, to find the value of K_{i1}, $(i = 1,2)$ from the normal table, and hence find $\alpha(K_{i1})$.

(e) Repeat steps from (b) to (d) until no change occurs in the values of q^{ig} and K_{ig}.

Step 3. Using the values of q^{ig*} and K_{ig}^* determine r^*, (i=1,2) from Eqs. 39, 44 and 49.

Step4. Compute $JTC(q^{ig*}, K_{ig}^*, l_g, r^*, n)$ using the values of q^{ig*}, r^* and K_{ig}^*.

Step 5. Set $n = n + 1$, repeat the Steps from 2 to 4.

Step 6. Find the value of $min_{g=0,1,2...,n} JTC(q^{ig*}, K_{ig}^*, l_g, r^*, n)$ and set

$JTC(q^{iS*}, K_{iS}^*, l_S, r^*, n)$

$= min_{g=0,1,2...,n} JTC(q^{ig*}, K_{ig}^*, l_g, r^*, n)$ then

$(q^{iS*}, K_{iS}^*, l_S, r^*, n)$ is the optimal solution.

6.Numerical example

The data gathered here are used to illustrate the suggested model's ideal values for numerical investigations. $D = 200$ units/year, $S_v = \$4000$ /setup, $O = \$100$/setup, $H_v = \$11$/units/week, $H_b = \$12$/units/week, $A_1 = 35 \times 10^3$, $A_2 = 0.1$,$C_0 = 12$, $\mu_1 = 15$, $\mu = 5$, $K_1 = 5000$, $t_e = 5$,$\phi_1 = 5$, $\phi_2 = 2$, $e_s = 2$, $e_{ps} = 8$,$e_p = 5$,$\mu_2 = 0.01$, $C_1 = 5$, $K_2 = 10000$,$\lambda = 2$, $u = 500$, $y = 5$,$\sum = 9$, $B_1 = 3$, $e_t = 0.1$, $e_{tp} = 7$, $m = 6$. For linear production, we take $\frac{1}{F(r)} = \frac{1}{10^{-4}r}$. The lead time has three components with data shown in Table 2. We obtain the optimal solutions of Example for normal distribution by executing the proposed Algorithm. The optimal results are presented in Table 3 and Figures 15-20 show the convexity of the total cost function.

The vendor's green investment is also a key contributor to the sustainability of the paper. Based on the three carbon plans, the limited carbon emission policy offers the lowest JTC cost possible. For instance, if the chemical or pharmaceutical industries produce greater carbon emissions, the supplier or producer must spend in reducing the emissions. The sustainability investment reduces the carbon emission and yearly total cost in this scenario. Finally, this paper's findings become ones that may be used by industries to save costs and boost profits.

Table 2: Lead time data.

Lead time component	Normal duration (days)	Minimum duration (days)	Unit crashing cost
1	20	6	0.4
2	20	6	1.2
3	20	6	5.0

6.1 Sensitivity analysis

The effects of changing the parametric values shown in Table 3. This analysis is performed by altering parameters ranges S_v, H_v, O and H_b by ?50%, −25%, +25%, and +50%. From Table 4, Table 5 and Table 6, we analysed that S_v, H_v, H_b are more sensitive than O. In Figures 1-14 is provided to view sensitivity of parameters S_v, H_v, O and H_b while altering the values with respect to the joint total expected cost. From the graphical representations, it is understood that the parameters S_v, H_v and H_b are directly proportional to the joint total cost of the supply chain. i.e. As the values of S_v, H_v, O and H_b increases/decreases, the total cost of the supply chain increases/ decreases. Under the carbon constrained integrated single vendor-buyer model establishes a link between the rate of production and the quality of the product. The following are the major repercussions of this study.

6.2 Managerial Implications

1. As the rate of production increases the quality of the product gradually decreases.
2. Assuming the quality functions of various carbon policies to be linear, the manager can control the production of defective products.
3. Crashing costs are added to reduce lead time to provide better service to their customers.
4. Centralized decision-making plays a significant role in minimizing total collective expected cost.
5. Based on sensitivity analysis, the seller's total expected cost gradually increases by raising the parameter values of setup costs S_v and holding costs H_v.
6. The buyer's anticipated total cost rises along with their holding cost H_b.

7. Conclusion

7.1 Finding

The proposed research analyses the effects of linear production rate on the quality of items under an integrated two-level supply chain model with different carbon policies. In order to make the model more accurate, this research is thought to analyze the combined total estimated cost of the complete system under difficult conditions. The usage of SI technology helps to decrease carbon emission and damaged goods. The energy consumption was created to increase energy efficiency, and it is divided into categories based on the manufacturing and reworking processes.

7.2 Recommendations for future research.

Future extensions of this research may taking into account lateral supply multi-echelon inventory systems using a parallel series system. A traditional inventory and vendor managed inventory model with centralised and decentralised agreement under closed-loop green supply chain is another extension that might be made.

Competing interests

The authors declare that they have no competing interests.

Funding

This research received no external funding.

Data Availability

All data generated and analyzed during the study are presented in the manuscript.

Author's contributions

All authors have read and agreed to the published version of the manuscript.

Acknowledgements

The authors are grateful to the anonymous reviewers and the editor for their insightful and constructive comments and helpful suggestions, which have led to a significant improvement in the earlier version of the paper.

Table 3: The optimal solution of Example for normal distribution model

			Linear production		Quadratic production	
	n	l_b	q	$JTC(q,R,l_b,r,n)$	q	$JTC(q,R,l_b,r,n)$
Carbon Tax	2	3	1265.28	$1.80e^{10}$	638.25	$3.96e^{10}$
Cap-and-Trade	2	3	1260.43	$1.78e^{10}$	638.33	$3.96e^{10}$
Limited Emission	2	3	1259.43	$1.77e^{10}$	638.25	$3.97e^{10}$

Table 4: Effect of change in various parameters of Carbon Tax.

		Linear production		Quadratic production	
Parameter	% changes	q	$JTC(q,R,l_b,r,n)$	q	$JTC(q,R,l_b,r,n)$
S_v	-50%	1265.73	18009219239	637.53	39684489266
	-25%	1265.28	18009218739	638.10	39684488766
	25%	1265.52	18009220739	638.12	39684490766
	50%	1265.28	18009221239	638.25	39684491266
H_v	-50%	1265.73	18009220234	637.73	39684490261
	-25%	1265.28	18009220231	636.45	39684490258
	25%	1265.52	18009220241	638.20	39684490268
	50%	1265.28	18009220245	638.36	39684490272
O	-50%	1265.73	18009220189	638.24	39684490216
	-25%	1265.73	18009220164	638.21	39684490191
	25%	1265.73	18009220214	638.45	39684490241
	50%	1265.73	18009220289	638.85	39684490316
H_b	-50%	1265.73	18009208531	637.89	39684467842
	-25%	1265.73	18009202677	638.12	39684456630
	25%	1265.73	18009226093	638.31	39684501478
	50%	1265.73	18009231947	638.41	39684512690

Table 5: Effect of change in various parameters of cap-and-Trade.

		Linear production		Quadratic production	
Parameter	% changes	q	$JTC(q,R,l_b,r,n)$	q	$JTC(q,R,l_b,r,n)$
S_v	-50%	1260.85	17843161405	638.25	39684476200
	-25%	1260.80	17843160905	638.20	39684475700

	25%	1260.75	17843162905	638.38	39684477700
	50%	1260.78	17843163405	638.41	39684478200
H_v	-50%	1260.85	17843162399	638.20	39684477195
	-25%	1260.80	17843162396	638.31	39684477192
	25%	1260.78	17843162407	638.36	39684477203
	50%	1260.76	17843162410	638.40	39684477192
O	-50%	1260.85	17843162355	638.20	39684477150
	-25%	1260.75	17843162330	638.26	39684477125
	25%	1260.70	17843162380	638.50	39684477175
	50%	1260.85	17843162455	638.51	39684477250
H_b	-50%	1260.80	17843150829	638.10	39684454776
	-25%	1260.75	17843145041	638.15	39684443564
	25%	1260.85	17843168192	638.28	39684488413
	50%	1260.88	17843173980	638.30	39684499625

Table 6: Effect of change in various parameters of Limited Emission

		Linear production		Quadratic production	
Parameter	% changes	q	$JTC(q, R, l_b, r, n)$	q	$JTC(q, R, l_b, r, n)$
S_v	-50%	1259.43	178431864337	637.80	39684501229
	-25%	1259.41	17843185933	637.90	39684500729
	25%	1259.40	17843187933	638.28	39684502729
	50%	1259.44	17843188433	638.75	39684503229
H_v	-50%	1259.43	17843187427	637.60	39684502224
	-25%	1259.42	17843187424	637.75	39684502221
	25%	1259.41	17843187435	638.30	39684502231
	50%	1259.43	17843187438	638.45	39684502235
O	-50%	1259.44	17843187383	638.1	39684502179
	-25%	1259.42	17843187358	638.4	39684502154
	25%	1259.41	17843187408	638.30	39684502204
	50%	1259.43	17843187483	638.36	39684502279
H_b	-50%	1259.45	17843175857	638.20	39684479805
	-25%	1259.42	17843170069	638.15	39684468593

	25%	1259.41	17843193220	638.24	39684513441
	50%	1259.43	17843199008	638.41	39684524653

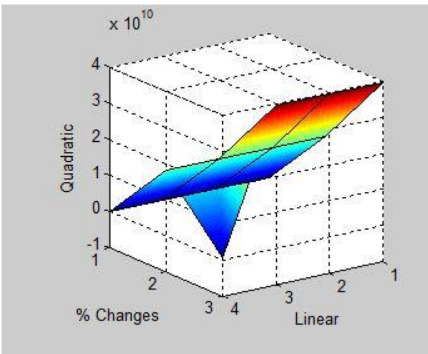


Figure1:*Sensitivity analysis of* O *in Carbon Tax*

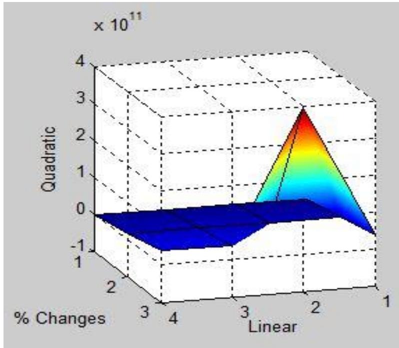


Figure 2: *Sensitivity analysis of* O *in Cap and Trade*

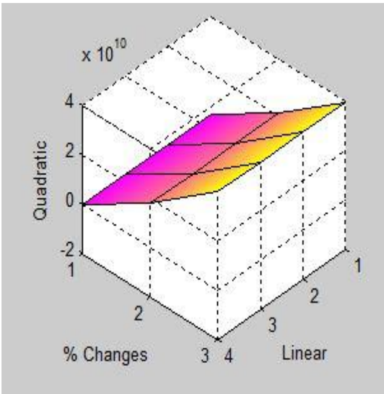


Figure 3: *Sensitivity analysis of* O *in Limited Emission*

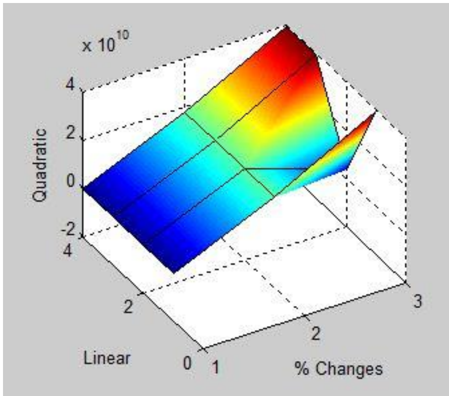


Figure 4: *Sensitivity analysis of* H_v *in Carbon Tax*

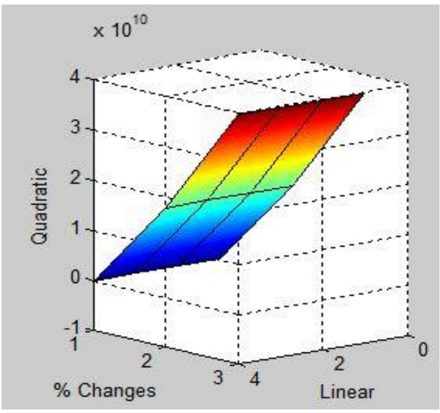


Figure 5: *Sensitivity analysis of* H_v *in Carbon Cap and Trade*

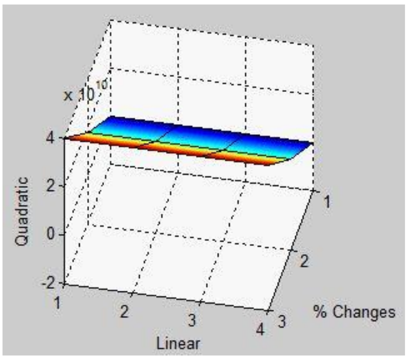


Figure 6: *Sensitivity analysis of* H_v *in Limited Emission*

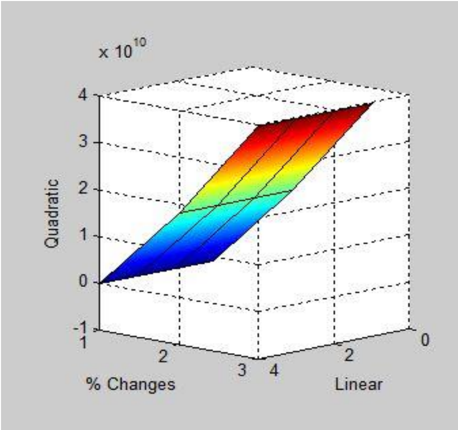


Figure 7: *Sensitivity analysis of* A_b *in Carbon Tax*

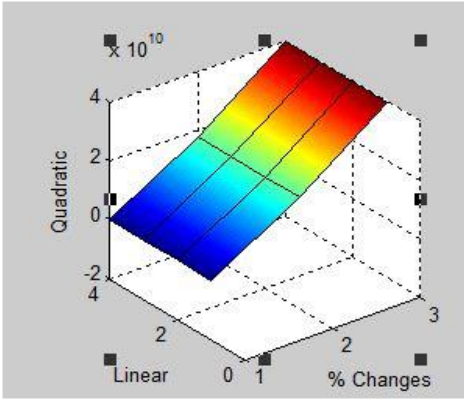


Figure 8: *Sensitivity analysis of* A_b *in Carbon Cap and Trade*

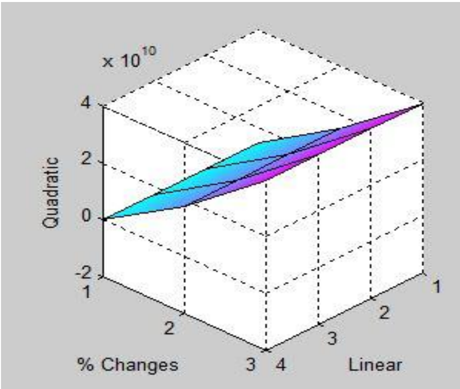


Figure 9: *Sensitivity analysis of* A_b *in Limited Emission*

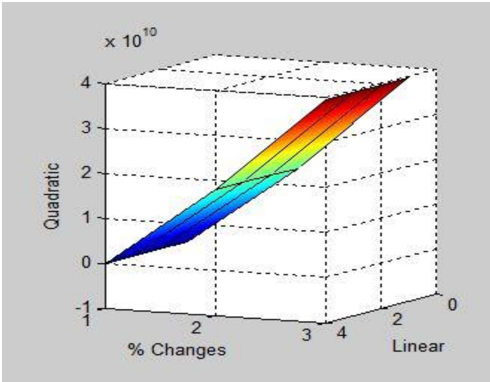


Figure 10: *Sensitivity analysis of* H_b *in Carbon Tax*

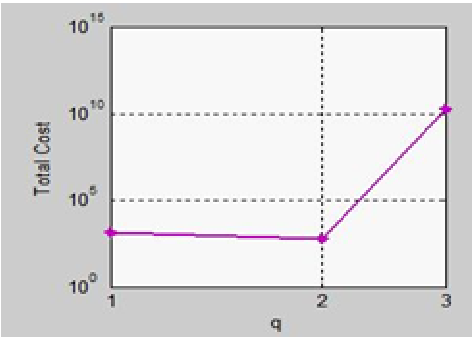


Figure 13: *Sensitivity analysis of* H_b *in Limited Emission*

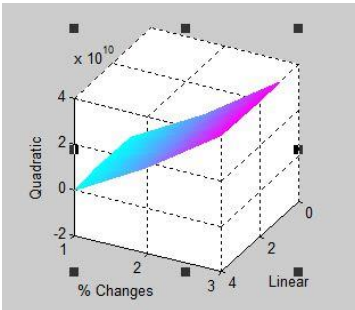


Figure 12: *Sensitivity analysis of* H_b *in Carbon Cap and Trade*

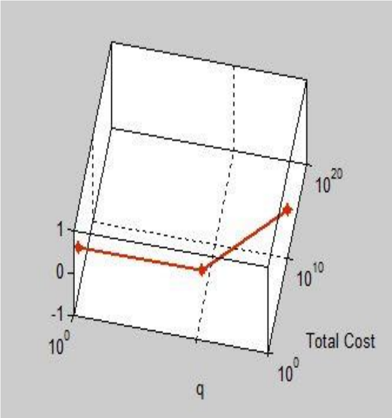


Figure 14: *Optimal solution for Carbon Tax Linear production*

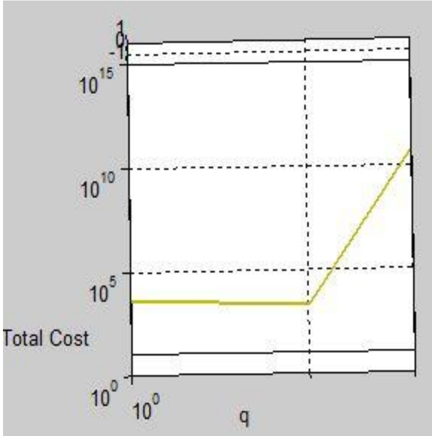


Figure 15: *Optimal solution for Carbon Tax quadratic production*

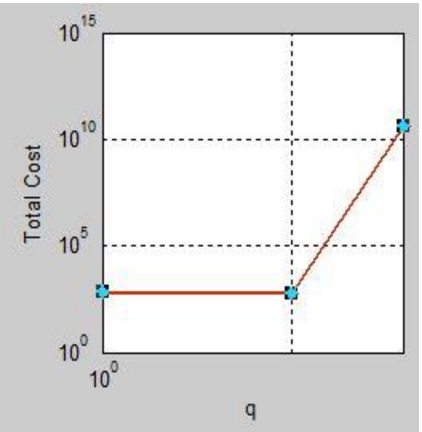
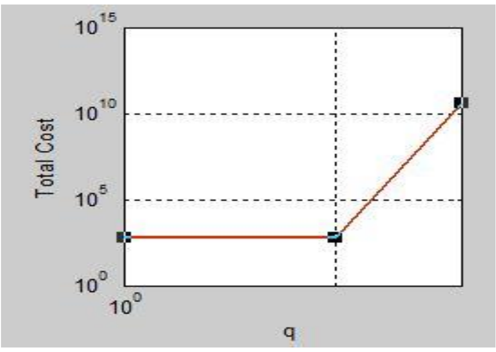


Figure 16:*Optimal solution for Carbon Cap and Trade Linear production*

Figure 18: *Optimal solution for Carbon Cap and Trade quadratic production*



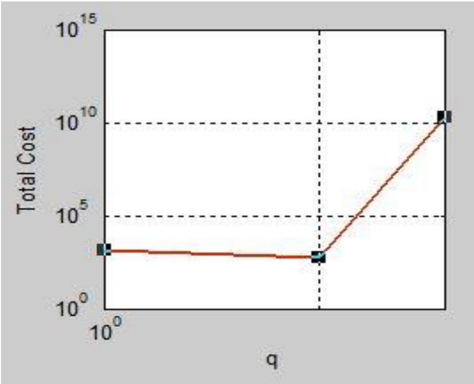


Figure 19:*Optimal solution for Limition Emission Linear production*

Figure 20: *Optimal solution for Limited Emission quadratic production*

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Appendix

We want to prove the Hessian matrix of $JTC(q,R,K,G,l_b,r,n)$ for fixed l_b is positive definite. We first obtain the Hessian matrix H as follows:

$$H = \begin{pmatrix} \frac{\partial^2 JTC(.)}{\partial q^2} & \frac{\partial^2 JTC(.)}{\partial q \partial K} & \frac{\partial^2 JTC(.)}{\partial q \partial r} & \frac{\partial^2 JTC(.)}{\partial q \partial G} \\ \frac{\partial^2 JTC(.)}{\partial K \partial q} & \frac{\partial^2 JTC(.)}{\partial K^2} & \frac{\partial^2 JTC(.)}{\partial K \partial r} & \frac{\partial^2 JTC(.)}{\partial K \partial G} \\ \frac{\partial^2 JTC(.)}{\partial r \partial q} & \frac{\partial^2 JTC(.)}{\partial r \partial K} & \frac{\partial^2 JTC(.)}{\partial K^2} & \frac{\partial^2 JTC(.)}{\partial r \partial G} \\ \frac{\partial^2 JTC(.)}{\partial G \partial q} & \frac{\partial^2 JTC(.)}{\partial G \partial K} & \frac{\partial^2 JTC(.)}{\partial G \partial r} & \frac{\partial^2 JTC(.)}{\partial G^2} \end{pmatrix}.$$

$$\frac{\partial JTC(.)}{\partial q^2} = \frac{D}{q^3}\left[O + b(l) + \pi \sum \sqrt{l_b \lambda}\alpha(K) + \frac{S_v}{n} + \frac{t_e}{n}(e_{ps} + ne_t d)\right]$$

$$\frac{\partial^2 JTC(.)}{\partial K^2} = \pi \frac{D}{q} \sum \sqrt{\lambda l_b}(\Phi(K))$$

$$\frac{\partial^2 JTC(.)}{\partial G^2} = 2t_e \mu_2$$

$$\frac{\partial^2 JTC(.)}{\partial r^2} = -2H_v \frac{q}{2}\frac{nD}{r^3} + 2H_v \frac{q}{2}\frac{2D}{r^3} + 2\frac{DA_1}{r^3}$$

$$\frac{\partial^2 JTC(.)}{\partial q \partial K} = \frac{\partial^2 JTC(.)}{\partial K \partial q} = -\pi \frac{D}{q^2} \sum \sqrt{\lambda l}(\Phi(K) - 1)$$

$$\frac{\partial^2 JTC(.)}{\partial q \partial r} = \frac{\partial^2 JTC(.)}{\partial r \partial q} = \frac{H_v nD}{2r^2} - \frac{H_v D}{r^2} + \frac{C_1 Dyrb_1}{2r^2} - \frac{C_1 Dyb_1 r}{2r^2}$$

$$\frac{\partial^2 JTC(.)}{\partial K \partial r} = \frac{\partial^2 JTC(.)}{\partial r \partial K} = 0$$

For principal minor of H is

$$|H_{11}| = \frac{\partial JTC(.)}{\partial q^2} = \frac{D}{q^3}\left[O + b(l) + \pi \sum \sqrt{l_b \lambda}\alpha(K) + \frac{S_v}{n} + \frac{t_e}{n}(e_{ps} + ne_t d)\right] > 0$$

Second principal minor of H is

$$|H_{22}| = [(\frac{\partial JTC(.)}{\partial q^2}) \times (\frac{\partial JTC(.)}{\partial K^2})] - [(\frac{\partial^2 JTC(.)}{\partial q \partial K} \times \frac{\partial^2 JTC(.)}{\partial K \partial q})]$$

$$= \quad [\frac{D}{q^3}\left[O + b(l) + \pi \sum \sqrt{l_b \lambda} \alpha(K) + \frac{S_v}{n} + \frac{t_e}{n}\left(e_{ps} + n e_t d\right)\right] \times \pi \frac{D}{q} \sum \sqrt{\lambda l_b}(\Phi(K))] - [-\pi \frac{D}{q^2} \sum \sqrt{\lambda l}(\Phi(K) - 1)] > 0$$

Similarly by expanding and simplifying the third and forth principal minor determinant of H. Where $JTC(.)$=$JTC(q, R, l_b, r, n)$Therefore, all principal minors of H are positive definite.