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# Fuzzy $\tilde{g}$-Continuous Functions and Fuzzy $\tilde{g}$-Irresolute Continuous Functions in Fuzzy Topological Spaces

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***Abstarct:***
In present of this paper, new classes of generalized fuzzy continuous functions called fuzzy $\tilde{g}$-continuous functions and fuzzy $\tilde{g}$-irresolute continuous functions are defined and studied. Also, several examples are given.

***Key words and phrases****: fuzzy continuous functions, fuzzy $\tilde{g}$-continuous functions and fuzzy $\tilde{g}$-irresolute continuous functions*
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## 1. Introduction

In 1965, Zadeh [43] was introduced and discussed the novel model of a fuzzy subsets. The consequent research behavior in this area and the linked areas have originate relevance in various branches of science and engineering. In 1968, Chang [9] by the idea of generalization of fuzzy topological spaces. Another researchers similar to Azad [2], Sinha [8], Wong [42] and any more authors donate to the growth of fuzzy sets and fuzzy functions in fuzzy topological spaces and so on. In present of this paper, new classes of generalized fuzzy continuous functions called fuzzy $\tilde{g}$-continuous functions and fuzzy $\tilde{g}$-irresolute continuous functions are defined and studied.

## Preliminaries

Throughout this paper $(X, F_\tau)$ (briefly, X) will denote fuzzy topological spaces or space $(X, F_\tau)$. We remember the following basic definitions which are apply in this paper.

**Definition 2.1** A fuzzy subset A of a fuzzy topological space $(X, \tau)$ is called:

1. fuzzy semi-open set [2] if $A \leq cl(int(A))$.
2. fuzzy $\alpha$-open set [8] if $A \leq int(cl(int(A)))$.
3. fuzzy semi-preopen set [36] if $A \leq cl(int(cl(A)))$.
4. fuzzy regular open set [2] if $A = int(cl(A))$.

The complements of the above mentioned fuzzy open sets are called their respective fuzzy closed sets.

**Definition 2.2** A fuzzy subset A of a fuzzy topological space $(X, \tau)$ is called:

1. a fuzzy generalized closed (briefly fg-closed) set [3] if $cl(A) \leq U$ whenever $A \leq U$ and U is fuzzy open in $(X, \tau)$. The complement of fg-closed set is called fg-open set;

2. a fuzzy semi-generalized closed (briefly fsg-closed) set [15] if $scl(A) \leq U$ whenever $A \leq U$ and U is fuzzy semi-open in $(X, \tau)$. The complement of fsg-closed set is called fsg-open set;

3. a fuzzy generalized semi-closed (briefly fgs-closed) set [15] if scl(A) ≤ U whenever A ≤ U and U is fuzzy open in (X, τ). The complement of fgs-closed set is called fgs-open set;

4. a fuzzy α-generalized closed (briefly fαg-closed) set [26] if αcl(A) ≤ U whenever A ≤ U and U is fuzzy open in (X, τ). The complement of fαg-closed set is called fαg-open set;

5. a fuzzy generalized semi-preclosed (briefly fgsp-closed) set [10] if spcl(A) ≤ U whenever A ≤ U and U is fuzzy open in (X, τ). The complement of fgsp-closed set is called fgsp-open set;

6. a fuzzy pre-semi-generalized closed (briefly fpsg-closed) set [29] if spcl(A) ≤ U whenever A ≤ U and U is fuzzy semi-open in (X, τ). The complement of fpsg-closed set is called fpsg-open set;

7. a fuzzy ω-closed set (briefly fω-closed) [32] if cl(A) ≤ U whenever A ≤ U and U is fuzzy semi-open in (X, τ). The complement of fω-closed set is called fω-open set;

8. a fuzzy $\psi$-closed set (briefly f $\psi$-closed)[28] if $scl(A) \leq U$ whenever A≤U and U is fsg-open in (X,$\tau$). The complement of fuzzy $\psi$-closed set is called fuzzy $\psi$-open set.

The collection of all fuzzy closed sets, fg-closed, fgs-closed, fsg-closed, $f\psi$-closed, $f\omega$-closed, $f\alpha g$-closed, fuzzy semi-closed, fuzzy $\alpha$-closed, fuzzy semi-preclosed of a fuzzy topological spaces (X, $\tau$) are denoted by FC(X), FGC(X), FGSC(X), FSGC(X), F$\psi$C(X), $F\omega$C(X), $F\alpha G$C(X), FSC(X), F$\alpha$C(X) and FSPC(X) respectively. The collection of all coressponding open sets are denoted by FO(X), FGO(X), FGSO(X), FSGO(X), F$\psi$O(X), $F\omega$O(X), $F\alpha G$O(X), FSO(X), F$\alpha$O(X) and, FSPO(X) respectively.

**Definition 2.3** A fuzzy function $f:(X,\tau) \to (Y,\sigma)$ is called
1. fuzzy continuous [9] if $f^{-1}$(V) is a fuzzy closed set in (X, $\tau$) for every fuzzy closed set V of (Y, $\sigma$).
2. fuzzy g-continuous [3] if $f^{-1}$(V) is a fg-closed set in (X, $\tau$) for every fuzzy closed set V of (Y, $\sigma$).
3. fuzzy gsp-continuous [28] if $f^{-1}$(V) is a fgsp-closed set in (X, $\tau$) for every fuzzy closed set V of (Y, $\sigma$).
4. fuzzy gs-continuous [?] if $f^{-1}$(V) is a fgs-closed set in (X, $\tau$) for every fuzzy closed set V of (Y, $\sigma$).
5. fuzzy sg-continuous [10] if $f^{-1}$(V) is a fsg-closed set in (X, $\tau$) for every fuzzy closed set V of (Y, $\sigma$).
6. fuzzy semi-continuous [2] if $f^{-1}$(V) is a fuzzy semi-open set in (X, $\tau$) for every fuzzy open set V of (Y, $\sigma$).
7. fuzzy $\alpha$-continuous [35] if $f^{-1}$(V) is a fuzzy $\alpha$-closed set in (X, $\tau$) for every fuzzy closed set V of (Y, $\sigma$).
8. fuzzy $\omega$-closed [32] if the image of every fuzzy closed set in (X, $\tau$) is $f\omega$-closed in (Y, $\sigma$).

**Definition 2.4** [9] A fuzzy function $f:(X,\tau) \to (Y,\sigma)$ is called
1. fuzzy closed if the image of every fuzzy closed set of X is a fuzzy closed in Y.
2. fuzzy open if the image of every fuzzy open set of X is a fuzzy open in Y.
3. fuzzy homeomorphism if f is bijective and both f and $f^{-1}$ are fuzzy continuous.

**Definition 2.5** A fuzzy function $f:(X,\tau) \to (Y,\sigma)$ is called
1. fuzzy sg-irresolute [26] if the inverse image of every fsg-closed (resp. fsg-open) set in (Y, $\sigma$) is fsg-closed (resp. fsg-open) in (X, $\tau$).
2. fuzzy gc-irresolute [3] if the inverse image of every fg-closed set in (Y, $\sigma$) is fg-closed in (X, $\tau$).
3. fuzzy gs-irresolute [10] if the inverse image of every fgs-closed (resp. fgs-open) set in $(Y,\sigma)$ is fgs-closed (resp. fgs-open) in (X, $\tau$).

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## 3 Fuzzy $\tilde{g}$-continuous functions

We introduce the following definitions.

**Definition 3.1** Let $f:(X,F_\tau)\to(Y,F_\sigma)$ be a fuzzy function. Then $f$ is said to be fuzzy $\tilde{g}$-continuous if the inverse image of every fuzzy closed set in $(Y,F_\sigma)$ is $f\tilde{g}$-closed set in $(X,F_\tau)$.

**Example 3.2** Consider $X=Y=\{m,n\}$ and $F_\tau=\{0_X,A,1_X\}$ where $A$ is a fuzzy set in X defined by $A(m)=1, A(n)=0$ and $F_\sigma=\{0_Y,B,1_Y\}$ where $B$ is a fuzzy set in $Y$ defined by $B(m)=0.5, B(n)=0$. In the space $(X,F_\tau)$ and $(Y,F_\sigma)$. Let $f:(X,F_\tau)\to(Y,F_\sigma)$ be the identity fuzzy function. Thus f is fuzzy $\tilde{g}$-continuous.

**Proposition 3.3** Every fuzzy continuous function is fuzzy $\tilde{g}$-continuous but not conversely.

**Example 3.4** Fuzzy $\tilde{g}$-continuous function $\nrightarrow$ Fuzzy continuous function.
The fuzzy function f in Example 3.2 is fuzzy $\tilde{g}$-continuous but not fuzzy continuous function.

**Proposition 3.5** Every fuzzy $\tilde{g}$-continuous function is fuzzy $\tilde{g}_\alpha$-continuous but not conversely.

**Example 3.6** Fuzzy $\tilde{g}_\alpha$-continuous function $\nrightarrow$ Fuzzy $\tilde{g}$-continuous function.
Consider $X=Y=\{m,n\}$ and $F_\tau=\{0_X,A,1_X\}$ where $A$ is a fuzzy set in $X$ defined by $A(m)=0.6, A(n)=0.5$ and $F_\sigma=\{0_Y,B,1_Y\}$ where $B$ is fuzzy set in $Y$ defined by $B(m)=0.6, B(n)=0.6$. In the space $(X,F_\tau)$ and $(Y,F_\sigma)$. Let $f:(X,F_\tau)\to(Y,F_\sigma)$ be the identity fuzzy function. Thus $f$ is fuzzy $\tilde{g}_\alpha$-continuous but not fuzzy $\tilde{g}$-continuous.

**Definition 3.7** Let $f:(X,F_\tau)\to(Y,F_\sigma)$ be a fuzzy function. Then $f$ is said to be fuzzy $\psi$-continuous if the inverse image of every fuzzy closed set in $(Y,F_\sigma)$ is $f\psi$-closed set in $(X,F_\tau)$.

**Proposition 3.8** Every fuzzy $\tilde{g}$-continuous function is fuzzy $\psi$-continuous but not conversely.

**Example 3.9** Fuzzy $\psi$-continuous function $\nrightarrow$ Fuzzy $\tilde{g}$-continuous function.
Consider $X=Y=\{m,n\}$ and $F_\tau=\{0_X,A,1_X\}$ where $A$ is a fuzzy set in $X$ defined by $A(m)=1, A(n)=0$ and $F_\sigma=\{0_Y,B,1_Y\}$ where $B$ is a fuzzy set in $Y$ defined by $B(m)=1, B(n)=0.5$. In the space $(X,F_\tau)$ and $(Y,F_\sigma)$. Let $f:(X,F_\tau)\to(Y,F_\sigma)$ be the identity fuzzy function. Thus $f$ is fuzzy $\psi$-continuous but not fuzzy $\tilde{g}$-continuous.

**Proposition 3.10** Every fuzzy $\tilde{g}$-continuous function is fuzzy $\omega$-continuous but not conversely.

**Example 3.11** Fuzzy $\omega$-continuous function $\nrightarrow$ Fuzzy $\tilde{g}$-continuous function.
Consider $X=Y=\{m,n\}$ and $F_\tau=\{0_X,A,1_X\}$ where $A$ is a fuzzy set in $X$ defined by $A(m)=A(n)=0.5$ and $F_\sigma=\{0_Y,B,1_Y\}$ where $B$ is a fuzzy set in $Y$ defined by $B(m)=1, B(n)=0.6$. In the space $(X,F_\tau)$ and $(Y,F_\sigma)$. Let $f:(X,F_\tau)\to(Y,F_\sigma)$ be the identity fuzzy function. Thus $f$ is fuzzy $\omega$-continuous but not fuzzy $\tilde{g}$-continuous.

**Proposition 3.12** Every fuzzy $\tilde{g}$-continuous function is fuzzy $g$-continuous but not conversely.

**Example 3.13** Fuzzy $g$-continuous function $\nrightarrow$ Fuzzy $\tilde{g}$-continuous function.
Consider $X=Y=\{m,n\}$ and $\mathrm{F}_\tau=\{0_X,A,1_X\}$ where $A$ is a fuzzy set in $X$ defined by $A(m)=0.4, A(n)=0.5$ and $F_\sigma=\{0_Y,B,1_Y\}$ where $B$ is a fuzzy set in $Y$ defined by $B(m)=0.5, B(n)=0.5$. In the space $(X,F_\tau)$ and $(Y,F_\sigma)$. Let $f:(X,F_\tau)\to(Y,F_\sigma)$ be the identity fuzzy function. Thus $f$ is fuzzy g-continuous but not fuzzy $\tilde{g}$-continuous.

**Definition 3.14** Let $f:(X,F_\tau)\to(Y,F_\sigma)$ be a fuzzy function. Then $f$ is called a fuzzy $\alpha gs$-continuous if the inverse image of every fuzzy closed set in $(Y,F_\sigma)$ is fuzzy $\alpha gs$-closed set in $(X,F_\tau)$.

**Proposition 3.15** Every fuzzy $\tilde{g}$-continuous function is fuzzy $\alpha gs$-continuous but not conversely.

**Example 3.16** Fuzzy $\alpha gs$-continuous $\nrightarrow$ Fuzzy $\tilde{g}$-continuous function.
Consider $X=Y=\{m,n\}$ and $F_\tau=\{0_X,A,1_X\}$ where $A$ is a fuzzy set in $X$ defined by $A(m)=1, A(n)=0$ and $F_\sigma=\{0_Y,B,1_Y\}$ where $B$ is a fuzzy set in $Y$ defined by $B(m)=1, B(n)=0.5$. In the space $(X,F_\tau)$ and $(Y,F_\sigma)$. Let $f:(X,F_\tau)\to(Y,F_\sigma)$ be the identity fuzzy function. Thus $f$ is fuzzy $\alpha gs$-continuous but not fuzzy $\tilde{g}$-continuous.

**Proposition 3.17** Every fuzzy $\tilde{g}$-continuous function is fuzzy $\alpha g$-continuous but not conversely.

**Example 3.18** Fuzzy $\alpha g$-continuous function $\nrightarrow$ fuzzy $\tilde{g}$-continuous function.
Consider $X=Y=\{m,n\}$ and $F_\tau=\{0_X,A,1_X\}$ where $A$ is a fuzzy set in $X$ defined by $A(m)=A(n)=0.5$ and $F_\sigma=\{0_Y,B,1_Y\}$ where $B$ is fuzzy set in $Y$ defined by $B(m)=B(n)=0.4$. In the space $(X,F_\tau)$ and $(Y,F_\sigma)$. Let $f:(X,F_\tau)\to(Y,F_\sigma)$ be the identity fuzzy function. Thus $f$ is fuzzy $\alpha$g-continuous but not fuzzy $\tilde{g}$-continuous.

**Proposition 3.19** Every fuzzy $\tilde{g}$-continuous function is fuzzy $gs$-continuous but not conversely.

**Example 3.20** Fuzzy $gs$-continuous continuous $\nrightarrow$ Fuzzy $\tilde{g}$-continuous function. The identity fuzzy function f in Example 3.18 is fuzzy gs-continuous but not fuzzy $\tilde{g}$-continuous.

**Proposition 3.21** Every fuzzy $\tilde{g}$-continuous function is fuzzy $gsp$-continuous but not conversely.

**Example 3.22** Fuzzy $gsp$-continuous continuous $\nrightarrow$ Fuzzy $\tilde{g}$-continuous function. The identity fuzzy function f in Example 3.18 is fuzzy $gsp$-continuous but not fuzzy $\tilde{g}$-continuous.

**Remark 3.23** The following Examples shows that fuzzy $\tilde{g}$-continuity is independent of fuzzy $\alpha$-continuity and fuzzy semi-continuity.

**Example 3.24** Consider $X=Y=\{m,n\}$ and $F_\tau=\{0_X,A,1_X\}$ where $A$ is a fuzzy set in $X$ defined by $A(m)=1, A(n)=0$ and $F_\sigma=\{0_Y,B,1_Y\}$ where $B$ is a fuzzy set in $Y$ defined by $B(m)=0.5, B(n)=1$. In the space $(X,F_\tau)$ and $(Y,F_\sigma)$. Let $f:(X,F_\tau)\to(Y,F_\sigma)$ be the identity fuzzy function, then

1. fuzzy $\tilde{g}$-continuous but it is neither fuzzy $\alpha$-continuous nor fuzzy semi-continuous.

2. both fuzzy $\alpha$-continuous and fuzzy semi-continuous but it is not fuzzy $\tilde{g}$-continuous.

**Proposition 3.25** A fuzzy function $f:(X,F_\tau)\to(Y,F_\sigma)$ the following statements are equivalent.

1. $f$ is fuzzy $\tilde{g}$-continuous.

2. $F^{-1}(U)$ is $f\tilde{g}$-open in $(X,F_\tau)$ for every fuzzy open set $U$ in $(Y,F_\sigma)$.

**Proof.** (1) $\Rightarrow$ (2). Let $f:(X,F_\tau)\to(Y,F_\sigma)$ be fuzzy $\tilde{g}$-continuous and $U$ be fuzzy open set in $(Y,F_\sigma)$. Then $U^c$ is fuzzy closed in $(Y,F_\sigma)$ and since $f$ is fuzzy $\tilde{g}$-continuous, $f^{-1}(U^c)$ is fuzzy $\tilde{g}$-closed in $(X,F_\tau)$. But $f^{-1}(U^c)=(f^{-1}(U))^c$ and so $f^{-1}(U)$ is $f\tilde{g}$-open in $(X,F_\tau)$.
(2) $\Rightarrow$ (1). Assume that $f^{-1}(U)$ is $f\tilde{g}$-open in $(X,F_\tau)$ for each fuzzy open set $U$ in $(Y,F_\sigma)$. Let $F$ be a fuzzy closed set in $(Y,F_\sigma)$. Then $F^c$ is fuzzy open in $(Y,F_\sigma)$ and by assumption, $f^{-1}(F^c)$ is $f\tilde{g}$-open in $(X,F_\tau)$. Since $f^{-1}(F^c)=(f^{-1}(F))^c$, we have $f^{-1}(F)$ is $f\tilde{g}$-closed in $(X,F_\tau)$ and so $f$ is fuzzy $\tilde{g}$-continuous.

**Remark 3.26** The following example shows that the composition of two fuzzy $\tilde{g}$-continuous functions need not be a fuzzy $\tilde{g}$-continuous function.

**Example 3.27** Consider $X = Y = Z = \{m, n\}$ with $F_\tau = \{0_X, A, B, 1_X\}$ where $A$ and $B$ are fuzzy subsets in $X$ defined by $A(m) = 0.5, A(n) = 0$ and $B(m) = 1, B(n) = 0, F_\sigma = \{0_Y, C, 1_Y\}$ where $C$ is a fuzzy set in Y defined by $C(m) = 1, C(n) = 0$ and $F_\eta = \{0_Z, D, 1_Z\}$ where $D$ is a fuzzy set in $Z$ defined by $D(m) = 0.4, D(n) = 0$. In the space $(X, F_\tau), (Y, F_\sigma)$ and $(Z, F_\eta)$. Let $f: (X, F_\tau) \to (Y, F_\sigma)$ be the identity fuzzy function and $g: (Y, F_\sigma) \to (Z, F_\eta)$ be the identity fuzzy function. Clearly both $f$ and $g$ are fuzzy $\tilde{g}$-continuous but their composition $gof: (X, F_\tau) \to (Z, F_\eta)$ is not a fuzzy $\tilde{g}$-continuous.

**Proposition 3.28** If $f: (X, F_\tau) \to (Y, F_\sigma)$ is fuzzy $\tilde{g}$-continuous and $g: (Y, F_\sigma) \to (Z, F_\eta)$ is fuzzy continuous, then their composition $gof: (X, F_\tau) \to (Z, F_\eta)$ is fuzzy $\tilde{g}$-continuous.

**Proof.**

Let $A$ be any fuzzy closed set in $(Z, F_\eta)$. Since $g: (Y, F_\sigma) \to (Z, F_\eta)$ is fuzzy continuous, $g^{-1}(A)$ is fuzzy closed in $(Y, F_\sigma)$. Since $f: (X, F_\tau) \to (Y, F_\sigma)$ is fuzzy $\tilde{g}$-continuous, $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is $f\tilde{g}$-closed in $(X, F_\tau)$ and so $gof$ is fuzzy $\tilde{g}$-continuous.

**Proposition 3.29** Let $S$ be $f\tilde{g}$-closed in $(X, F_\tau)$. If $f: (X, F_\tau) \to (Y, F_\sigma)$ is fuzzy $sg$-irresolute and fuzzy closed, then $f(S)$ is $f\tilde{g}$-closed in $(Y, F_\sigma)$.

**Proof.**

Let $G$ be any $fsg$-open in $(Y, F_\sigma)$ such that $f(S) \le G$. Then $S \le f^{-1}(G)$ and by hypothesis, $C(S) \le f^{-1}(G)$. Thus $f(C(S)) \le G$ and $f(C(S))$ is a fuzzy closed set. Now, $C(f(S)) \le C(f(C(S))) = f(C(S)) \le G$. i.e., $C(f(S)) \le G$ and so $f(S)$ is $f\tilde{g}$-closed.

**Theorem 3.30** If $f: (X, F_\tau) \to (Y, F_\sigma)$ is fuzzy $\tilde{g}$-continuous and fuzzy pre-$sg$-closed and if $S$ is an $f\tilde{g}$-open (or $f\tilde{g}$-closed) subset of $(Y, F_\sigma)$, then $f^{-1}(S)$ is $f\tilde{g}$-open (or $f\tilde{g}$-closed) in $(X, F_\tau)$.

**Proof.**

Let $S$ be a $f\tilde{g}$-open set in $(Y, F_\sigma)$ and $G$ be any $fsg$-closed set in $(X, F_\tau)$ such that $G \le f^{-1}(\mathrm{S})$. Then $f(G) \le S$. By hypothesis, $f(G)$ is $fsg$-closed and $S$ is $f\tilde{g}$-open in $(Y, \sigma)$. Therefore, $f(G) \le I(\mathrm{S})$ and hence $G \le f^{-1}(I(S))$. Since $f$ is fuzzy $\tilde{g}$-continuous and $I(S)$ is fuzzy open in $(Y, F_\sigma), f^{-1}(I(S))$ is $f\tilde{g}$-open in $(X, F_\tau)$. Thus $G \le I(f^{-1}(I(S))) \le I(f^{-1}(S))$. i.e., $F \le I(f^{-1}(S))$ and $f^{-1}(\mathrm{S})$ is $f\tilde{g}$-open in $(X, F_\tau)$. By taking complements, we can show that if $S$ is $f\tilde{g}$-closed in $(Y, F_\sigma)$, $f^{-1}(S)$ is $f\tilde{g}$-closed in $(X, F_\tau)$.

**Corollary 3.31** If $f: (X, F_\tau) \to (Y, F_\sigma)$ is fuzzy continuous and fuzzy pre-$sg$-closed and if $B$ is a $f\tilde{g}$-closed (or $f\tilde{g}$-open) subset of $(Y, F_\sigma)$, then $f^{-1}(B)$ is $f\tilde{g}$-closed (or $f\tilde{g}$-open) in $(X, F_\tau)$.

**Proof.**

Follows from Proposition 3.29 and Theorem 3.30.

**Corollary 3.32** Let $(X, F_\tau), (Y, F_\sigma)$ and $(Z, \mathrm{F}_\eta)$ be any three fuzzy topological spaces. If $f: (X, F_\tau) \to (Y, F_\sigma)$ is fuzzy $\tilde{g}$-continuous and fuzzy pre-$gs$-closed and $g: (Y, F_\sigma) \to (Z, F_\eta)$ is fuzzy $\tilde{g}$-continuous, then their composition $gof: (X, F_\tau) \to (Z, F_\eta)$ is fuzzy $\tilde{g}$-continuous.

**Proof.**

Let $A$ be any fuzzy closed set in $(Z, F_\eta)$. Since $g: (Y, F_\sigma) \to (Z, F_\eta)$ is fuzzy $\tilde{g}$-continuous, $g^{-1}(A)$ is $f\tilde{g}$-closed in $(Y, F_\sigma)$. Since $f: (X, F_\tau) \to (Y, F_\sigma)$ is fuzzy $\tilde{g}$-continuous and fuzzy pre-$sg$-closed, by Theorem 3.30, $f^{-1}(g^{-1}(F)) = (gof)^{-1}(A)$ is $f\tilde{g}$-closed in $(X, F_\tau)$ and so $gof$ is fuzzy $\tilde{g}$-continuous.

**Fuzzy $\tilde{g}$-irresolute functions**

We introduce the following definition.

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**Definition 4.1** Let $f:(X,F_\tau)\to(Y,F_\sigma)$ be a fuzzy function. Then $f$ is said to be a fuzzy $\tilde{g}$-irresolute if the inverse image of every $f\tilde{g}$-closed set in $(Y,F_\sigma)$ is $f\tilde{g}$-closed in $(X,F_\tau)$.

**Remark 4.2** The following example shows that the concepts of fuzzy $sg$-irresolute and the concepts of fuzzy $\tilde{g}$-irresolute are independent of each other.

**Example 4.3** 1. Fuzzy $\tilde{g}$-irresolute $\nrightarrow$ Fuzzy $sg$-irresolute.
Consider $X=Y=\{m,n\}$ and $F_\tau=\{0_X,A,1_X\}$ where $A$ is a fuzzy set in $X$ defined by $A(m)=1, A(n)=0$ and $F_\sigma=\{0_Y,B,1_Y\}$ where $B$ is a fuzzy set in $Y$ defined by $B(m)=0.5, B(n)=0$. In the space $(X,F_\tau)$ and $(Y,F_\sigma)$. Let $f:(X,F_\tau)\to(Y,F_\sigma)$ be the identity fuzzy function. Then $f$ is fuzzy $\tilde{g}$-irresolute but it is not fuzzy $sg$-irresolute.

2. Fuzzy $sg$-irresolute $\nrightarrow$ Fuzzy $\tilde{g}$-irresolute.
Consider $X=Y=\{m,n\}$ and $F_\tau=\{0_X,A,1_X\}$ where $A$ is a fuzzy set in $X$ defined by $A(m)=0.4, A(n)=0.5$ and $F_\sigma=\{0_Y,B,1_Y\}$ where $B$ is a fuzzy set in Y defined by $B(m)=0.6, B(n)=0.5$. In the space $(X,F_\tau)$ and $(Y,F_\sigma)$. Let $f:(X,F_\tau)\to(Y,F_\sigma)$ be the identity fuzzy function. Then $f$ is fuzzy $sg$-irresolute but it is not fuzzy $\tilde{g}$-irresolute.

**Proposition 4.4** A fuzzy function $f:(X,F_\tau)\to(Y,F_\sigma)$ is fuzzy $\tilde{g}$-irresolute if and only if the inverse of every $f\tilde{g}$-open set in $(Y,F_\sigma)$ is $f\tilde{g}$-open in $(X,F_\tau)$.
**Proof.**
Similar to Proposition 3.25.

**Proposition 4.5** If a fuzzy function $f:(X,F_\tau)\to(Y,F_\sigma)$ is fuzzy $\tilde{g}$-irresolute then it is fuzzy $\tilde{g}$-continuous but not conversely.

**Example 4.6** Consider $X=Y=\{m,n\}$ and $F_\tau=\{0_X,A,B,1_X\}$ where $A$ is a fuzzy set in $X$ defined by $A(m)=0.5, A(n)=0$ and $F_\sigma=\{0_Y,B,1_Y\}$ where $B$ is a fuzzy set in $Y$ defined by $B(m)=1, B(n)=0$. In the space $(X,F_\tau)$ and $(Y,F_\sigma)$. Let $f:(X,F_\tau)\to(Y,F_\sigma)$ be the identity fuzzy function. Thus $f$ is fuzzy $\tilde{g}$-continuous but it is not fuzzy $\tilde{g}$-irresolute.

**Proposition 4.7** If $f:(X,F_\tau)\to(Y,F_\sigma)$ is bijective fuzzy pre-$sg$-open and fuzzy $\tilde{g}$-continuous then $f$ is fuzzy $\tilde{g}$-irresolute.
**Proof.**
Let $S$ be $f\tilde{g}$-closed set in $(Y,F_\sigma)$. Let $G$ be any $fsg$-open set in $(X,F_\tau)$ such that $f^{-1}(S)\le G$. Then $S\le f(G)$. Since $S$ is $f\tilde{g}$-closed and $f(G)$ is $fsg$-open in $(Y,F_\sigma)$, $C(S)\le f(G)$ holds and hence $f^{-1}(C(S))\le G$. Since $f$ is fuzzy $\tilde{g}$-continuous and $C(S)$ is fuzzy closed in $(Y,F_\sigma)$, $f^{-1}(C(S))$ is $f\tilde{g}$-closed and hence $C(f^{-1}(C(S)))\le G$ and so $C(f^{-1}(S))\le G$. Therefore, $f^{-1}(S)$ is $f\tilde{g}$-closed in $(X,F_\tau)$ and hence $f$ is fuzzy $\tilde{g}$-irresolute.
The following Example shows that no assumption of Proposition 4.7 can be removed.

**Example 4.8** The identity fuzzy function in Example 4.6 is fuzzy $\tilde{g}$-continuous and fuzzy bijective but not fuzzy pre-$sg$-open and so $f$ is not fuzzy $\tilde{g}$-irresolute.

**Example 4.9** Consider $X=Y=\{m,n\}$ and $F_\tau=\{0_X,A,1_X\}$ where $A$ is a fuzzy set in $X$ defined by $A(m)=0.6, A(n)=0.5$ and $F_\sigma=\{0_Y,B,1_Y\}$ where $B$ is a fuzzy set in $Y$ defined by $B(m)=0.4, B(n)=0.5$. In the space $(X,F_\tau)$ and $(Y,F_\sigma)$. Let $f:(X,F_\tau)\to(Y,F_\sigma)$ be the identity fuzzy function. Then $f$ is fuzzy bijective and fuzzy pre-$sg$-open but not fuzzy $\tilde{g}$-continuous and so $f$ is not fuzzy $\tilde{g}$-irresolute.

**Proposition 4.10** If $f:(X,F_\tau)\to(Y,F_\sigma)$ is bijective fuzzy closed and fuzzy $sg$-irresolute then the inverse function $f:(X,F_\tau)\to(Y,F_\sigma)$ is fuzzy $\tilde{g}$-irresolute.

**Proof.**

Let $S$ be $f\tilde{g}$-closed in $(X, F_\tau)$. Let $(f^{-1})^{-1}(S) = f(S) \leq G$ where $G$ is $fsg$-open in $(Y, F_\sigma)$. Then $S \leq f^{-1}(G)$ holds. Since $f^{-1}(G)$ is $fsg$-open in $(X, F_\tau)$ and $S$ is $f\tilde{g}$-closed in $(X, F_\tau), C(S) \leq f^{-1}(G)$ and hence $f(C(S)) \leq G$. Since $f$ is fuzzy closed and $C(S)$ is fuzzy closed in $(X, F_\tau), f(C(S))$ is fuzzy closed in $(X, F_\tau)$ and so $f(C(S))$ is fuzzy $\tilde{g}$-closed in $(Y, F_\sigma)$. Therefore $C(f(C(S))) \leq G$ and hence $C(f(S)) \leq G$. Thus $f(S)$ is $f\tilde{g}$-closed in $(Y, F_\sigma)$ and so $f^{-1}$ is fuzzy $\tilde{g}$-irresolute.

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