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Emerging Theorems: An Amalgamative Outcome of Simple, 0-Simple, Semigroups, Principal factors an Expounding Assortment and Automation of semi-groups with space valued Fuzzy weakly inside principles

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Abstract: -Before tackling the question we should perhaps begin by saying what a semigroup is. A non-empty set S endowed with a single binary operation $\circ$ is known as a semigroup if for evey x,y,z [2][3]in S, $(xy)z = x(yz)$. If in addition there exists 1 in S such that, for every x in S, $1x = x1 = z$ we say that S is a semigroup with identity or (more usually) a monoid. In this article, we give a definition of an space valued fuzzy weakly inside ideal. We study some interesting properties [4] [5] [6] of space valued fuzzy weakly inside ideals and the relationship between space valued fuzzy weakly interior ideals and space valued fuzzy principles. We characterize some semigroups by using interval valued fuzzy weakly interior principles. Moreover, we found theorems of the homorphic image and the preimage of an space valued fuzzy weakly inside principle in semigroups [7]. The 0-simple semigroups begins with some elementary results on simple and 0-simple semigroups and a decomposition theorem for semigroups in general that indicates why and understanding of simple and 0-simple semigroups is important. The main result of the chapter is a structure theorem due to Rees (1940) [2, 31] which applies to 0-simple semigroups satisfying both the minimal conditions $min_L \ and \ min_R$ -- what are called completely 0-simple semigroups. Rees himself used a different definition, but we shall see below theorems that the two definitions are equivalent [1].

Keywords: *Normal semigroup, intra-normal semigroup, semisimple semigroup, Semigroup Codes, Algebraic theoryof Semigroups, Finite Automata, Coset Semigroup and Semigroup Forum.*

1. Introduction

Suppose S is a semigroup with identity monoid, we shall be confining ourself today to semigroups that have no additional structure [1]. Thus, though semigroups feature quite prominently in parts of functional analysis, the algebraic structure of those semigroups is usually very straightforward and so they scarely rate a mention in any algebraic theory. Equally, although they are often of greater algebraic interest, We shall say nothing about topological semigroups. Assusme that begin by answering a slightly different question: Who studies semigroups? Sectopm 20 in Mathematical Reviews is entitled " Groups and Generalizatrions" and has two **leper**

colonies at the end, is known as 20M semigroups and 20N other generalizations. In the introduction to their algebraic theory of semigroups in 1961 Clifford and Preston [2] remarked that about thirty papers on semigroups per year were currently appearing. Incidentally, comparable figures for other generalizations are about on third of these. So it is clear that of all generalizations of the group concept the semigroup is the one that has attracted the most interest by far. We shall in due course hazard a guess as to why this is so. Mathematics are rightly a bit suppicious of theories whose only motive seems to be to generalize existing theories-and if the only motivation for semigroup theory were to examine group theoretical results with a view to generalization, then we would have no very convincing answer to the question of in this title.

Perhaps unfortunately, the word "simple" as used in semigroup theory does not have the same import as in group theory or ring theory, where it implies the total absence of non-trivial homomorphic images. By contrast with ring theory, not every congruence on a semigroup is associated with an ideal [3, 4, 5], and so it is normally the case that a simple or 0-simple semigroup has non-trivial congruences. Using the Rees structure theorem we describe a classification of the congruences on a completely 0-simple semigroup. A result on the nature of the lattice of congruences readily follows, and so also does a classification of the finite *congruence-free* truly simple semigroups [7].

II Simple And 0-Simple Semigroups; Principal factors

A semigroup S without zero is known as simple if it has no proper ideals. A semigroup S with zero is known as 0-simple if $(i)\ \{0\}$ and S are the only ideals and $(ii)\ S^2 \neq \{0\}$. The latter condition serves only to exclude the two-element null semigroup from the class of 0-simple semigroups, since any larger num semigroup fails to quality on the grounds of having proper ideals [6, 7].

It is easy to see that S is simple iff $f = S \times S.$ The corresponding criterion for 0-simplifying is that $S^2 \neq \{0\}$ and that $\{0\}, S\backslash\{0\}$ are the only I$-Classes.$

A simple semigroup can be made into a 0-simple semigroup by merely adjoining a zero element. Not all 0-simple semigroups arise from simple semigroups in this way, however; the zero element of a 0-simple semigroup S can be removed to leave behind a simple semigroup only if it is a "prime" element in the sense that $ab = 0 \Rightarrow a = 0\ or\ b = 0,$ -------------------- (1.1.1) [8,9,]and this is not always the case, as will be clear by the end of the part. If the implication (1.1.1) does hold, we say that S has no proper zero-divisors. It will always be possible to deduce a theorem about simple semigroups from one about 0-simple semigroups, and for this reason we shall turn our attention primarily towards the 0-simple case [8,10].

Definition 1: For clearly every ring (R,+, .) [2] is a semigroup if we simply neglect the operation + . The converse is certainly not true: that is, there are semigroups (S, .) [3] with zero on which it is not possible to define an operation + so as to create a ring (S, +, .) . The easiest way to see this is to recall the known result that a ring (R, +, .) with the property that $x^2 = x\ for\ all\ x\ in\ R$ is necessarily commutative satisfies $xy = yx\ for\ every\ x\ in\ R.$ Assume that $S = (A \times B) \cup \{0\},\ where\ A, B\ are\ non-empty\ sets\ and$ 0 does not belongs to $A \times B, and\ make\ S$ into a semigroup with zero by defining $(a, b)0 = 0(a, b) = 00 = 0, (a_1, b_1)(a_2, b_2) = (a_1, b_2).$

Definition 2: Suppose that a semigroup (S,.) is normal iff $(for\ each\ a \in S)(\exists\ a'\ belongs\ S)\ aa'a = a, a'aa' = a'\ so, simply\ we\ take\ a' = xax\ we\ have\ aa'a = a, a'aa' = a'.$

The element $a' is\ usually\ is\ known\ as\ an\ inverse\ of\ a,$ but it should be noted that this is a weaker concept of inverse than the one used in group theory: Observe that four element semigroup with Cayley table as hold [4] [5]:

	w	x	y	z
w	w	x	y	z
x	w	x	w	x
y	y	z	y	z
z	y	z	y	z

It is easy to check that every element is an inverse of every other element.

Theorem 1: The following conditions on a regular semigroup S are equivalent:

(i) Idempotent commute;
(ii) (ii) Inverse are unique.

Proof: Suppose that idempotents commute. Assume that $a', a^*\ be\ an\ inverse\ of\ a, Then\ a' = a'aa' = a'aa^*aa' = a'aa^*aa^*\ aa' = a^*aa'aa^*aa' = a^*aa'aa'aa^* = a^*aa^* = a^*$

Assume that $e, f\ \ be\ idempotents\ and\ let\ x\ be\ the\ unique\ invese\ of\ ef: efxef = ef,$

$xefx = x. Then\ fxe\ is\ idempotenet, since\ (fxe)^2 = f(xefx)e = fxe; and\ ef\ is\ an$ inverse of $fxe: (fxe)(ef)(fxe) = f(xefx)e = fxe, (ef)(fxe)(ef) = efxef - ef$ But an idempotent $i\ is\ its\ own\ unique\ inverse\ (iii = i, iii = i) and\ so\ ef = fxe, and\ idempotent.$ Similarly $fe\ is$ idempotent.

The unique inverse of $ef\ \ is\ thus\ ef\ itself.$ On the other hand $fe\ is\ an\ inverse\ of\ ef, since$

$(ef)(fe)(ef) = (ef)^2 = ef, (fe)(ef)(fe) - (fe)^2 = fe.$

It follows that $ef\ = fe, as\ required.$

That argument goes back to the early 1950s, to some basic work by Vagner and [16] [17] Preston [12] [13] [14]. Regular semi groups satisfying either one of the conditions are satisfied. This is known as inverse semi group. Let me give a not very well known example due to Schein and McAlister. Let G be a group and let $K(G)$ be the set of all right semigroups of $G.$ This includes G itself and also the semigroups of the subgroup1, which are effectively the elements of $G.$ By the definition of an operation * on $K(G)$ by $Ha * kb = (H \vee aka^{-1})ab.$ This is a natural definition: [11] it is not hard to check that $Ha * kb$ is the smallest coset containing the product $Hakb.$ clearly $Hakb = \ (H \vee aka^{-1})ab \subseteq (H \vee aka^{-1})ab.$ Conversely, suppose that $Hakb \subseteq Pc(\in K(G)).$ then in particular $ab \in Pc$ and so $Pc = Pab.$ Now $Had \subseteq Hakb \subseteq Pab$ and so H ⊆ P; also $(aka^{-1})ab = akb \subseteq Hakb \subseteq Pab$ and so $aka^{-1} \subseteq P.$ Thus $H \vee aka^{-1} \subseteq P$ and so $(H \vee aka^{-1})ab \subset Pab = Pc.$ It is a routine matter to check that * is an associative [15] operation and that $(a^{-1}Ha)a^{-1}$ is an inverse of Ha in the semi group$(K(G),\ *)$. Now suppose that Ha is idempotent: [10]

$Ha = Ha * Ha = (H \vee aHa^{-1})a^2$. In fact the idempotent of $(K(G),\ *)$ are precisely the subgroups of G. for any two subgroups $H, K\ of\ G$ then $H * K = H \vee K = K * H$. Thus idepotents commute and so $(K(G),\ *)$ is an inverse semigroup. The normal subgroups $N\ of\ G$ are such that, for all $Ha\ in\ K(G),\ N * Ha = (N \vee H)a = (NH)a.\ Ha * N = (H \vee aNa^{-1})a = (H \vee N)a = (NH)a,$ and so are central idempotents in $(K(G),\ *)$. Conversely, if N is a central idempotent then for all $a\ in\ G. Na = N * 1a = 1a * N = (1 \vee aNa^{-1})a = aN\ and\ so\ N\ is\ normal.$

The main reason that semi groups turn up in mathematics is that one is very often interested in self-maps of a set of one kind or another, and whenever f, g, h are such maps it is automatically the case that $(f \circ g) \circ h = f \circ (g \circ h)$.

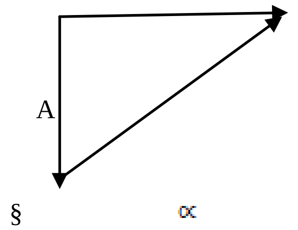
There is another pure mathematical reason for being interested in semi groups. It is possible to take a very general standpoint in algebra and to discussed is known as $\Omega - algebra\ A\ haviang\ a\ family\ \Omega = \{w_i : i \in I\}$ of the operations, where $w_i : A^n i\ tends\ to\ A$ is an n_i – array operation. If $\emptyset : A \to B\ is\ a\ map\ between\ \Omega - algebras$ then we say that $\emptyset\ is\ a\ morphism\ if\ \emptyset(w_i(a_1, \dots, a_{n_i})) = w_i(\emptyset(a_1, \dots, \emptyset\ (a_{n_i}))$.

If we regard $\emptyset$ as applying to $A^n i$ in an obvious way then we can express this property succinctly as a commuting condition $\emptyset \circ w_i = w_i \circ \emptyset$. A congruence on A is an equivalence relation $\sim$ with the property that hold:

$a_1 \sim a_1', \dots, a_{n_i}, a_{n_i}' = w_i(a_1, \dots, a_{n_i}) \sim w_i(a_1',, \dots, a_{n_i}')$ consider the quotient set $\frac{A}{\sim}$, whose elements are equivalence classes [a] = $\{x \in A : x \sim a\}$. The congruence property means that $\frac{A}{\sim}$ inherits the $\Omega - algebra$ structure from A : we simply defined as $w_i([a_1], \dots, [a_{n_i}])$ = $[w_i(a_1, \dots, a_{n_i})]$ .b And the compatibility condition ensures that the definition makes sense. There is a natural map § :$A\ tends\ to\ A/\sim$ defined by §(a) = [a] , $(a \in A)$ and the definition can be interpreted as saying that $w_i \circ$ § = § $\circ\ w_i$; hence, comparing with the § is the morphism. Now suppose that $\emptyset$ is a morphism from $A\ onto\ B,$ we say that B is a morphic image of A. Define $\sim$ on A by the rule that $a \sim a'\ iff\ \emptyset(a) = \emptyset(a')$ It is easy to verify that $\sim$ is a congruence. The first isomorphic theorem for $\Omega - algebras$ is then as follows:

Theorem 2: Suppose that $A, B\ be\ \Omega - algebrs, assume\ that\ \emptyset : A \to B$ be a morphism with $im\ \emptyset = B, and\ let \sim$ be the congruence on A defined as an isomorphism $\propto : \frac{A}{\sim} \to B$ such that the diagram commutes.

Ø B



$\frac{A}{\sim}$

Proof: This is, is one form or another, one the cornerstones of abstract algebra. It says in effect that an $\Omega - algebra\ A$ carries its morphic images within itself and that to reveal them we need only consider the quotients of A by its various congruences.

The result applies to groups, of course, but it is not usually stated in quite this way. This is because for a group A there is a one-one correspondence between congruences $\sim$ and normal subgroups N given by $N = \{a \in A: a{\sim}1\}, or$

$a{\sim}b\ iff\ if\ ab^{-1} \in N.$ The quotient $\frac{A}{\sim}$ is always denoted by $\frac{A}{\sim}$. Similarly, for a ring A there is a one – one correspondence between congruences $\sim$ and two sided ideals $I\ given\ by\ I = \{a \in A: a{\sim}0\}, or\ a{\sim}b\ iff\ a - b \in I,$

And the quotient $\frac{A}{\sim}\ is\ always\ written\ \frac{A}{\sim}.$

II Applied Mathematical reasons

Assume that reasons for studying semigroups. One of the striking aspects of semigroup conferences these days is that many of the participants, between a third and a half, at a guess, come from departments of computer science. The reason is that semigroups have found significant applications in the theory of automata, languages and codes.

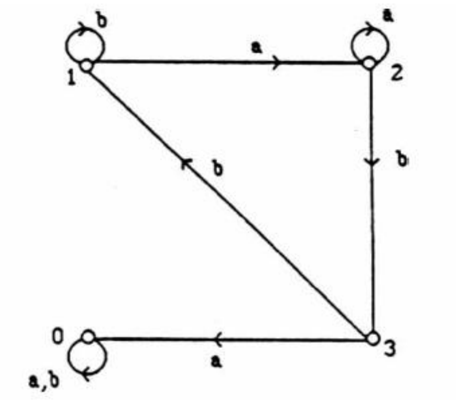
If A is a non-empty set then the set of all finite words $w = a_1 a_2\ \ldots\ a_n$ in the alphabet A is a semi group if we define multiplication by juxtaposition. Denote the length of $w\ by\ |w|.$ If we include the empty word 1 with $|1| = 0$ then we obtain a monoid., which we denote by $A^*.$ This is a free monoid geberated by A. The set of non-emtpy words in A^* is usually denoted by $A^+.$ A subset of $A^*\ is$ known as a automata fuzzy language.

Now aussume that Q be a finite non-empty set and suppose that we have a mapping $f: Q\ \times A \to Q;$ we normally write $f(q, a)$ simply as qa and think of A as $acting\ on\ Q.$ The function f can be extended to $Q \times\ \ A^*$ by defining as

$q1 = a\{q \in Q\}, q(wa) = (qw)a\ (q \in Q, w \in A^*, a \in A). We\ say\ that\ A = (Q, f)$ is an $A^* -$ automation. Eilenberg [4] this is a complete deterministic automaton, we say think of it as a very rudimentary machine whose states the elements of Q can be altered by various input the elements of A.

Suppose now that among the elements of Q there is an element i which we call the initial state and that there is a subset $T\ of\ Q$ is known as the set of terminal states. Assume that $L = \{w \in A^*\ ; iw \in T\}.$ Then we say that L is the language recognized by the automaton A. A language L is known as recognizable if there exists an automaton A recognizing L.

Example 1: We can picture an automaton via its state graph. If $A = \{a\ b\}, Q = \{0,1,2,3\}, 0a = 0b = 0, 1a = 2a = 2, 3a = 0, 1b = 1, 2b = 3, 3b = 1,$ then we can draw the picture as follows:



Assume that $i = 1, T = \{1,2,3\}$. It is easy to see that 0 is a **Sink** state from which no escape is possible, and that $1aba = 2aba = 0$. In fact L, the language recognized by this automaton, consists of all words in A^* not containing aba as a segment.

If $L_1, L_2 \subseteq A^*$ then $L_1.L_2$ is defined as $\{w_1w_2 : w_1 \in L_1, w_2 \in, L_2\}. If\ L \subseteq A^*$ then $< L >$ denotes the submonoid of A^* generated by L. Assume that F be the set of all finited subsets of A^*. Then the set Rat A^* of rational subsets of A^* is the set of subsets of A^* obtained from F by means of the operations U finite union, . and $< \ >$. This leads to an important characterization of recognizable subsets of A^*:

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