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Micro Binary Semi Open Sets in Micro Binary Topological Spaces

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Abstract:- A Micro-Binary structure that satisfies specific axioms that are comparable to topological axioms is said to have a Micro-Binary topology from a nonempty set X to a nonempty set Y. The notation of micro binary open sets, micro binary closed sets, micro binary interior and micro binary closure are introduced and their basic properties are discussed with the suitable examples. In this research, we present and examine Micro-Binary semi open sets in Micro Binary topological spaces.

Keywords: Micro Binary topological spaces, Micro Binary semi-open, Micro Binary interior, Micro Binary closure, Micro Binary subspace.

1. Introduction

The authors [4] introduced the concept of binary topology and discussed some of its basic properties. Norman Levine introduces semi-open sets in topological spaces [5]. Nano topology was introduced by Thivagar [3] in the year 2013. Nano topology is based on the concept of lower approximation, upper approximation, and boundary region. Nano topology has a maximum of five Nano open sets and a minimum of three Nano open sets including U, ϕ suppose we want to add some more open sets, for that time we can use Levine's simple extension concept in Nano topology we can extend some more open sets that topology is called micro topology[1]. Every Nano topology is Micro topology. This paper introduces Micro Binary semi-open sets in Micro Binary topological spaces, and their basic properties are studied. Section 2 deals with basic concepts. Micro Binary semi-open sets in Micro Binary topological spaces are discussed in section 3. Throughout the paper, $\rho(X)$ denotes the power set of X.

2. Preliminaries

Let X and Y be any two nonempty sets. A binary [4] topology [2] from X to Y is a binary structure $M \subseteq \rho(X)\times\rho(Y)$ that satisfies the axioms namely

(i) (ϕ, ϕ) and $(X, Y) \in M$

(ii) $(A_1 \cap A_2, B_1 \cap B_2) \in M$ whenever $(A_1, B_1) \in M$ and $(A_2, B_2) \in M$, and

(iii) If $(A\alpha, B\alpha)$: $\alpha \in \Delta$ is a family of members of M, then $(\cup_{\alpha\in\Delta} A\alpha, \cup_{\alpha\in\Delta} B\alpha) \in M$. If M is a binary topology from X to Y then the triplet (X, Y, M) is called a binary topological space and the members of M are called the binary open subsets of the binary topological space (X, Y, M). The elements of $X \times Y$ are called the binary points of the binary topological space (X, Y, M). If Y=X then M is called a binary topology on X in which case we write (X, M) as a binary topological space. The examples of binary topological spaces are given in [2].

2.1. Definition [4]

Let X and Y be any two nonempty sets and let (A, B) and $(C, D) \in \rho(X) \times \rho(Y)$. We say that $(A, B) \subseteq (C, D)$ if $A \subseteq C$ and $B \subseteq D$.

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2.2. Definition [1]

Let U be a universe, R be an equivalence relation on U, and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where X ⊆ U satisfies the following axioms:

1. U, $\phi \in \tau_R(X)$

2. The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$

3. The intersection of the elements of any finite sub-collection of $\tau_R(X)$ is in $\tau_R(X)$. Then $\tau_R(X)$ is called the Nano topology on U for X. The space $(U, \tau_R(X))$ is the Nano topological space. The elements of $\tau_R(X)$ are called Nano open sets.

2.3. Definition [3]

$(U, \tau_R(X))$ is a Nano topological space here $\mu_R(X) = \{L \cup (L' \cap \mu)\}$: L, L′ $\in \tau_R(X)$ and is called Micro topology of $\tau_R(X)$ by μ where $\mu \notin \tau_R(X)$. The Micro topology $\mu_R(X)$ satisfies the following axioms:

1. U, $\emptyset \in \mu_R(X)$.

2. The union of the elements of any sub-collection of $\mu_R(X)$ is in $\mu_R(X)$.

3. The intersection of the elements of any finite sub-collection of $\mu_R(X)$ is in $\mu_R(X)$. Then $\mu_R(X)$ is called the Micro Topology on U for X. The triplet $(U, \tau_R(X), \mu_R(X))$ is called Micro topological spaces and the elements of $\mu_R(X)$ are called Micro open sets and the complement of a Micro open set are called a Micro closed set.

2.4. Definition

Let $((A_1, A_2), (\phi, \phi), \tau_R(x_1, x_2), \mu_R(x_1, x_2))$ be a Micro Binary [6] topological space and $A \subseteq A_1$, $B \subseteq A_2$. Then (A, B) is called Micro Binary closed in Micro Binary topological space.
$((A_1, A_2), (\phi, \phi), \tau_R(x_1, x_2), \mu_R(x_1, x_2))$ if $(A_1/A, A_2/B) \in \mu_R(x_1, x_2)$.

2.5. Definition

Let $((A_1, A_2), (\phi, \phi), \tau_R(x_1, x_2), \mu_R(x_1, x_2))$ be a micro binary topological space and $(A, B) \subseteq (A\alpha, B\alpha)$ and $(A, B)^{2*} = \cap B\alpha:(A\alpha, B\alpha)$ is micro binary closed and $(A, B) \subseteq (A\alpha, B\alpha)$. Then $((A, B)^{1*}, (A, B)^{2*})$ is micro binary closed and $(A, B) \subseteq ((A, B)^{1*}, (A, B)^{2*})$. We call $((A_1, A_2), (\phi, \phi), \tau_R(x_1, x_2), \mu_R(x_1, x_2))$ as the Micro Binary Topological Spaces. The elements of $\mu_R(x_1, x_2)$ are called as Micro Binary open sets and it is denoted by M_B open sets. Their complement is called M_B closed sets.

2.6. Definition

The ordered pair $((A, B)^{1*}, (A, B)^{2*})$ is called micro binary closure of (A, B), denoted by μ_R-cl(A, B) in the micro binary space $((A_1, A_2), (\phi, \phi), \tau_R(x_1, x_2), \mu_R(x_1, x_2))$ where $(A, B) \subseteq (A_1, A_2)$.

2.7. Definition

Let X and Y be any two nonempty sets and let (A, B) and (C, D) $\in \rho(A_1)\times\rho(A_2)$. We say that (A, B) and (C, D) if one of the following holds:

(i) $A \subseteq C$ and $B \not\subset D$

(ii) $A \not\subset C$ and $B \subseteq D$

(iii) $A \not\subset C$ and $B \not\subset D$

2.8. Definition

(i) $(A, B)^{1*} = \cup\{A\alpha: (A\alpha, B\alpha)$ is Micro Binary open and $(A\alpha, B\alpha) \subseteq (A, B)\}$.

(ii) $(A, B)^{2*} = \cup\{B\alpha: (A\alpha, B\alpha)$ is Micro Binary open and $(A\alpha, B\alpha) \subseteq (A, B)\}$.

2.9. Definition

Let $((A_1, A_2), (\phi, \phi), \tau_R(x_1, x_2), \mu_R(x_1, x_2))$ be a Micro Binary topological space and $(A, B) \subseteq (A_1, A_2)$. The ordered pair Methods $((A, B)^{1o}, (A, B)^{2o})$ is called the Micro Binary interior of (A, B), denoted by μ_R-int(A, B).

2.10. Definition

Let $((A_1, A_2), (\phi, \phi), \tau_R(x_1, x_2), \mu_R(x_1, x_2))$ be a Micro Binary topological space. Let $(A, B) \subseteq (A_1, A_2)$.
Define $\mu_{R(A, B)}(x_1, x_2) = \{(A\cap U, B\cap V) \in \mu_R(x_1, x_2))\}$. Then $\mu_{R(A, B)}(x_1, x_2)$ is a Micro Binary topology from A to B. The Micro Binary topological space $((A, B), (\phi, \phi), \tau_{R(A, B)}(x_1, x_2), \mu_{R(A, B)}(x_1, x_2))$ is called a Micro Binary subspace of $((A_1, A_2), (\phi, \phi), \tau_R(x_1, x_2), \mu_R(x_1, x_2))$.

2.11. Definition [4]

Let $f : Z \rightarrow X \times Y$ be a function. Let $A \subseteq X$ and $B \subseteq Y$. We define $f^{-1}(A, B) = \{z \in Z : f(z) = (x, y) \in (A, B)\}$.

2.12. Definition

Let $((X, Y), (\phi, \phi), \tau_R(x_1, x_2), \mu_R(x_1, x_2))$ be a micro binary topological space and let $(Z, \tau_R(x), \mu_R(x))$ be a micro binary topological space. Now, let $f : (Z, \tau_R(x), \mu_R(x)) \rightarrow X \times Y$ be a function, then f is said to be micro binary continuous if $f^{-1}(A, B)$ is open in $(Z, \tau_R(x), \mu_R(x))$ for every micro binary open set (A, B) in $X \times Y$.

3. Micro Binary Semi - Open Sets

In this section, we begin with the definition of a Micro Binary semi-open set in a Micro binary topological space.

3.1. Definition

Let $((A_1, A_2), (\phi, \phi), \tau_R(x_1, x_2), \mu_R(x_1, x_2))$ be a Micro Binary topological space. Let $(A, B) \subseteq (A_1, A_2)$. Then (A, B) is called Micro Binary semi-open such that $(A, B) \subseteq \mu_R$-cl(μ_R-int(A, B)). The collection of all micro binary semi-open sets and micro binary semi-closed sets are denoted by MBSO$((A_1, A_2), (\phi, \phi), \tau_R(x_1, x_2), \mu_R(x_1, x_2))$ and MBSC$((A_1, A_2), (\phi, \phi), \tau_R(x_1, x_2), \mu_R(x_1, x_2))$, respectively.

3.2. Example

Consider $A_1 = \{1, 2, 3, 4, 5, 6\}$ and $A_2 = \{p, q, r, s, t, u\}$.
Clearly $\mu_R(x_1, x_2) = \{(\phi, \phi), (A_1, A_2), (\{5\}, \{r\}), (\{3\}, \{u\}), (\{3, 5\},\{u, r\}), (\{3, 4, 6\},\{u, t, s\}), (\{3, 4, 5, 6\},\{u, t, r, s\}), (\{4, 6\},\{t, s\}), (\{4, 5, 6\},\{t, r, s\})$ is a Micro Binary topology from A_1 to A_2.
Also $(\phi, \phi), (A_1, A_2), (\{1, 2, 3, 4, 6\}, \{p, q, s, t, u\}), (\{1, 2, 4, 5, 6\}, \{p, q, r, s, t\}), (\{1, 2, 4, 6\}, \{p, q, s, t\}), (\{1, 2, 5\}, \{p, q, r\}), (\{1. 2, 3, 5\}, \{p, q, r, u\})$, and $(\{1, 2, 3\}, \{p, q, u\})$ are M_B closed sets in $((A_1, A_2), (\phi, \phi), \tau_R(x_1, x_2), \mu_R(x_1, x_2))$. Consider $(A, B) = (\{2, 5\}, \{q, r\})$. Clearly a M_B open set $(\{5\}, \{r\}) \subseteq (\{2, 5\}, \{q, r\}) \subseteq \mu_R(\{5\}, \{r\})$. Since, $\mu_R(\{5\}, \{r\}) = ((\{5\}, \{r\})^{1*}, (\{5\},\{r\})^{2*})$. Hence, $(A, B) = (\{2, 5\}, \{q, r\})$ is Micro Binary semi open.

3.3. Proposition

Let $((A_1, A_2), (\phi, \phi), \tau_R(x_1, x_2), \mu_R(x_1, x_2))$ be a Micro Binary topological space and $A \subseteq A_1$, $B \subseteq A_2$. If (A, B) is Micro Binary open in $((A_1, A_2), (\phi, \phi), \tau_R(x_1, x_2), \mu_R(x_1, x_2))$, then A is semi-open in $(A_1, \mu_{RA1}(x_1, x_2))$ and B is semi-open in $(A_2, \mu_{RA2}(x_1, x_2))$.

Proof

We have, $\mu_{RA1} = \{A \subseteq A_1: (A, B) \in \mu_R(x_1, x_2)$ for some $B \subseteq A_2\}$ is a topology on A_1 and

$\mu_{RA2}(x_1, x_2) = \{B \subseteq A_2: (A, B) \in \mu_R(x_1, x_2)$ for some $A \subseteq A_1\}$ is a topology on A_2. Since (A, B) is Micro Binary open in $((A_1, A_2), (\phi, \phi), \tau_R(x_1, x_2), \mu_R(x_1, x_2))$, we have $A \in \mu_{RA1}(x_1, x_2)$ and $B \in \mu_{RA2}(x_1, x_2)$. That is, A is open in $(A_1, \mu_{RA1}(x_1, x_2))$ and B is open in $(A_2, \mu_{RA2}(x_1, x_2))$. Since every open set is semi-open, we have A is semi-open in $(A_1, \mu_{RA1}(x_1, x_2))$ and B is semi-open in $(A_2, \mu_{RA2}(x_1, x_2))$. The converse of the above proposition need not be true which is shown in Example 3.4.

3.4. Example

Let $A_1 = \{(\phi, \phi), (A_1, A_2), (\{5\}, \{r\}), (\{3\}, \{u\}), (\{3, 5\}, \{u, r\}), (\{3, 4, 6\}, \{u, t, s\}), (\{3, 4, 5, 6\}, \{u, t, r, s\}), (\{4, 6\}, \{t, s\}), (\{4, 5, 6\}, \{t, r, s\})$ is a Micro Binary topology from A_1 to A_2. Also,

$\mu_{RA1}(x_1, x_2) = (A_1, \phi, \{4\}, \{3, 5\}, \{3, 4, 5\}, \{1, 2, 3, 5\}, \{1, 2, 3, 4, 5\}, \{1, 2\}, \{1, 2, 4\})$ is a topology on A_1

$\mu_{RA2}(x_1, x_2) = (\phi, A_2, \{p\}, \{u\}, \{p, u\}, \{q, r, u\}, \{p, q, r, u\}, \{q, r\}, \{p, q, r\})$ is a topology on A_2.

Consider $A = \{3, 5\}$ and $B = \{u\}$. Then A is open in A_1 and B is open in A_2. Therefore, A is semi open in A_1 and B is semi open in A_2. But (A, B) is not Micro Binary open. The proof of the following Proposition is obtained directly from Example 3.2.

3.5. Proposition

If (U, V) is Micro Binary open in a Micro Binary topological space $((A_1, A_2), (\phi, \phi), \tau_R(x_1, x_2), \mu_R(x_1, x_2))$, then (U, V) is Micro Binary semi-open in $((A_1, A_2), (\phi, \phi), \tau_R(x_1, x_2), \mu_R(x_1, x_2))$. The converse of Proposition 3.5 is not true. From Example 3.2, we can easily see that the Micro Binary set ({2, 5}, {q, r}) is Micro Binary semi-open but not Micro Binary open.

3.6. Proposition

Every micro binary open set is micro binary semi-open.

Proof

Let (A, B) be a micro binary open set in $((A_1, A_2), (\phi, \phi), \tau_R(x_1, x_2), \mu_R(x_1, x_2))$, then $(A, B) \subseteq \mu_R\text{-Cl}(\mu_R\text{-Int}(A, B))$, since (A, B) is a micro binary open set, Int(A, B) = (A, B), then $(A, B) \subseteq Cl(A, B)$, now either Cl(A, B) = (X, Y) or Cl(A, B) = (C, D), where $A \subseteq C$ and $B \subseteq D$, therefore (A, B) is binary semi-open. The converse of the above Proposition need not be true as can be seen in the Example 3.4.

3.7. Remark

Let $((A_1, A_2), (\phi, \phi), \tau_R(x_1, x_2), \mu_R(x_1, x_2))$ be a Micro Binary topological space and $A \subseteq A_1$, $B \subseteq A_2$ is Micro Binary semi open in $((A_1, A_2), (\phi, \phi), \tau_R(x_1, x_2), \mu_R(x_1, x_2))$, then A need not be semi-open in $(A_1, \mu_{RA2}(x_1, x_2))$ and B need not be semi open in $(A_2, \mu_{RA2}(x_1, x_2))$. The following proposition gives a characterization of Micro Binary semi-open sets.

3.8. Proposition

Let $((A_1, A_2), (\phi, \phi), \tau_R(x_1, x_2), \mu_R(x_1, x_2))$ be a Micro Binary topological space. Let $(A, B) \subseteq (A_1, A_2)$. Then (A, B) is Micro Binary semi-open if and only if $(A, B) \subseteq \mu_R\text{-cl}(\mu_R\text{-int}(A, B))$.

Proof

Let (A, B) be Micro Binary semi-open. Then there exists a Micro Binary open set (U, V) such that

$(U, V) \subseteq (A, B)\mu_R\text{-cl}(U, V)$. Now, $(U, V) \subseteq (A, B)$implies $\mu_R\text{-int}(A, B)$. Since (U, V) is Micro Binary open,

we have $(U, V) = \mu_R\text{-int}(A, B)$. Hence, $(U, V) \subseteq \mu_R\text{-int}(A, B)$.

This implies $\mu_R\text{-cl}(U, V) \subseteq \mu_R\text{-cl}(\mu_R\text{-int}(A, B))$. Thus, $(A, B) \subseteq \mu_R\text{-cl}(U, V) \subseteq \mu_R\text{-cl}(\text{int}(A, B))$.

That is, $(A, B) \subseteq \mu_R\text{-cl}(\mu_R\text{-int}(A, B))$. Conversely, assume that $(A, B) \subseteq \mu_R\text{-cl}(\mu_R\text{-int}(A, B))$. Then for

$(U, V) = \mu_R\text{-int}(A, B)$, we have, $\mu_R\text{-int}(A, B) \subseteq (A, B) \subseteq \mu_R\text{-cl}(\mu_R\text{-int}(A, B))$. This implies that $(U, V) \subseteq (A, B) \subseteq \mu_R\text{-cl}(U, V)$. Hence, (A, B) is Micro Binary semi-open.

3.9. Proposition

Let (A, B) be a Micro Binary semi-open set in a Micro Binary topological space

$((A_1, A_2), (\phi, \phi), \tau_R(x_1, x_2), \mu_R(x_1, x_2))$ and suppose $(A, B) \subseteq (C, D) \subseteq \mu_R\text{-cl}(A, B)$. Then (C, D) is Micro Binary semi-open.

Proof

Since (A, B) is Micro Binary semi-open, there exists a Micro Binary open set (U, V) such that $(U, V) \subseteq (A, B) \subseteq \mu_R\text{-cl}(U, V)$. Therefore, $(U, V) \subseteq (C, D)$ and $\mu_R\text{-cl}(A, B) \subseteq \mu_R\text{-cl}(\text{cl}(U, V))$. Since $(C, D) \subseteq \mu_R\text{-cl}(A, B)$, we have $(C, D) \subseteq \mu_R\text{-cl}(U, V)$. Thus $(U, V) \subseteq (C, D) \subseteq \mu_R\text{-cl}(U, V)$. Hence, (C, D) is Micro Binary semi-open.

3.10. Proposition

Let $((A_1, A_2), (\phi, \phi), \tau_R(x_1, x_2), \mu_R(x_1, x_2))$ be a Micro Binary topological space and $((A, B), (\phi, \phi), \tau_{R(A,B)}(x_1, x_2), \mu_{R(A,B)}(x_1, x_2))$. Let (c, d) be a Micro Binary semi-open set in $((A_1, A_2), (\phi, \phi), \tau_R(x_1, x_2), \mu_R(x_1, x_2))$ and $(C, D) \subseteq (A, B)$. Then (C, D) is Micro Binary semi-open in $((A, B), (\phi, \phi), \tau_{R(A, B)}(x_1, x_2), \mu_{R(A, B)}(x_1, x_2))$.

Proof

Since (C, D) is a Micro Binary semi-open set in $((A_1, A_2), (\phi, \phi), \tau_R(x_1, x_2), \mu_R(x_1, x_2))$, we have $(U, V) \subseteq (C, D) \subseteq \mu_R\text{-cl}(U, V)$ where $(U, V) \in \mu_R(x_1, x_2)$.

Since $(U, V) \subseteq (A, B)$, $(U, V) = (U \cap A, V \cap B) \subseteq (C \cap A, D \cap B) \subseteq \mu_R\text{-cl}_{(A, B)}(U, V)$. Also, since $(U, V) = (U \cap A, V \cap B)$, we have (U, V) open in $((A, B), (\phi, \phi), \tau_{R(A, B)}(x_1, x_2), \mu_{R(A, B)}(x_1, x_2)$. This implies that $(U, V) \subseteq (C, D) \subseteq \mu_R\text{-cl}_{(A, B)}(U, V))$.

Conclusion

Semi-open sets in topological spaces are extended to Micro Binary topological spaces. In this paper, we introduced Micro Binary semi-open sets in Micro Binary topological spaces and their properties are discussed.

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