Hamiltonian Graph Theory Using Multiplicative Matrix Square Divisor Cordial Labeling for Complex Network Structures

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Abstract: - Recently, networks have become increasingly complex, requiring advanced mathematical solutions to optimize communication. This paper proposes a mathematical solution for complex network structures based on Hamiltonian Graph Theory (HGT) and Multiplicative Matrix Square Divisor Cordial Labeling (MMSDCL) models. Graph theory provides a framework for representing and analyzing networks, with graph-theoretic measures characterizing the network structure. The proposed model improves network construction performance and offers a redundant solution for finding the least distance to connect network nodes for optimal communication. Compared to previous graph theory models, the proposed solution simplifies network connections, enhancing network communication. Mathematical models of complex networks aid in designing efficient algorithms for routing traffic and reducing complex network structures.

Keywords: graph theory, square matrix divisor, labelling, network queuing theory, Distance theory, network path discovery.

1. Introduction

Recently, networks become more sophisticated in the advanced human life cycle by connecting more structured principles in the communication environment. The mathematical study of complex networks is a rapidly growing field. New mathematical tools and techniques are being developed to help us understand these complex systems. This research has a major impact on many fields, providing solutions to complex network structures using the Hamiltonian approach [1]. The development of network structure creates complex structures due to inappropriate mathematical solutions to finding the distance theory to connect the nodes in the network based on Hamiltonian cycles using tree graphs [2]. Understanding the mathematical principles that govern these network structures is crucial for predicting the routing problems and controlling the behaviour of complex networks based on the distance theory.

This research proposes a mathematical solution for complex network structures based on HGT [3] and MMSDCL [4, 5, 6] models implemented. Graph theory provides a framework for representing and analyzing networks as mathematical objects of network nodes. This Graph theory consists of nodes (or vertices) and edges (or links) that connect the nodes based on the Least distance model. The structure of a network can be characterized by graph theory metrics such as the number of nodes and edges, average size of nodes, clustering coefficient, and length of the shortest path between two nodes.

A complex network can be viewed as a set of interacting nodes whose edges represent interactions. Maximal Queuing Little Theory (MQLT) can be used to study the phase network transitions that occur in complex networks based on Euclidian Distance Theory (EDT) and Linguistic probabilistic search model to reduce the maximum distance with support of the Transition Queuing Labeling Model (TLQM) such as the transition from a disordered to an ordered state. In addition to graph theory and statistical mechanics, it creates a dynamical system to provide a redundant network framework for understanding the evolution of complex networks over time of changing dynamic nodes. Based on the Queuing Information Theory (QIT) provides a framework for quantifying the amount of information that is contained in a network to be connected to solve the connective node edges problems.

This proposed model supports improving the performance of network construction and gives the mathematical redundant solution to find the least distance to connect the network nodes for the best communication. Compared to the previous graph theory models and mathematical solutions, the proposed solution makes network connections less complex and improves network communication. The mathematical models of complex networks have been used to design efficient algorithms for routing traffic and making connective node edges to reduce complex network structures.

2. Related Preliminaries

The concept of Hamiltonian cycles and paths has been a subject of significant interest in graph theory and its applications in various domains. This essay explores the application of Hamiltonian cycle theory in computer science and information systems, drawing insights from recent research and publications in the field.

Hamiltonian Cycle in Computer Science: The study by V. Traneva and S. Tranev, titled "An Intuitionistic Fuzzy Hamiltonian Cycle by Index Matrices," was presented at the 2020 15th Conference on Computer Science and Information Systems (FedCSIS) in Sofia, Bulgaria. The novel analyzes the use of intuitionistic fuzzy sets and index matrices in Hamiltonian cycles. The authors provide a novel approach to modelling and analyzing Hamiltonian cycles in fuzzy environments, offering valuable contributions to computer science and information systems.

Furthermore, the analysis results in "Hamiltonian Circuits in Tree Graphs", published by R. Cummins in IEEE Transactions on Circuit Theory in 1966, revealed the fundamental properties of tree graphs. This classic study remains relevant in understanding the structural characteristics of Hamiltonian cycles and their significance in graph theory.

Applications in Information Systems In the realm of information systems, J. B. Leite and J. R. S. Mantovani's research, "Distribution System State Estimation Utilizing the Hamiltonian Cycle Theory," published in the IEEE Transactions on Smart Grid in 2016, demonstrates the practical application of Hamiltonian cycle theory in evaluating the state of a distribution system. The authors propose an innovative approach that leverages the principles of Hamiltonian cycles to enhance the efficiency and accuracy of state estimation in smart grid systems.

Implications and Future Directions The studies above underscore the diverse applications of Hamiltonian cycle theory in computer science and information systems. From theoretical advancements in fuzzy environments to practical implementations in smart grid systems, the relevance of Hamiltonian cycles transcends traditional graph theory and extends into real-world problem-solving domains.

Future research should integrate the Hamiltonian cycle theory with modern technologies such as artificial intelligence, machine learning, and optimization algorithms. By harnessing the inherent properties of Hamiltonian cycles, researchers can unlock new avenues for addressing complex computational and system optimization challenges. The application of Hamiltonian cycle theory in computer science and information systems continues to yield valuable insights and practical solutions across diverse domains. The seminal works discussed in this essay exemplify the enduring relevance of Hamiltonian cycles, inspiring further exploration and innovation in the intersection of graph theory, computational algorithms, and system optimization.

3. Proposed system

The Hamiltonian Graph Theory (HGT) model is utilized to identify if a network contains a Hamiltonian cycle (According to Wadee et al., 2016); each node will be visited once. This is essential for comprehending the connectivity and traversal characteristics of a network. The MMSDCL model is employed to label the nodes and edges of a network in a way that allows for analysis and understanding of specific network properties to find the routes along cyclic graphs with the least distance.

The proposed mathematical solution offers a comprehensive and efficient approach to analyzing complex network structures by integrating these two models.

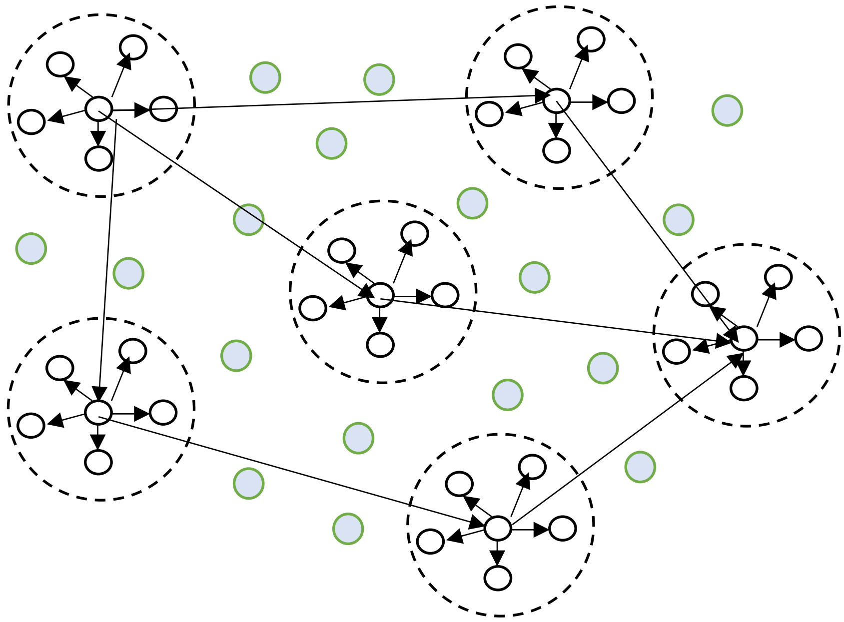

Fig. 1. Dynamic Node Construction Hamiltonian Graph Theory

Implementing this solution involves developing algorithms and computational methods to efficiently identify the Hamiltonian cycle in a network and assign the MMSDCL labels to the nodes and edges. Figure 1 shows the Dynamic Node construction HGT. These algorithms and methods can then analyze the network properties and predict their behaviour.

In summary, the proposed mathematical solution based on HGT and MMSDCL models provides a powerful tool for understanding and analyzing complex network structures. By leveraging the principles of graph theory and advanced labelling techniques, this solution aims to aid in developing more efficient and reliable methods for network analysis.

A. Hamiltonian Cycle for Network Path Identification

A Hamiltonian cycle is a concept in graph theory that describes a closed path in an undirected graph, where each vertex is visited exactly once and the path ends at the initiating vertex, as Traneva et al. (2020) discussed. In other words, the loop in the graph passes through each vertex exactly once and returns to the first vertex, forming a closed loop systematically [1]. Furthermore, G-undirected graph, V-vertices and E-edges are Hamiltonian cycles.

Each vertex in Y $\{Y_1, Y_2, Y_3, Y_4 \ldots Y_h\}$ occurs sequentially to analyze accuracy. Furthermore, every consecutive pair of vertices $(Y_a, Y_a + 1)$ in M $(Y_a, Y_a + 1)$ is an edge. Moreover, the edge(Y_f, Y_a) must be there to complete the loop. Given an undirected graph C with vertices Y and edges M, a Hamiltonian path is a sequence of vertices (Y_1, Y_2, Y_3, Y_4) such that each vertex of Y appears exactly once.

Theorem 1: For each odd integer $f \geq 5$ such that, $f - 1 = 1 \bmod 4, b_f$ is $L^{+1} - \text{Hypo} - \text{edge} - \text{Hamiltonian} - p - \text{laceable}$ where $1 \leq p \leq \text{diamb}_f$[4]

Theorem 2: The flower snarkb_f, if for every positive integer, $f \geq 7$ such that$n - 1 = 1 \bmod 4$, then b_f is $L^{+1} - \text{Hypo} - \text{edge} - \text{Hamiltonian} - p^* - \text{laceable}$ where $1 \leq p \leq \text{diam}b_f$[4]

Theorem 3: For every positive integer $f \geq 2$, j_{2n} is $L^{+0} - \text{Hypo} - \text{edge} - \text{Hamiltonian} - p^* - \text{laceable}$ for $p = 1$ and L^{+1} $\text{Hypoedge} - \text{hamiltonion}$ for $2 \leq p \leq \text{diam}b_{2f}$[5]

Theorem 4: Let t_e and t_f be two paths. If e and f are odd integers such that $(e, f) \geq 5$ and $(e \geq f)$ then the Cartesian product $t_e X t_f$ is L^{+1} hypo edge $\text{Hamiltonion} - p^* - \text{laceable}$ for odd $p(1 \leq p \leq \text{Diam}C)$ [5]

Theorem 5: Let t_e and t_f be two paths. If e and f are even integers such that $(e, f) \geq 4$ and $(e \geq f)$ then the Cartesian product $t_e X t_f$ is L^{+1} hypo edge $\text{Hamiltonion} - p^* - \text{laceable}$ for odd $p(1 \leq p \leq \text{Diam}C)$ [5]

B. Constructing Tree Graphs of Separable Node groups

According to Cummins et al, 2018 assume a separable network f consisting of $f_1, f_2, \dots, f_L$ elements. Let t, $t_1, \dots,$ andt_s be the network tree and the set of its constituent sub-networks, respectively. Then, an equal statement of the Percival theorem [2] is provided by the outer creation of the set t.

$$t = t_1 \times t_2 \times .., t_s$$

For the case S=2, if $t_1 = \{q_1, q_2, \dots, q_L\}$ and $t^2 = \{q^1, q^2, \dots, q^3\}$then

$$t = \{q_1 q^1; q_1 q^2; \dots; q_L q^3\}$$

Or

$$t = \{q_{1,1}; q_{1,2}; \dots, q_{L,O}\}$$

Where

$$q_{u.v} = q_u q^v$$

(In the representation of set theory, $q_{u,v} = q_u \cup q^v$. The algebraic representation is used to manage fundamental tree transformations) in this case, L is sett$_u$, and s are setst$_v$, Described as:

$$s_u = \{q_{u,v} | q_u \varepsilon t_1, q^v \varepsilon t_2, v = 1,2,..,o\}$$

and

$$s^v = \{q_{u,v} | q_u \varepsilon t_1, q^v \varepsilon t_2, u = 1,2,..,kL\}$$

The following attributes presently:

A) $s_u \cap s^v = q_{u.v}$;

B) $s_\alpha \cap s_\beta = \phi$ for $\alpha \neq \beta, \alpha, \beta = 1,2 \dots, L$;

C) $s^\alpha \cap s^\beta = \phi$ for $\alpha \neq \beta, \alpha, \beta = 1,2 \dots, o$;

D)$\bigcup_{u=1}^{o} s_u = \bigcup_{u=1}^{o} s^u = t$

Moreover, the relation $q_{u.v} = q_u q^v$establishes a one- to-one equality between the components $q^v \varepsilon t_2$ and the elements $q_{u,v}$ of $s_{u,}$ as well as between the features q_u of set t_1 and the elements $q_{u.v}$ of s^v.

Now, let's implement the properties of a tree graph [2] associated with a group of trees with the graph related to the set of trees mentioned above and a subscript.

Each vertex in the graph corresponds to one tree in the collection. For each, the corresponding vertices of the tree and the tree graph are denoted by the same symbols;

When trees q_i andq_j, are related by a base tree transformation, there is an edge (q_i, q_j) that intersects the vertices of q_i and q_j.

$$q^{\alpha} = (m_a/m_b)q^{\beta}$$

The vertices of graph t_2 correspond one-to-one with the vertices of graphs_u, as there is a correspondence between the sets t_2 and s_u. If t^2 has an edge (q^{α}, q^{β}), then there is an essential tree transition between t_2 and the tree $q^{\alpha}$of q^{β}.

Sets_u contains $s_{u,\alpha}$and $s_{u,\beta}$

$$q_{u,\alpha} = q_u q^{\alpha} = q_u(m_a/m_b)q^{\beta} = (m_a/m_b)q_u q^{\beta}(m_a/m_b)q_{u,\beta}$$

Thus, graph s_u contains edges$(q_{u,\alpha}; q_{u,\beta})$, establishing a one-to-one correspondence between the edges of t^2 and s_u, preserving the lookalike relationship. Similarly, each graph s^v is isomorphic to t_1. Therefore, graphss_u and s^v have the following properties:

(G) Let $s_u \sim t_2$ and $s^v \sim t_1$ for $u = 1, 2, \dots, L$ and $v = 1, 2, \dots, o$ every edge of graph t is in one of the subgraphs of $s_u$or s^v. It can be seen that some network elementsm_a, m_v if $(q_i,\ q_j)$ is an edge of t.

$$q_i = (m_a/m_b)q_j$$

But

$$q_i = q_{u,v} = q_u q^v, \text{and } q_j = q_{u',v'} = q_{u'} q^{v'},$$

So that,

$$q_u q^v = (m_a|m_b)q_{u'} q^{v'}$$

Since f is separate, m_a and m_b must belong to the same n element; hence, $u = u'$ or$v = v'$. Edge (q_i, q_j) is accordingly confined in either $s_u$ors^v.

(H) if (q_i, q_j) is an edge in graph t, then there exists an integer $u, 1 \le u \le L$ such that (q_i, q_j) is in subgraphs_u, or there exists an integer, $1 \le v \le o$, such that $(q_i,\ q_j)$ is in subgraphs^v.

C. Dynamic Node $B_{n,n}$is a Square Multiplicative Graph

Multiplicative matrix square divisor cordial labels are used to assign labels to the vertices of a graph. These labels are set so that Reena et al. (2014) cover the maximally adjacent node at some conditional distance. In this labelling, the vertices are labelled with positive integers as support nodes, and the labels must satisfy a specific property related to the graph's adjacency matrix. The concept of matrix multiplication and square divisor cordial labelling has applications in connecting the edges of a node.

Evaluate $B_{n,n}$ with vertex set $\{u, v, u_i, v_j, 1 \le i \le d\}$ where u_i, v_j pendant vertices are. If $C = B_{n,n}$ then $|V(C)| = 2d + 2$ and$|G(C)| = 2_d + 1$. We define vertex labelling $h: V(C) \to \{1, 2, \dots, 2_d + 2\}$ as follows$n(x) = 1$,

$h\ (v) = 2_d + 1,$

$h(v_i) = 1 + i;\ 1 \le i \le d$

$h(v_i) = d + 1 + i;\ 1 \le i \le f - 1$

$h\,(v_d) = 2_d + 2$

The prompted function $h *: G_{(C)} \to d$ defined by $h * (uv) = h(u)^2 . h(v)^2$ for every $uv \in G_{(e)}$ −distinct. Hence, Bistar B_{nn} −square multiplicative graph.

D. Distance Theory for Connecting Node Topology

Nedjatia et al. (2020) demonstrate that a Hamiltonian graph is a path in a graph that visits every vertex exactly once. This concept is fundamental in graph theory and has practical applications in various fields, such as computer science and telecommunications. A Hamiltonian path can be constructed using little equations and algorithms to determine the optimal path efficiently. By utilizing these methods, the network path can be built with precision and accuracy, ensuring the efficient traversal of the graph.

In Little Queuing theory, the average number of nodes, the average time a node spends in the current transmission medium, and the average arrival rate of nodes in the current medium are considered. The equation based on Little's Law is:

$$K = \sum_{a=0}^{f} \lambda q$$

Where: K- Average number of nodes, λ = average arrival rate of nodes, q = Average time spent by a node on the system.

$$K = \sum_{a=0}^{f} w\,.\,\lambda q\; h(v_d)$$

This Dynamic rule of the sum of all nodes in little theory $K \to f$ nodes in the equation provides a powerful tool for understanding and analyzing systems with queues to evaluate the routes by the dynamic distribution having the nodes f+1 in Hamiltonian path optimized.

Algorithm: 1

```
Hamiltonian (w−→ k)
        {
                Repeat
                {
                        For each Next Value (w)
                        If ((X[w]  = 0) then, return)
                        Else
                                Hamiltonian(w + 1);
                }
                        Until (F_alse)
        }
        Algorithm Next Value (w)
        {
                Repeat
                {
                        x[w]  = x[w + 1];
                        If (x [w]  = 0) the return
                            S_um = 0
                        For each j = 1 to w − 1 do
                        {
```

```
                    Sum = Sum([w])then break;
                    If (x[j] = x[w]) then break;
            }
            Return (x[w].........x[w − 1] [x w])
        }
        Until (False)
    }
```

The Proposed distance theory refers to measuring the distance between nodes in a network; the humiliation cycles concern the overall structure and efficiency of the network to formalize the network solutions. Distance theory is critical for connecting node topology as it helps determine the most optimal way to connect nodes in a network. By calculating the distance between nodes, network topology minimizes latency, maximizes bandwidth utilization, and ensures efficient communication between nodes.

E. Dynamic Path Construction

Dynamic path construction based on graph cycles is a process used in network routing and mathematics to determine the shortest path between two points in a graph that have cycles. A cycle is a sequence of vertices that form a closed loop in a graph, where the start and end vertices are the same. In dynamic path construction, the algorithm continuously updates the shortest path between two points as it explores the graph, considering the presence of cycles. This is achieved by keeping track of the distances between vertices and updating them whenever a shorter path is found. One method for establishing a path between two points in a graph with negative edge weights is affinity labelling. This technique can identify the shortest path between two points on the graph's edge. The algorithm iterates through this process until the shortest route is determined.

The vertex transformation of the cycle with the triangle $m_d\ (1, 1, d - 5)$ allows for cordial labelling of the square divisor.

Proof. Consider that triangle $m_d\ (1, 1, d - 5)$ represents the cycle C; Let us assume that the switching vertices of e are $u_1, u_2, \ldots, u_d$. Assume that the chords of e are $g_1 = v_1 v_{d-1},\ g_2 = v_1 v_3, g_3 = v_{d-1} v_3$.

Vertices u_i and u_j of degree 2, which are isomorphic to all i and j, are switched to construct the graph. Similarly, the isomorphic 4th degree's vertices, u_i andu_j, are switched to create the graph. Hence, we can consider two scenarios: (i) permuting any 2-degree vertex of e and (ii) permuting any 4-degree vertex of e. Furthermore, the generality of the vertices of e for u_1 can be applied to the switches of e_{u1} without changing the permuted vertex u_1.

To define the labelling functiond: $v(e) \rightarrow \{1, 2, \ldots, |v(e)|\}$, we assume the subsequent points.

Case 1: The degree of $u_1$is 2, and its vertices f and total edges $2f - 2$

Subcase I: When n is an even number.

$h(u_1) = 1, h(u_2) = 2, h(u_3) = 4,$

$h(u_4) = 3, h(u_5) = 6, h(u_6) = 5,$

$h(u_d) = d - 1, h(u_{d-1}) = d, h(u_{d-3}) = d - 3, h(u_{d-3}) = d - 2,$

$h(u_i) = i, 7 \leq i \leq d - 4.$

Subcase II: When h is an odd number.

$h\ (u_1) = 1, h(u_2) = 2, h(u_3) = 4,$

$h(u_4) = 3, h(u_d) = d - 1, h(u_{d-1}) = d,$

$h(u_i) = i, 5 \leq i \leq d - 2.$

Case 2: u^1 has a degree of 4. In this case, f stands for the number of vertices, and 2_{d-6} connects multiple points with corresponding edges and the number of edges of the graph.

Subcase I: When n is an even number.

$h(u_1) = 1, h(u_2) = 2, h(u_3) = 4,$

$h(u_4) = 3, h(u_5) = 6, h(u_6) = 5,$

$h(u_i) = i, 7 \leq i \leq d$

Subcase II: When d is odd.

$h(u_1) = 1, h(u_2) = 2, h(u_3) = 4,$

$h(u_4) = 3, h(u_5) = 6, h(u_6) = 5,$

$h(u_d) = d - 1, h(u_{d-1}) = d,$

$h(u_i) = i, 7 \leq i \leq d - 2.$

As a result, in each case, $|g_h(0) - g_h(1)| \leq 1$. Thus, square divisor cordial labelling is made possible by swapping the cycle's vertex with trianglem_d $(1, 1, d - 5)$. It efficiently calculates the shortest route between two points on a map with cycles. Continuously updating the path as the algorithm explores the graph ensures an optimal solution is found.

According to the difference, the labels on node findings are covered by the shortest paths to connect the edges to form the topology. In its dynamic nature, each node varies at different times to create a topology. Due to inconsistent variation in path construction, the distance theory finds the support vertex on connecting edges to construct the path. A network simulator performs the performance evaluation to marginalize the node construction in the route arrangement, which will be evaluated using throughput performance. This parameter indicates the rate at which transmitted packets are successfully delivered and can be expressed as follows:

Throughput performance=$\frac{\sum \text{Recevied bytes of packets}}{\text{data transfer duration}} * 100$

To assess a network's packet transmission efficiency, it's imperative to evaluate its throughput.

Latency $= \frac{\sum D_i^R - S_i^S}{F}$

Let's assume D_i^R represents destination node (D) received (R) with the packet, S_i^S represents source node (S) sent (s) with the packet, and F refers to successfully received packets.

Routing overhead $= \frac{\sum \text{Total number of control packets}}{\sum \text{Data packets}}$

Similar to the Packet drop ratio performed by successive taken packets in best routing evaluation. Packet drop ratio $= \sum \frac{P_s - P_r}{\text{Total}(P_s)} * 100$ Transmission time is a term used to describe the delay in delivering a packet from a sending node to a destination node. The proposed applied mathematical solution to the network performs efficient path construction to improve the network routing performance.

4. Conclusion

To conclude, the proposed graph theory and labelling provide the best path construction to reduce the network computation time to provide quality service. This mathematical solution attains the redundant solution to solve the distance problem to construct the best path. The escalating complexity of networks demands sophisticated mathematical solutions to enhance communication efficiency. This paper advocates a mathematical approach for intricate network configurations using the HGT and MMSDCL models. Graph theory is a fundamental tool for depicting and evaluating networks, with graph-theoretic metrics delineating network architecture. The proposed model enhances network construction efficacy and furnishes a fail-safe method for determining the shortest path to link network nodes, thereby optimizing communication channels. Through the amalgamation of HGT and MMSDCL models, a robust solution is presented to address the intricate challenges modern network structures pose.

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