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An Eoq (Economic Order Quantity) Model for Linearly Time Dependent Deterioration with Quadratic Time Dependent Demand and Quadratic Time Dependent Shortages

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Abstract:- In this study, An EOQ model is developed with quadratic time dependent demand. Deterioration is consider linearly time dependent. Shortages are allowed with quadratic time dependent to meet the demand. The mathematical model is developed for finding the optimal total inventory cost. Numerical example is given to authenticate to proposed the model.

Keywords: Inventory, deterioration, shortages, holding cost time dependent.

1. Introduction

Managing and controlling inventory poses several challenges for businesses. These include errors in manual data entry which can lead to inaccuracies, issues with maintaining optimal inventory levels to prevent stock outs or excess stock, which can tie up capital and lead to storage costs. Additionally, incomplete or outdated inventory information can hinder decision-making, while communication gaps between departments can result in coordination issues. Furthermore, delays in receiving goods from suppliers and reliance on unreliable suppliers can disrupt production schedules and increase the risk of shortages. Consequently, effectively managing inventory, especially for perishable items, is a complex task for decision-makers. We consider some basic definitions.

Harris (1915) introduced the initial inventory model known as Economic Order Quantity [4], further developed by Wilson (1934) who provided a formula for its calculation. Ghare and Schrader (1963) introduced a model for inventory subject to exponential decay [3]. Dave and Patel (1981) pioneered the examination of deteriorating inventory with linearly increasing demand, where shortages are not allowed. Recent advancements in this field include studies by Chung and Ting (1993) [2]. And Wee (1995) on inventory models involving deteriorating items. In 1999, Chang and Dye formulated an inventory model considering time-varying demand and partial backlogging [1]. Goyal and Giri (2001) presented contemporary trends in deteriorating item inventory modeling, categorizing models based on demand variations and constraints. Ouyang and Cheng (2005) developed an inventory model for deteriorating items with exponential declining demand and partial backlogging [10]. Alamri and Balkhi (2007) investigated the impact of learning and forgetting on determining the optimal production lot size for deteriorating items facing time-varying demand and deterioration rates. In 2007, Dye and colleagues identified optimal selling price and lot size considering variable deterioration rates and exponential partial backlogging. They suggested that the fraction of customers backlogging orders increases exponentially as waiting time for replenishment decreases.

In 2008, Roy developed a deterministic inventory model where the deterioration rate increases proportionally with time [12]. The demand rate depends on the selling price, and the holding cost fluctuates over time. Liao (2008) introduced an economic order quantity (EOQ) model incorporating non-immediate receipt and exponential item deterioration, while also considering a two-level trade credit system.

In 2009, Pareek and co-authors introduced a deterministic inventory model designed for deteriorating items, incorporating salvage value and addressing shortages [11]. Skouri (2009) presented an inventory model characterized by ramp-type demand, partial backlogging [13], and Weibull's deterioration rate. Mishra and Singh (2010) developed a deteriorating inventory model involving waiting time partial backlogging, with constant demand and deterioration rates [8]. Yang and Wee (2001) devised a joint inventory model considering unit deterioration, catering to scenarios involving multiple consumers and a single merchant. Wee (2001) explored supply chain inventory models where multiple consumers and a single merchant utilized information technologies to enhance coordination and mechanization, reducing ordering costs [14]. Zavanella and Zanonia (2009) presented an analytical model based on an industrial case, considering scenarios with a single merchant and multiple consumers. In 2011, Shah developed a joint inventory model within a supply chain system, incorporating quadratic demand and involving multiple consumers and a single merchant. Ghiami and Williams (2015) analyzed inventory models with a two-stage creation process involving multiple consumers, where decaying items with a fixed production rate are dispatched by the manufacturer in predefined order quantities for a set time period, with any surplus inventory utilized for subsequent deliveries.

In 2010, Mandal introduced an Economic Order Quantity (EOQ) inventory model tailored for deteriorating items following a Weibull distribution, accounting for ramp-type demand and shortages [6]. Mishra and Singh (2011) devised an inventory model considering ramp-type demand, time-dependent item deterioration with salvage value, and shortages. They also developed a deteriorating inventory model accommodating time-dependent demand and holding cost, while incorporating partial backlogging. Hung (2011) proposed an inventory model featuring generalized demand, deterioration, and back order rates [5]. Tripathi RP devised an EOQ Model with Cash Flow Oriented and Quantity Dependent Under Trade Credits [15]. Tripathi RP and Shweta developed an Establishment of EOQ Model with Quadratic Time-Sensitive Demand, Parabolic-Time Linked Holding Cost with Salvage Value [16]. Goyal and S.K. devised an Economic Order Quantity under Conditions of Permissible Delay in Payments [17]. Gupta PN and Agarwal introduced An Order Level Inventory Model with Time Dependent Deterioration [18]. Hariga M and Ben-Daya study on Some stochastic inventory models with deterministic variable lead time [19]. Hollier R.H. and Mak K.L. Researched An Inventory Model for Deteriorating Items with Generalised Exponential Decreasing Demand, Constant Holding Cost and Time-Varying Deterioration Rate [20].

The remaining part of the paper is organized as follows. In the next section we describe the notation and assumptions used in the whole paper. In section 3 mathematical formulation is given.

2. Assumptions and Notations

1. Deterioration rate is linear with time dependent.
2. Deterioration rate = $\alpha + \beta\, t; \alpha, \beta > 0$.
3. Demand rate is time dependent and quadratic, i.e., $D(t) = a + bt + ct^2$; $a, b, c > 0$ and a, b, c are constants.
4. Shortages are time dependent and quadratic, i.e., $S = a + bt + ct^2$; $a, b, c > 0$ and a, b, c are constants.
5. $0 < \gamma < 1$ is the backlogging rate.
6. Holding cost is per unit time and constant assumed $h(t) = h$.
7. Q is the maximum inventory level during $(0, t_1)$.
8. T is the length of the cycle.
9. Replenishment is instantaneous; lead time is zero.
10. There is no repair or replenishment of deteriorating item during the period under consideration.
11. C_2 is the shortage cost per unit per unit time.
12. C_3 is purchase cost per unit per unit time.
13. A is ordering cost per unit per unit time.

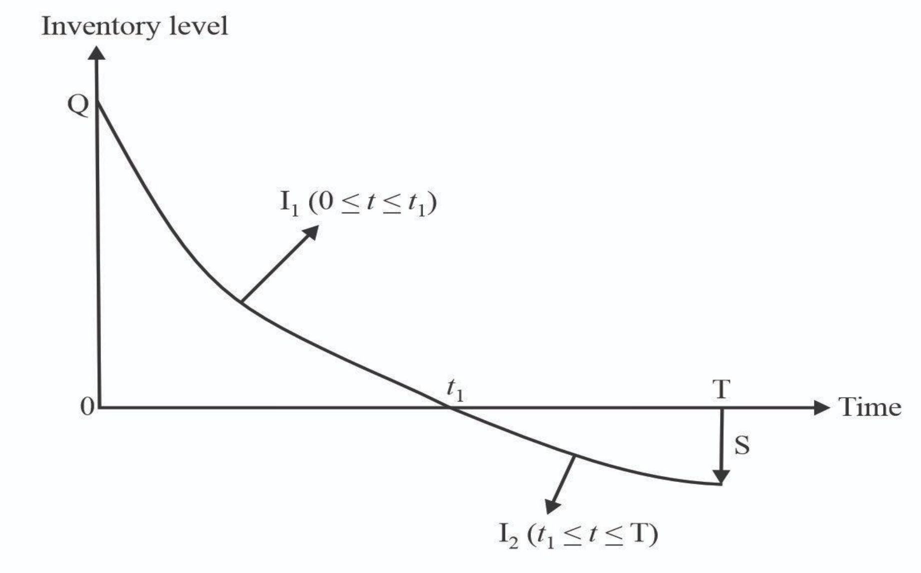

Figure 1: Graphical Representation of the inventory System

3. Mathematical formulation and solution

The mathematical formulation, the rate of change of the inventory $I_1(t)$ during period $(0, t_1)$ and $I_2(t)$ during the period (t_1, T) i.e. shortage by the following differential equations.

$$\frac{dI_1(t)}{dt} = -(a + bt + ct^2) - (\alpha + \beta\, t)I_1(t);\ \ 0 \le\ t \le\ t_1 \tag{1}$$

and

$$\frac{dI_2(t)}{dt} = -\gamma\,(a + bt + ct^2);\ \ t_1 \le\ t \le\ T \tag{2}$$

where $a, b, c, \alpha, \beta, \gamma$ are constants.

The initial conditions for the inventory level is Q when time t=0, and also when $t = t_1$

the inventory level Q becomes zero with deterioration rate . Shortage is accumulated which is fully backlogged at the rate γ. In cycle time inventory reaches maximum shortage level and backlogged and again raises inventory level to Q.

Thus, boundary conditions are as follows:

$$I_1(0) = Q, I_1(t_1) = 0, I_2(t_1) = 0, I_2(T) = -S$$

where $S = a + bt + ct^2$

The solution of equation (1) with boundary conditions

Which is linear whose integrating factor is

$$e^{\int (\alpha+\beta\, t)dt} \;=\; e^{\left(\alpha t+\frac{\alpha t^2}{2}\right)}$$

solution is

$$e^{\left(\alpha t+\frac{\alpha t^2}{2}\right)} I_1(t) \;=\; -\int (a+bt+ct^2)\, e^{\left(\alpha t+\frac{\alpha t^2}{2}\right)} \;+\; C \tag{3}$$

where $a, b, c, \alpha, \beta, \gamma$ are constants.

The initial conditions for the inventory level is Q when time $t = 0$, and also when $t = t_1$

the inventory level Q becomes zero with deterioration rate . Shortage is accumulated which is fully backlogged at the rate γ. In cycle time inventory reaches maximum shortage level and backlogged and again raises inventory level to Q.

Thus, boundary conditions are as follows:

$I_1(0) = Q,\; I_1(t_1) = 0,\; I_2(t_1) = 0,\; I_2(T) = -S$

where $S = a + bt + ct^2$

The solution of equation (eq. 1) with boundary conditions

Which is linear whose integrating factor is

$$e^{\int (\alpha+\beta\, t)dt} \;=\; e^{\left(\alpha t+\frac{\alpha t^2}{2}\right)}$$

solution is

$$e^{\left(\alpha t+\frac{\alpha t^2}{2}\right)} I_1(t) \;=\; -\int (a+bt+ct^2)\, e^{\left(\alpha t+\frac{\alpha t^2}{2}\right)} \;+\; C \tag{4}$$

using initial condition $I_1(0) = Q$ then we get $C \;=\; Q$

using this value in equation (eq. 3) we get particular solution

$$I_1(t) = \; -e^{\left(-\alpha t+\beta\frac{t^2}{2}\right)}\left[a\left(t+\alpha\frac{t^2}{2}+\beta\frac{t^3}{6}\right)+b\left(\left(\frac{t^2}{2}+\alpha\frac{t^3}{3}+\beta\frac{t^4}{8}\right)\right)+c\left(\left(\frac{t^3}{3}+\alpha\frac{t^4}{4}+\beta\frac{t^5}{10}\right)\right)\right]+Q \tag{5}$$

where Q is the maximum inventory level during $(0, t_1)$ i.e.

$$Q \;=\int_0^{t_1} (a+bt+ct^2)\, e^{\left(\alpha t+\frac{\alpha t^2}{2}\right)} dt$$

$$Q = \frac{at_1^3}{3}+\frac{bt_1^2}{2}+ct_1+\frac{a\alpha t_1^4}{4}+\frac{b\alpha t_1^3}{3}+\frac{c\alpha t_1^2}{2}+\frac{a\beta t_1^5}{6}+\frac{b\beta t_1^4}{8}+\frac{c\beta t_1^3}{6}$$

Solution of eq. 2 is

$$I_2(t) = -\gamma\left(at+\frac{bt^2}{2}+\frac{ct^3}{3}\right)$$

$$I_2(t) = -\gamma\left[a(T-t_1) + \frac{b}{2}(T^2 - t_1^2) + \frac{c}{3}(T^3 - t_1^3)\right]; \quad t_1 \le t \le T$$

(6)

3(a) Holding cost:

Let h is the holding cost per unit time.

$$HC = \int_0^{t_1} hI_1(t)dt$$

$$HC = h\left[-a\left(\frac{t_1^2}{2} + \frac{\alpha t_1^3}{6} + \frac{\beta t_1^4}{24}\right) - b\left(\frac{t_1^3}{6} + \frac{\alpha t_1^4}{12} + \frac{\beta t_1^5}{40}\right) - c\left(\frac{t_1^4}{12} + \frac{\alpha t_1^5}{20}\right) + a\alpha\left(\frac{t_1^3}{3} + \frac{\alpha t_1^4}{8} + \frac{\beta t_1^5}{30}\right) + b\alpha\left(\frac{t_1^4}{8} + \frac{\alpha t_1^5}{15}\right) + c\alpha\left(\frac{t_1^5}{15}\right) + \frac{a\beta}{2}\left(\frac{t_1^4}{4} + \frac{\alpha t_1^5}{10}\right) + \frac{b\beta}{2}\left(\frac{t_1^5}{10}\right) + Qt_1\right]$$

(7)

(Neglecting higher powers of t greater than t^5)

3(b) Shortage cost:

Let C_2 is the shortage cost per unit time.

$$SC = C_2\int_{t_1}^{T} (-I_2(t))dt$$

$$SC = C_2\gamma\left[\frac{a}{2}(T^2 - t_1^2) + \frac{b}{6}(T^3 - t_1^3) + \frac{c}{12}(T^4 - t_1^4)\right]$$

(8)

3(c) Lost sale cost is:

$$LSC = S\int_{t_1}^{T} [1 - \gamma D(t)]dt$$

$$LSC = S(1-\gamma)\left[a(T - t_1) + \frac{b}{2}(T^2 - t_1^2) + \frac{c}{3}(T^3 - t_1^3)\right]$$

(9)

3(d) Purchase cost:

Let C_3 is purchase cost per unit time.

$$PC = C_3\left(Q + \int_{t_1}^{T} \gamma D(t)dt\right)$$

$$PC = C_3Q + C_3\gamma\left[a(T - t_1) + \frac{b}{2}(T^2 - t_1^2) + \frac{c}{3}(T^3 - t_1^3)\right]$$

(10) 1

4. Total Cost

The total cost is TC i.e. $TC = OC + PC + HC + SC + LSC$

$$TC = A + C_3Q + C_3\gamma\left[a(T - t_1) + \frac{b}{2}(T^2 - t_1^2) + \frac{c}{3(T^3 - t_1^3)}\right] + h\left[-a\left(\frac{t_1^2}{2} + \frac{\alpha t_1^3}{6} + \frac{\beta t_1^4}{24}\right) - b\left(\frac{t_1^3}{6} + \frac{\alpha t_1^4}{12} + \frac{\beta t_1^5}{40}\right) - c\left(\frac{t_1^4}{12} + \frac{\alpha t_1^5}{20}\right) + a\alpha\left(\frac{t_1^3}{3} + \frac{\alpha t_1^4}{8} + \frac{\beta t_1^5}{30}\right) + b\alpha\left(\frac{t_1^4}{8} + \frac{\alpha t_1^5}{15}\right) + c\alpha\left(\frac{t_1^5}{15}\right) + \frac{a\beta}{2}\left(\frac{t_1^4}{4} + \frac{\alpha t_1^5}{10}\right) + \frac{b\beta}{2}\left(\frac{t_1^5}{10}\right) + Qt_1\right] + C_2\gamma\left[\frac{a}{2}(T^2 - t_1^2) + \frac{b}{6}(T^3 - t_1^3) + \frac{c}{12(T^4 - t_1^4)}\right] + S(1-\gamma)\left[a(T - t_1) + \frac{b}{2}(T^2 - t_1^2) + \frac{c}{3(T^3 - t_1^3)}\right] \tag{11}$$

Differentiating equation (11) with respect to t_1 and T then we get the following

$$\frac{\partial TC}{\partial t_1} \; and \; \frac{\partial TC}{\partial T}$$

To minimize the total cost $TC(t_1, T)$ per unit time, the optimum value of T and t_1 can be obtained by solving the following equations.

$$\frac{\partial TC}{\partial t_1} = 0 \; and \; \frac{\partial TC}{\partial T} = 0 \tag{12}$$

Providing that the equation (11) satisfies the following conditions.

$$\left(\frac{\partial^2 TC}{\partial t_1^2}\right)\left(\frac{\partial^2 TC}{\partial T^2}\right) - \left(\frac{\partial^2 TC}{\partial t_1 \partial T}\right)^2 > 0 \; and \; \left(\frac{\partial^2 TC}{\partial t_1^2}\right) > 0 \tag{13}$$

It is difficult to find equation (13) due to large power of t_1 and T. It becomes complicated and lengthy to remove the complexity. we can show the minimum total cost by graph.

5. Results and Discussion

For the numerical and graphical analysis, we considered as inputs of the parameters in proper unit of the model, $A = 100, C_3 = 5,\ \gamma = 3,\ \alpha = 4,\ \beta = 1.2,\ h = 30,\ a = 0.5,\ b = 1.5,\ c = 5,\ S = 10,\ C_2 = 29,$. The output of the model by using a mathematical software (the optimal value of the total cost, the time when the inventory level reaches zero and the time when the maximum shortage occur) is $TC = 99.814, t_1 = 0.0230$, and $T = 0.0638$.

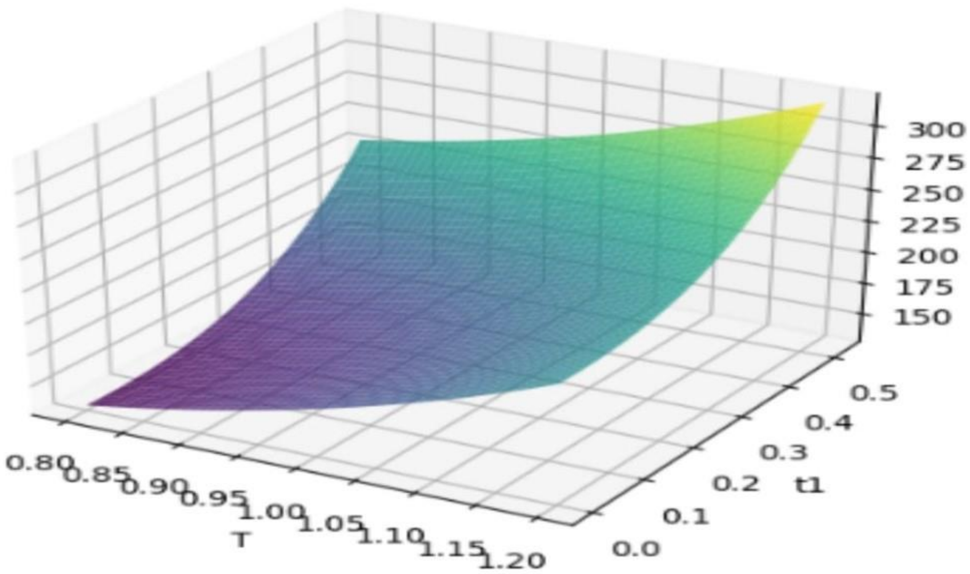

Figure2: Total Cost function in 3D

If we plot the total cost function (eq. 11) with some values of t_1 and T such that T at 0.0638 then t_1 varies from 0.00 to 0.5 and at $t_1 = 0.0230$ then T varies from 0.80 to 1.20 therefore we get the strictly convex graph of the total cost function (TC) which is given by figures 2.

From the figures 2 the observation is that the total cost function of the model gives that the total inventory cost per unit time of the inventory system is minimum.

6. Conclusion

The study of this paper presents an EOQ inventory model of direct application to the business enterprises that consider the fact that the storage item is deteriorated during storage periods and in which demand is time dependent and quadratic, deterioration is linear, and holding cost is constant. In this paper, we developed an EOQ deterministic inventory model with demand is time dependent and quadratic and holding cost is per unit time where deterioration rate is linear and time dependent. The model allows for shortages where shortage are time dependent with quadratic and fully backlogged. The model is solved with the help of numerical example by minimizing the total inventory cost.

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