Performance Analysis of Single Server Fuzzy Queues with Trapezoidal Fuzzy Numbers Using Lr Method

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***Abstract*:** Queueing system environment has been analyzed in this document where the queueing model is described under fuzzy theory. The performance measure of single server fuzzy queues are calculated with trapezoidal fuzzy numbers using LR method as well as alpha-cut method for comparison. LR representation for trapezoidal fuzzy numbers are calculated to find the expected customer count and mean waiting period of customers in FM/FM/1 queueing pattern. Important concepts such as functional materials, simulation materials can be formulated in Fuzzy queues using LR method. To illustrate the validity of LR method, an arithmetical example is solved for trapezoidal fuzzy numbers.

1. Introduction

Queueing model applications are of great importance in areas like manufacturing industry, logistics, health systems, information processing techniques and computing. Various customers receive service according to queue discipline where the interval time and service times follow certain queueing distributions. Queuing models will have wider application under fuzzy set theory. Fuzzy Queueing models are studied by many researchers, Negi and Lee [1], Li and Lee[2], B. Palpandi and G. Geetha Ramani [3] analysed the performance measure for bulk arrival queueing system withparameters of fuzzy using RRT. Also LR method was proposed by researchers, J.P. Mukeba Kanyinda et al[4] for solving triangular fuzzy numbers. Also Shanmugasundari and S. Aarthi [5] studied a different approach for solving Fuzzy Queueing system, G. Geetharamani, B. Palpandi and J.Arun Pandian [7] analysed performance system measures for Heterogeneous Computing Network by using Fuzzy Queueing system.

In this paper, FM/FM/1 Queueing pattern is studied for analyzing the performance measure of single server fuzzy queues where the expected customer count and expected delay period are calculated by using trapezoidal numbers with fuzzy boundaries of LR type. The objective of this document is to emphasize LR method using trapezoidal numbers with fuzzy boundaries of LR type which gives a platform for other Queueing model problems since LR method takes less time in solving problems.

2. Preliminaries

2.1Basic operations in trapezoidal numbers with fuzzy boundaries of LR type :

Let $\check{T}= (t_1,t_2,t_3,t_4)$ & $\tilde{V}= (v_1, v_2, v_3, v_4)$ & 'λ' is a parameter, then

(i) $\check{T} + \tilde{V}= (t_1, t_2, t_3, t_4) + (v_1, v_2, v_3, v_4) = (t_1+v_1, t_2+v_2, t_3+v_3, t_4+v_4)$
(ii) $\check{T} - \tilde{V}=(t_1, t_2, t_3, t_4) - (v_1, v_2, v_3, v_4) =(t_1-v_2,t_2-v_1,t_3+v_4, t_4+v_3)$
(iii) $\check{T}\tilde{V}=(t_1, t_2, t_3, t_4). (v_1, v_2, v_3, v_4)=(t_1v_1,t_2v_2,t_1v_3 + t_3v_1, t_2v_4+v_2t_4)$
(iv) $\frac{\check{T}}{\tilde{V}}=\frac{(t_1,t_2,t_3,t_4)}{(v_1,v_2,v_3,v_4)}= (\frac{t_1}{v_2},\frac{t_2}{v_1},\frac{t_3}{v_4},\frac{t_4}{v_3})$
(v) $\lambda (t_1, t_2, t_3, t_4) = (\lambda t_1, \lambda t_2, \lambda t_3, \lambda t_4)$

(vi) $1/\lambda (t_1, t_2, t_3, t_4) =(t_1/\lambda, t_2/\lambda, t_3/\lambda, t_4/\lambda)$

3. Expected customer count and Expected delay period in a fuzzy queue:

3.1.Representation of trapezoidal numbers with fuzzy boundaries of LR type

Trapezoidal numbers with fuzzy boundaries are noted as Ŧ(t, v, m, s). In LR-type representation, this can be written as Ŧ(t, v, m, s) = <v, m, v - t, s - m>. Hence the arrival rate $\lambda = (t_1, t_2, t_3, t_4)$ and the service rate $\mu = (v_1, v_2, v_3, v_4)$ are taken as $\lambda = (t_2, t_3, t_2-t_1, t_4-t_3)_{LR}$ and $\mu = (v_2, v_3, v_2-v_1, v_4-v_3)_{LR}$ in LR representation.

3.2.Theorem

If a fuzzy queueing pattern FM/FM/1 is considered with customer entry rate and customer service rate taken as positive trapezoidal numbers with fuzzy boundaries having $\lambda = (t_1, t_2, t_3, t_4)$ and $\mu = (v_1, v_2, v_3, v_4)$ with $t_4<v_1$, then the customer countexpected in the system at steady state is the approximate number N with fuzzy parameters of LR type trapezoidal fuzzy values given by $N = (\alpha_N, \beta_N, \gamma_N, \eta_N)$ whose (i) Most possible value is the real number $\alpha_N = (\frac{t_2}{v_3-t_2})$ (ii) Left and Right spreads are given by two cases:

Case 1: If $(\gamma_N = \eta_N) < \beta_N$, then $\alpha_N - (\gamma_N = \eta_N)$ and $\beta_N + (\gamma_N = \eta_N)$ indicate spreads.

Case 2: If $(\gamma_N = \eta_N) > \beta_N$, then $(\gamma_N = \eta_N) - \beta_N$ and $(\gamma_N = \eta_N) + \alpha_N$ indicate spreads.

Proof:

In LR method, trapezoidal fuzzy numbers are given by replacing its suitable LR fuzzy numbers, $\lambda = (t_2, t_3, t_2-t_1, t_4-t_3)_{LR}$ and $\mu = (v_2, v_3, v_2-v_1, v_4-v_3)_{LR}$. Let 'N' be the expected customer count in the system at steady state.

$$N = \left(\frac{\lambda}{\mu-\lambda}\right)$$

$$= \frac{(t_2, t_3, t_2-t_1, t_4-t_3)}{(v_2, v_3, v_2-v_1, v_4-v_3)-(t_2, t_3, t_2-t_1, t_4-t_3)}$$

$$= \frac{(t_2, t_3, t_2-t_1, t_4-t_3)}{(v_2-t_3, v_3-t_2, v_2-v_1+t_4-t_3, v_4-v_3+t_2-t_1)}$$

$$N = \left(\frac{t_2}{v_3-t_2}, \frac{t_3}{v_2-t_3}, \frac{t_2-t_1}{v_4-v_3+t_2-t_1}, \frac{t_4-t_3}{v_2-v_1+t_4-t_3}\right),$$

Where $\alpha_N = \frac{t_2}{v_3-t_2}$, $\beta_N = \frac{t_3}{v_2-t_3}$, $\gamma_N = \frac{t_2-t_1}{v_4-v_3+t_2-t_1}$, $\eta_N = \frac{t_4-t_3}{v_2-v_1+t_4-t_3}$

Hence the most possible value is given by $\alpha_N = (\frac{t_2}{v_3-t_2})$

The left and right spreads are given by $\alpha_N - (\gamma_N = \eta_N)$ and $\beta_N + (\gamma_N = \eta_N)$ if $(\gamma_N = \eta_N) < \beta_N$ and $(\gamma_N = \eta_N) - \beta_N$ and $(\gamma_N = \eta_N) + \alpha_N$ if $(\gamma_N = \eta_N) > \beta_N$.

3.3.Theorem

If a fuzzy queueing pattern FM/FM/1 is considered with customer entry rate and customer service rate taken as positive trapezoidal numbers with fuzzy boundarieshaving $\lambda = (t_1, t_2, t_3, t_4)$ and $\mu = (v_1, v_2, v_3, v_4)$ with$t_4<v_1$, then the expected delay period in the queue at steady state is the approximatenumber T with fuzzy parameters of LR type trapezoidal fuzzy values given by $T = (\alpha_T, \beta_T, \gamma_T, \eta_T)$ whose (i) Most possible value is the real number $\alpha_T = (\frac{t_2}{t_3(v_3-t_2)})$ (ii) Left and Right spreads are given by two cases:

Case 1: If $(\gamma_T = \eta_T) < \beta_T$, then $\alpha_T - (\gamma_T = \eta_T)$ and $\beta_T + (\gamma_T = \eta_T)$ indicate spreads.

Case 2: If $(\gamma_T = \eta_T) > \beta_T$, then $(\gamma_T = \eta_T) - \beta_T$ and $(\gamma_T = \eta_T) + \alpha_T$ indicate spreads.

Proof:

In LR method, trapezoidal fuzzy numbers are given by replacing its suitable LR fuzzy numbers, $\lambda = (t_2, t_3, t_2-t_1, t_4-t_3)_{LR}$ and $\mu = (v_2, v_3, v_2-v_1, v_4-v_3)_{LR}$. Let 'T' be the expected delay period in the queue at steady state.

$T = \frac{N}{\lambda}$

$= \frac{(\alpha N, \beta N, \gamma N, \eta N)}{(t_2, t_3,\ t_2 - t_1, t_4 - t_3)}$

$= (\frac{\alpha_N}{t_3}, \frac{\beta_N}{t_2}, \frac{\gamma_N}{t_4 - t_3}, \frac{\eta_N}{t_2 - t_1})$

Where $\alpha_T = \frac{\alpha_N}{t_3}$, $\beta_T = \frac{\beta_N}{t_2}$, $\gamma_T = \frac{\gamma_N}{t_4 - t_3}$, $\eta_T = \frac{\eta_N}{t_2 - t_1}$

Hence the most possible value is given by $\alpha_T = (\frac{t_2}{t_3(v_3 - t_2)})$

The left and right spreads are given by $\alpha_T - (\gamma_T = \eta_T)$ and $\beta_T + (\gamma_T = \eta_T)$ if $(\gamma_T = \eta_T) < \beta_T$ and $(\gamma_T = \eta_T) - \beta_T$ and $(\gamma_T = \eta_T) + \alpha_T$ if $(\gamma_T = \eta_T) > \beta_T$.

4. Numerical Example:

About 35 people per hour seek photocopies from a Xerox shop. The owner of the Xerox shop supplied a high speed Xerox machine which may serve approximately 75 individuals per hour. The shop owner wants to make a comfortable waiting hall so that no one can wait in his shop. What is the customer count expected in hallfor a long time in service? Calculate the mean delay period of the customer in hall?

Solution:

The customer entry rate and the customer service rate are taken as trapezoidal numbers with fuzzy parameters $\lambda = (20,30,40,50)$ and $\mu = (60,70,80,90)$. The results are derived by alpha – cut method and LR method for comparison.

Alpha – cut Method:

The alpha cut for the customer entry or arrival rate and the customer service rate are given by

$[\lambda]_\alpha^L = (10\alpha + 20)$; $[\lambda]_\alpha^U = (50 - 10\alpha)$
$[\mu]_\alpha^L = (10\alpha + 60)$; $[\mu]_\alpha^U = (90 - 10\alpha)$

Expected customer count in the system:

$N = (\frac{\lambda}{\mu - \lambda})$

$N = [\frac{(10\alpha + 20,\ 50 - 10\alpha)}{(10\alpha + 60, 90 - 10\alpha) - (10\alpha + 20,\ 50 - 10\alpha)}]$

$N = [\frac{(10\alpha + 20), (50 - 10\alpha)}{(20\alpha + 10), (-20\alpha + 70)}]$

N = [min I, max I]

Where, $I = [\frac{10\alpha + 20}{20\alpha + 10}, \frac{10\alpha + 20}{70 - 20\alpha}, \frac{50 - 10\alpha}{20\alpha + 10}, \frac{50 - 10\alpha}{70 - 20\alpha}]$

Using parametric non-linear programming,

$N = [\frac{10\alpha + 20}{70 - 20\alpha}, \frac{50 - 10\alpha}{20\alpha + 10}]$

If $\alpha = 0$, $[\frac{20}{70}, \frac{50}{10}] = [0.2857, 5]$

If $\alpha = 1$, $[\frac{30}{50}, \frac{40}{30}] = [0.6, 1.3333]$

Trapezoidal Membership Function:

$$f(0.2857, 0.6, 1.3333, 5) = \begin{cases} \frac{x - 0.2857}{0.3143}, & 0.2857 \le x \le 0.6 \\ \frac{5 - x}{3.6667}, & 1.3333 \le x \le 5 \end{cases}$$

Expected delay period in the queue:

$T = \frac{N}{\lambda}$

$$T = \left[\frac{\left(\frac{10\alpha+20}{70-20\alpha}, \frac{50-10\alpha}{20\alpha+10}\right)}{(10\alpha+20,\ 50-10\alpha)} \right]$$

T = [min J, max J]

Where, $J = \left[\frac{1}{70-20\alpha}, \frac{10\alpha+20}{(70-20\alpha)(50-10\alpha)}, \frac{50-10\alpha}{(20\alpha+10)(10\alpha+20)}, \frac{1}{20\alpha+10} \right]$

Using parametric non-linear programming,

$T = \left[\frac{10\alpha+20}{(70-20\alpha)(50-10\alpha)}, \frac{50-10\alpha}{(20\alpha+10)(10\alpha+20)} \right]$

If α = 0, $\left[\frac{2}{350}, \frac{5}{20} \right]$ = [0.0057, 0.25]

If α = 1, $\left[\frac{3}{200}, \frac{4}{90} \right]$ = [0.015, 0.0444]

Trapezoidal Membership Function:

$$f(0.0057,0.015,0.0444,0.25)=\begin{cases} \frac{x-0.0057}{0.0093}, & 0.0057 \le x \le 0.015 \\ \frac{0.25-x}{0.2056}, & 0.0444 \le x \le 0.25 \end{cases}$$

LR Method:

The customer entry rate and the customer service rate are taken astrapezoidal numbers with fuzzy boundaries of LR type. By 3.1, λ = (30,40,10,10) and μ = (70,80,10,10).

Expected customer count in the system:

$N = \left(\frac{\lambda}{\mu-\lambda} \right)$

$N = \left[\frac{(30,40,10,10)}{(70,80,10,10)-(30,40,10,10)} \right]$

$= \left[\frac{(30,40,10,10)}{(30,50,20,20)} \right]$

= [0.6, 1.3, 0.5, 0.5]

By 3.2, the most possible value is 0.6 and the spreads to the left and right are 0.1 and 1.8.

Expected delay period in the queue:

$T = \frac{N}{\lambda}$

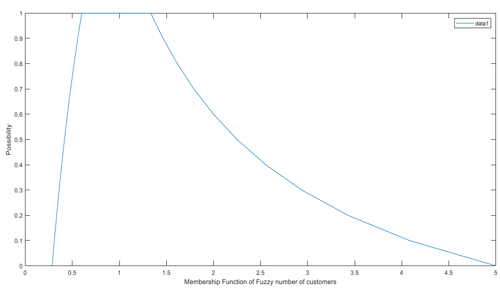
$= \left[\frac{\frac{(30,40,10,10)}{(30,50,20,20)}}{(30,40,10,10)} \right]$

$= \left[\frac{(0.6,1.3,0.5,0.5)}{(30,40,10,10)} \right]$

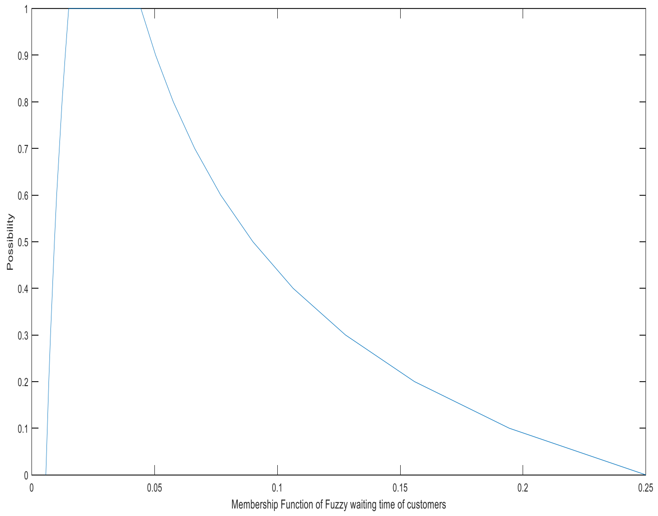
= [0.015, 0.0433, 0.05,0.05]

By 3.3, the most possible value is 0.015 and the spreads to the left and right are 0.0067 and 0.065.

Membership Function of Fuzzy customer count



Membership Function of Fuzzy delayperiod



5. Conclusion:

In this paper, expected customer count and mean waiting period of customers of FM/FM/1 pattern are calculated by both α-cut method and LR method. Using α-cut method, the values for expected customer count lie between 0.2857 to 5 and the most possible value is exactly 0.6. Similarly the mean waiting period of customers lie between 0.0057 and 0.25 and the most possible value is exactly 0.015.

Using LR method, the spreads to the left and right for the expected customer count are 0.1 and 1.8 and the most possible value is exactly 0.6. Similarly the spreads to the left and rightfor the mean waiting period of customers are 0.0067 and 0.065 and the most possible value is 0.015. Therefore compared to alpha-cut method, LR method is easier, comfortable and gives accurate material values in computing system performance measures.

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