Tuijin Jishu/Journal of Propulsion Technology
ISSN: 1001-4055
Vol. 45 No. 2 (2024)

Quadripartitioned Neutrosophic Vague Generalized Pre-Closed Sets in Quadripartitioned Neutrosophic Vague Topological Spaces

A.Sahaya Roseline Divya[1], Carmel Richard[2]

[1]*Department of Mathematics, Lady Doak College, Madurai- 625002, Tamilnadu, India*

[2]*Department of Mathematics, Lady Doak College, Madurai- 625002,Tamilnadu,India*

***Abstract*:-** The aim of this paper is to introduce and develop a new class of sets namely quadripartitioned neutrosophic vague generalized pre-closed sets in quadripartitioned neutrosophic vague topological space. Further we have analysed the properties of quadripartitioned neutrosophic vague generalized pre-open sets. Also some applications namely quadripartitioned neutrosophic vague $T_{1/2}$ space and quadripartitioned neutrosophic vague $pT_{1/2}$ space and quadripartitioned neutrosophic vague $gpT_{1/2}$ space are introduced.

***Keywords**:* Quadripartitioned neutrosophic vague topological space, quadripartitioned neutrosophic vague generalized pre-closed sets, quadripartitioned neutrosophic vague $T_{1/2}$ space, quadripartitioned neutrosophic vague $pT_{1/2}$ space and quadripartitioned neutrosophic vague $gpT_{1/2}$ space.

Introduction:

Fuzzy sets which allows the elements to have a degrees of membership in the set and it was introduced by Zadeh [30] in 1965. The degrees of membership lies in the real unit interval [0, 1]. Intuitionstic fuzzy set (IFS) allows both membership and non membership to the elements and this was introduced by Atanassov [4] in 1983. In the year 2005, Smarandache [27] extended the concept of intuitionistic fuzzy set by introducing the notion of neutrosophic set (NS). Later on, many researchers use NS in their theoretical and practical research. As an extension of fuzzy set theory in 1993, the theory of vague sets was first proposed by Gau and Buehre[9]. Shawkat Alkhazaleh [26] in 2015 introduced the concept of neutrosophic vague set as a combination of neutrosophic set and vague set. Neutrosophic vague theory is an effective tool to process incomplete, indeterminate and inconsistent information. In the year 2016, Chatterjee et. al. [5] grounded the idea of quadripartitioned neutrosophic set and defined several similarity measures between two quadripartitioned neutrosophic sets. Suman Das, Rakhal Das, and Carlos Granados[28] introduced the concept of Topology on Quadripartitioned Neutrosophic Sets. The idea of neutrosophic topological space (NTS) was presented by Salama and Alblowi [24] in the year 2012. Salama and Alblowi [25] also introduced Generalized neutrosophic set and generalized neutrosophic topological space. The neutrosophic semi-open mappings are studied by Arokiarani et. al. [3]. Afterwards, Iswaraya and Bageerathi [10] studied the concept of neutrosophic semi-open sets and neutrosophic semi-closed sets. Pushpalatha and Nandhini [20] grounded the idea of neutrosophic generalized closed sets in NTSs. The notion of neutrosophic b-open sets in NTSs was presented by Ebenanjar et al. [8]. Rao and Srinivasa [23] grounded the concept of pre open set and pre closed set via neutrosophic topological spaces. Maheswari, C., Sathyabama, M., & Chandrasekar, S [17] introduced by Neutrosophic generalized *b*-closed sets in neutrosophic topological spaces. Thereafter, Mohammed Ali Jaffer and Ramesh [19] studied the concept of neutrosophic generalized pre-regular closed sets. The generalized neutrosophic *b*-open sets in NTSs was introduced by Das and Pramanik [6]. T. Madhumathi and F.Nirmala Irudayam [16] introduced the idea of neutrosophic pre-open set in simple extended neutrosophic topology. A.Mary Margaret and M.Trinita Pricilla[18] introduced the concept of neutrosophic vague generalized pre-closed sets in neutrosophic vague topological spaces.

In this paper we introduce the concept of quadripartitioned neutrosophic vague generalized pre-closed sets and quadripartitioned neutrosophic vague generalized pre-open sets and their properties are obtained. Also its relationship with other existing sets are compared and discussed with examples.

2. Preliminaries

2.1 Definition[24]

Let U be a universe. A Neutrosophic set A on U can be defined as follows:

$$A = \{< x, T_A(x), I_A(x), F_A(x) >: x \in X\}$$

Where T_A , $I_A, F_A: U \to [0,1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$

Here,$T_A(x)$ is the degree of membership, $I_A(x)$ is the degree of inderminancy and $F_A(x)$ is the degree of non-membership.

2.2 Definition[5]

Let U be a universe. A Quadripartitioned neutrosophic set A with independent neutrosophic components on U is an object of the form

$$A = \{< x, T_A(x), C_A(x), U_A(x), F_A(x) >: x \in X\}$$

and $0 \leq T_A(x) + C_A(x) + U_A(x) + F_A(x) \leq 4$

Here,$T_A(x)$ is the truth membership, $C_A(x)$ is contradiction membership,$U_A(x)$ is ignorance membership and $F_A(x)$ is the false membership.

2.3 Definition[9]

A vague set V in a universe of discourse U is characterized by a true membership function t_v , and a false membership function f_v, as follows: $t_v : U \to [0, 1]$, $f_v: U \to [0, 1]$, and $t_v+f_v \leq 1$, where t_v (x) is a lower bound on the grade of membership of x derived from the evidence for x, and f_v (x) is a lower bound on the grade of membership of the negation of x derived from the evidence against x. The vague set A is written as,

A={ < x, t_A(x), 1-f_A(x)> |x ∈ X}.

2.4 Definition 2.1:[26] A neutrosophic vague set (NVS in short) on the universe of discourse U written as

$$A_{NV} = \{< x, \hat{T}_{A_{NV}}(x), \hat{I}_{A_{NV}}(x), \hat{F}_{A_{NV}}(x) >: x \in X\}$$

whose truth membership, indeterminacy membership and false membership functions is defined as:

$$\hat{T}_{A_{NV}}(x) = [T^-, T^+], \hat{I}_{A_{NV}}(x) = [I^-, I^+], \hat{F}_{A_{NV}}(x) = [F^-, F^+]$$

Where,

1) $T^+ = 1 - F^-$
2) $F^+ = 1 - T^-$ and
3) $^-0 \leq T^- + C^- + U^- + F^- \leq 2^+$

2.5 Definition: Let (X, τ) be topological space. A subset A of X is called:

1. semi closed set (SCS in short) [12] if int (cl (A)) ⊆ A,
2. pre- closed set (PCS in short) [15] if cl (int (A)) ⊆ A,

3. semi-pre closed set (SPCS in short) [1] if int (cl (int (A))) ⊆ A,
4. α-closed set (α CS in short) [21] if cl (int (cl (A))) ⊆ A,
5. regular closed set (RCS in short) [29] if A = cl (int (A)) .

2.6 Definition: Let (X, τ) be topological space. A subset A of X is called :

1. generalized closed (briefly, g-closed) [11] if cl (A) ⊆ U , whenever A ⊆ U and U is open in X.
2. generalized semi closed (briefly, gs-closed) [2] if scl (A) ⊆ U whenever A ⊆ U and U is open in X.
3. α -generalized closed (briefly, αg-closed) [13] if αcl (A) ⊆ U whenever A ⊆ U and U is open in X.
4. generalized pre-closed (briefly, gp-closed) [14] if pcl (A) ⊆ U whenever A ⊆ U and U is open in X.
5. generalized semi-pre closed (briefly, gsp-closed) [7] if spcl (A) ⊆ U whenever A ⊆ U and U is open in X.

3. Quadripartitioned neutrosophic vague topology

Definition 3.1

A quadripartitioned neutrosophicvauge set A_{QNV} on the universe of discourse X. written as $A_{QNV} = \left\{\left\langle x; \hat{T}_{A_{QNV}}; \hat{C}_{A_{QNV}}; \hat{U}_{A_{QNV}}; \hat{F}_{A_{QNV}} \right\rangle; x \in \mathrm{X}\right\}$

whose truth membership, contradiction membership, ignorance membership and false membership functions is defined as

$$\hat{T}_{A_{QNV}}(x) = [T^-, T^+],$$

$$\hat{U}_{A_{QNV}}(x) = [U^-, U^+],$$

$$\hat{C}_{A_{QNV}}(x) = [C^-, C^+],$$

$$\hat{F}_{A_{QNV}}(x) = [F^-, F^+]$$

where

1) $T^t = 1 - F$
2) $\mathrm{F}^+ = 1 - T$ and
3) $^-0 \leq T^- + C^- + U^- + F^- \leq 3^+$

Definition 3.2

Let A_{QNV} and B_{QNV} be two QNVSs of the universe U. If $\forall u_i \in U$, $\hat{T}_{A_{QNV}}(u_i) \leq \hat{T}_{B_{QNV}}(u_i)$; $\hat{C}_{A_{QNV}(u_i)} \leq \hat{C}_{B_{QNV}}(u_i)$, $\hat{U}_{A_{QNV}}(u_i) \geq \hat{U}_{B_{QNV}}(u_i)$; $\hat{F}_{A_{QNV}}(u_i) \geq \hat{F}_{\boldsymbol{B}_{QNV}}(u_i)$, then the QNVS AQNV Included by B_{QNV}whore $1 \leq i \leq n$.

Definition 3. 3

The complement of QNVS $A_{QNV}$is denotod by A^c_{QNV} and is defined by.

$$\hat{T}_{A_{QNV}}(x) = [1 - T^+, 1 - T^-],$$

$$\hat{C}_{A_{QNV}}(x) = [1 - C^+, 1 - C^-]$$

$$\hat{U}_{A_{QNV}}(x) = [1 - U^+, 1 - U^-],$$

$$\hat{F}_{A_{QNV}}(x) = [1 - F^+, 1 - F^-],$$

Definition 3.4

Let A_{QNV} be QNVS of the universe U where $\forall u_i \in U, \hat{T}_{A_{QNV}}(u) = [0,0]; \hat{C}_{A_{QNV}}(u) = [0,0]; \hat{U}_{A_{QNV}}(u) = [1,1]; \hat{F}_{A_{QNV}}(u) = [1,1]$.

Then, A_{QNV} is called null QNVS (0_{QNV} in Short). where $1 \leq i \leq n$.

Definition 3.5

Let A_{QNV} be QNUS of the universe U where $\forall u_i \in v, \hat{T}_{AQNV}(u) = [1,1]; \hat{C}_{AQNV}(u) = [1,$ $\hat{V}_{\mathrm{AONV}}(u) = [0,0]; \hat{F}_{AQNV}(u) = [1,1]$.

Then,A_{QNV} is called an absolute QNVS (1_{QNV}in short) whesene $1 \leq i \leq n$.

Definition 3.6

The union of two QNVSs A_{QNV} and B_{QNV} is QNVS D_{QNV}, written as $D_{QNV} = A_{QNV} \cup B_{QNV}$, whose truth membership, contradiction membership, ignorance membership and false membership functions are related to those of A_{QNV} and B_{QNV} given by

$$\hat{T}_{D_{QNV}}(x) = \left[\max\left(T^-_{A_{QNV_x}}, T^-_{B_{QNV_x}}\right), \max\left(T^+_{A_{QNV_x}}, T^+_{B_{QNV_x}}\right)\right]$$

$$\hat{C}_{D_{QNV}}(x) = \left[\max\left(C^-_{A_{QNV_x}}, C^-_{B_{QNV_x}}\right), \max\left(C^+_{A_{QNV_x}}, C^+_{B_{QNV_x}}\right)\right]$$

$$\hat{U}_{D_{QNV}}(x) = \left[\min\left(U^-_{A_{QNV_x}}, U^-_{B_{QNV_x}}\right), \min\left(U^+_{A_{QNV_x}}, U^+_{B_{QNV_x}}\right)\right]$$

$$\hat{F}_{D_{QNV}}(x) = \left[\min\left(F^-_{A_{QNV_x}}, F^-_{B_{QNV_x}}\right), \min\left(F^+_{A_{QNV_x}}, F^+_{B_{QNV_x}}\right)\right]$$

Definition 3.7

The intersection of two QNVS A_{QNV} and B_{QNV} is D_{QNV}, written as $D_{\mathrm{QNV}} = A_{\mathrm{QNV}} \cap B_{\mathrm{QNV}}$, whose truth membership, contradiction membership, ignorance membership and false membership functions are related to those of A_{QNV} and B_{QNV} given by.

$$\hat{T}_{D_{QNV}}(x) = \left[\min\left(T^-_{A_{QNV_x}}, T^-_{B_{QNV_x}}\right), \min\left(T^+_{A_{QNV_x}}, T^+_{B_{QNV_x}}\right)\right]$$

$$\hat{C}_{D_{QNV}}(x) = \left[\min\left(C^-_{A_{QNV_x}}, C^-_{B_{QNV_x}}\right), \min\left(C^+_{A_{QNV_x}}, C^+_{B_{QNV_x}}\right)\right]$$

$$\hat{U}_{D_{QNV}}(x) = \left[\max\left(U^-_{A_{QNV_x}}, U^-_{B_{QNV_x}}\right), \max\left(U^+_{A_{QNV_x}}, U^+_{B_{QNV_x}}\right)\right]$$

$$\hat{F}_{D_{QNV}}(x) = \left[\max\left(F^-_{A_{QNV_x}}, F^-_{B_{QNV_x}}\right), \max\left(F^+_{A_{QNV_x}}, F^+_{B_{QNV_x}}\right)\right]$$

Definition 3.8. Let X be a fixed set. A collection τ of some QNVSs over X is called a QNVT on X , if the following conditions holds:

(i) $1_{QNV}, 0_{QNV} \in \tau$;
(ii) $M_1 \cap M_2 \in \tau$ whenever $M_1, M_2 \in \tau$;
(iii) $\cup M_i \in \tau$,whenever $\{M_i : i \in \Delta\} \subseteq \tau$.

Then, (X, τ) is called a QNVTS. Every element of τ are called a quadripartitioned neutrosophic vague open set (QNVOS). If $M \in \tau$, then M^c is called a quadripartitioned neutrosophic vague closed set (QNVCS).

Definition 3.9. Let us consider a quadripartitioned neutrosophic vague subset X of a QNVTS , (X, τ). Then,the quadripartitioned neutrosophic vague closure (QNV-cl) of X is the intersection of all QNVCSs containing X and the quadripartitioned neutrosophic vague interior (QNV-int) of X is the union of all QNVOSs contained in X, i.e.

QNV-cl(*U*) = ∩ $\{W: U \subseteq W$ and W is a QNVCS in (X, τ) };

QNV-int(*U*) = ∪ $\{V: V \subseteq U$ and V is a QNVOS in $(X, \tau)\}$.

Definition 3.10. A QNVS $A_{QNV} = \left\{\left\langle x; \hat{T}_{A_{QNV}}; \hat{C}_{A_{QNV}}; \hat{U}_{A_{QNV}}; \hat{F}_{A_{QNV}} \right\rangle; x \in \mathrm{X}\right\}$in QNVTS (X, τ) is said to be

1. Quadripartitioned Neutrosophic Vague semi closed set (QNVSCS in short) if QNV int (QNV cl (A)) ⊆ A,
2. Quadripartitioned Neutrosophic Vague semi open set (QNVSOS in short) if A ⊆ QNV cl (QNV int (A)),
3. Quadripartitioned Neutrosophic Vague pre-closed set (QNVPCS in short) if QNV cl (QNV int (A)) ⊆ A,
4. Quadripartitioned Neutrosophic Vague pre-open set (QNVPOS in short) if A ⊆ QNV int (QNV cl (A)),
5. Quadripartitioned Neutrosophic Vague α-closed set (QNVαCS in short) if QNV cl (QNV int (QNV cl (A))) ⊆ A,
6. Quadripartitioned Neutrosophic Vague α-open set (QNVαOS in short) if A ⊆ QNV int (NV cl (QNV int (A))),
7. Quadripartitioned Neutrosophic Vague semi pre- closed set (QNVSPCS in short) if QNV int (QNV cl (QNV int (A))) ⊆ A,
8. Quadripartitioned Neutrosophic Vague semi pre-open set (QNVSPOS in short) if A ⊆ QNV cl (QNV int (QNV cl (A))),
9. Quadripartitioned Neutrosophic Vague regular open set (QNVROS in short) if A = QNV int (QNV cl (A)),
10. Quadripartitioned Neutrosophic Vague regular closed set (QNVRCS in short) if A = QNV cl (QNV int (A)).

Definition 3.11. Let A be QNVS of a QNVTS (X, τ) . Then the quadripartitioned neutrosophic vague semi interior of A (QNV sint (A) in short)

and quadripartitioned neutrosophic vague semi closure of A (QNV scl (A) in short) are defined by

1) QNV sint (A) = ∪$\{G/G$ is a QNVSOS in X and $G \subseteq A\}$,
2) QNV scl (A) = ∩ $\{K/K$ is a QNVSCS in X and $A \subseteq K\}$.

Result 3.12. Let A be QNVS of a QNVTS (X, τ) ,then

1) QNV scl (A) = A ∪ QNV int (QNV cl (A)) ,
2) QNV sint (A) = A ∩ QNV cl (QNV int (A)).

Definition 3.13. Let A be QNVS of a QNVTS (X, τ). Then the quadripartitioned neutrosophic vague alpha interior of A (QNV αint (A) in short) and quadripartitioned neutrosophic vague alpha closure of A (QNV αcl (A) in short) are defined by

1) QNV αint (A) = ∪$\{G/G$ is a QNVαOS in X and $G \subseteq A\}$,
2) QNV αcl (A) = ∩ $\{K/K$ is a QNVαCS in X and $A \subseteq K\}$.

Result 3.14. Let A be QNVS of a QNVTS (X, τ), then

1) QNV αcl (A) = A ∪ QNV cl (QNV int (QNV cl (A))) ,
2) QNV αint (A) = A ∩ QNV int (QNV cl (QNV int (A))).

Definition 3.15. Let A be QNVS of a QNVTS (X, τ). Then the quadripartitioned neutrosophic vague semi-pre interior of A (QNV spint (A) in short) and quadripartitioned neutrosophic vague semi-pre closure of A (QNV spcl (A) in short) are defined by

1) QNV spint (A)= ∪$\{G/G$ is a QNVSPOS in X and $G \subseteq A\}$,
2) QNV scl (A) = ∩ $\{K/K$ is a QNVSPCS in X and $A \subseteq K\}$.

Definition 3.16. A QNVS A of a QNVTS (X, τ) is said to be quadripartitioned neutrosophic vague generalized closed set (QNVGCS in short) if QNV cl (A) ⊆ U whenever A ⊆ U and U is QNVOS in X.

Definition 3.17. A QNVS A of a QNVTS (X, τ) is said to be quadripartitioned neutrosophic vague generalized semi closed set (QNVGSCS in short) if QNV scl (A) ⊆ U whenever A ⊆ U and U is QNVOS in X.

Definition 3.18. A QNV A of a QNVTS (X, τ) is said to be quadripartitioned neutrosophic vague alpha generalized closed set (QNVαGCS in short) if QNVαcl (A) ⊆ U whenever A ⊆ U and U is QNVOS in X.

Definition 3.19. A QNVS A of a QNVTS (X, τ) is said to be quadripartitioned neutrosophic vague generalized semi-pre closed set (QNVGSPCS in short) if QNV spcl (A) ⊆ U whenever A ⊆ U and U is QNVOS in X.

Definition 3.20. Let (X, τ) be a QNVTS and $A_{QNV} = \left\{\left\langle x; \hat{T}_{A_{QNV}}; \hat{C}_{A_{QNV}}; \hat{U}_{A_{QNV}}; \hat{F}_{A_{QNV}} \right\rangle; x \in \mathrm{X}\right\}$

be a QNVS in X. The quadripartitioned neutrosophic vague pre interior of A and denoted by QNV pint (A) is defined to be the union of all quadripartitioned neutrosophic vague pre-open sets of X which are contained in A. The intersection of all quadripartitioned neutrosophic vague pre-closed sets containing A is called the quadripartitioned neutrosophic pre-closure of A and

is denoted by QNV pcl (A).

1) QNV pint (A) = $\cup\{G/G$ is a QNVPOS in X and $G \subseteq A\}$,
2) QNV pcl (A) = $\cap\,\{K/K$ is a QNVPCS in X and $A \subseteq K\}$.

Result 3.21. Let A be QNVS of a QNVTS (X, τ) , then

1) QNV pcl (A) = A ∪ QNV cl (QNV int (A)) ,
2) QNV pint (A) = A ∩ QNV int (QNV cl (A)).

4. Quadripartitioned Neutrosophic Vague Generalized Pre-closed Sets

In this section we introduce quadripartitioned neutrosophic vague generalized pre-closed set and their properties are analysed.

Definition 4.1. A QNVS A is said to be quadripartitioned neutrosophic vague generalized pre-closed set (QNVGPCS in short) in (X,τ) if QNVpcl (A) ⊆ U whenever A ⊆ U and U is QNVOS in X. The family of all QNVGPCSs of a QNVTS (X,) is denoted by QNVGPC (X).

Example 4.2. Let X = {a, b} and let $\tau = \{0_{QNV}, G, 1_{QNV}\}$ is QNVT on X, where

G=$\left\{x, \frac{a}{[0.6,0.8],[0.1,0.2],[0.2,0.4],[0.2,0.4]}, \frac{b}{[0.4,0.5],[0.2,0.6],[0.2,0.4],[0.5,0.6]}\right\}$.

Then the QNVS

A=$\left\{x, \frac{a}{[0.6,0.7],[0.5,0.8],[0.2,0.4],[0.3,0.4]}, \frac{b}{[0.2,0.5],[0.4,0.6],[0.4,0.6],[0.5,0.8]}\right\}$ is QNVGPCS in X.

Theorem 4.3. Every QNVCS is QNVGCS but not conversely.

Proof. Let A be QNVCS in X. Suppose U is QNVOS in X, such that A ⊆ U. Then QNV cl (A) = A ⊆ U. Hence A is QNVGCS in X.

Example 4.4. Let X = {a, b,c} and let $\tau = \{0_{QNV}, G, 1_{QNV}\}$ is QNVT on X, where

G=$\left\{x, \frac{a}{[0.2,0.3],[0.2,0.4],[0.5,0.7],[0.7,0.8]}, \frac{b}{[0.1,0.4],[0.1,0.2],[0.8,0.9],[0.6,0.9]}, \frac{c}{[0.1,0.2],[0.1,0.3],[0.6,0.7],[0.8,0.9]}\right\}$.

Then the QNVS

A=$\left\{x, \frac{a}{[0.3,0.5],[0.2,0.3],[0.6,0.9],[0.3,0.7]}, \frac{b}{[0.2,0.3],[0.6,0.7],[0.5,1],[0.7,0.8]}, \frac{c}{[0.1,0.6],[0.1,0.2],[0.7,0.8],[0.4,0.9]}\right\}$ is

QNVGCS in X but not QNVCS in X.

Theorem 4.5. Every QNVCS is QNV αCS but not conversely.

Proof. Let A be QNVCS in X. Since QNV int (A) ⊆ A, and QNV cl (A) = A, which implies QNV int (QNV cl (A)) ⊆ QNV cl (A), so QNV cl (QNV int (QNV cl (A))) ⊆ A. Hence A is QNVαCS in X.

Example 4.6. Let X = {a, b,c} and let $\tau = \{0_{QNV}, G_1, G_2, 1_{QNV}\}$ be QNVT on X , where
G$_1$=$\left\{x, \frac{a}{[0.2,0.5],[0.6,0.7],[0.6,0.9],[0.8,0.9]}, \frac{b}{[0.1,0.2],[0.5,0.7],[0.1,0.3],[0.8,0.9]}, \frac{c}{[0.3,0.4],[0.1,0.3],[0.5,0.7],[0.6,0.7]}\right\}$,
G$_2$=$\left\{x, \frac{a}{[0.6,0.8],[0.7,0.8],[0.2,0.3],[0.2,0.4]}, \frac{b}{[0.5,0.7],[0.5,0.7],[0.1,0.2],[0.3,0.5]}, \frac{c}{[0.7,0.9],[0.1,0.3],[0.4,0.6],[0.1,0.3]}\right\}$.

Then the QNVS A=$\left\{x, \frac{a}{[0.1,0.4],[0.2,0.8],[0.3,0.4],[0.6,0.9]}, \frac{b}{[0.7,0.9],[0.1,0.4],[0.3,0.4],[0.1,0.3]}, \frac{c}{[0.1,0.2],[0.1,0.3],[0.3,0.6],[0.8,0.9]}\right\}$ is QNVαCS in X but not QNVCS in X.

Theorem 4.7. Every QNVCS is QNVPCS but not conversely.

Proof. Suppose A is QNVCS in X. Since QNV int (A) ⊆ A, QNV cl (QNV int (A)) ⊆ NV cl (A) = A, which implies QNV cl (QNV int (A)) ⊆ A. Thus A is QNVPCS in X.

Example 4.8. Let X = {a, b} and let $\tau = \{0_{QNV}, G_1, G_2, 1_{QNV}\}$ be QNVT on X , where
G$_1$=$\left\{x, \frac{a}{[0.5,0.8],[0.6,0.7],[0.2,0.3],[0.2,0.5]}, \frac{b}{[0.6,0.9],[0.5,0.7],[0.3,0.7],[0.1,0.4]}\right\}$,
G$_2$=$\left\{x, \frac{a}{[0.4,0.7],[0.6,0.8],[0.8,0.9],[0.3,0.6]}, \frac{b}{[0.2,0.5],[0.5,0.7],[0.6,0.8],[0.5,0.8]}\right\}$.

Then the QNVS A=$\left\{x, \frac{a}{[0.2,0.6],[0.2,0.8],[0.7,0.9],[0.4,0.8]}, \frac{b}{[0.3,0.8],[0.1,0.4],[0.5,0.8],[0.2,0.7]}\right\}$ is QNVPCS in X but not QNVCS in X.

Theorem 4.9. Every QNV αCS is QNVPCS but not conversely.

Proof. Assume that A is QNVαCS in X. Then QNV cl (QNV int (QNV cl (A))) ⊆ A. Since A ⊆ QNV cl (A), which implies QNV cl (QNV int (A)) ⊆ A. Hence A is QNVPCS in X.

Example 4.10.

Let X = {a, b,c} and G$_1$=$\left\{x, \frac{a}{[0.7,0.9],[0.6,0.7],[0.3,0.5],[0.1,0.3]}, \frac{b}{[0.6,0.8],[0.5,0.7],[0.1,0.3],[0.2,0.4]}, \frac{c}{[0.8,1],[0.1,0.3],[0.2,0.6],[0,0.2]}\right\}$,
G$_2$=$\left\{x, \frac{a}{[0.5,0.7],[0.4,0.6],[0.4,0.8],[0.5,0.3]}, \frac{b}{[0.4,0.6],[0.4,0.7],[0.3,0.7],[0.4,0.6]}, \frac{c}{[0.2,0.4],[0,0.2],[0.8,0.9],[0.6,0.8]}\right\}$.

Then $\tau = \{0_{QNV}, G_1, G_2, 1_{QNV}\}$ is QNVT on X.

The QNVS A=$\left\{x, \frac{a}{[0.3,0.6],[0.2,0.8],[0.1,0.2],[0.4,0.7]}, \frac{b}{[0.2,0.8],[0.1,0.4],[0.7,0.9],[0.2,0.8]}, \frac{c}{[0.4,0.7],[0.2,0.3],[0.1,0.3],[0.3,0.6]}\right\}$ is QNVPCS in X but not QNVαCS in X.

Theorem 4.11. Every QNVRCS is QNVCS but not conversely.

Proof. Let A be QNVRCS in X. Then A = QNV cl (QNV int (A)) which implies QNV cl (A) = QNV cl (QNV int (A)). Therefore QNV cl (A) =A. Hence, A is QNVCS in X.

Example 4.12.

Let X = {a, b,c} and G=$\left\{x, \frac{a}{[0.4,0.8],[0.6,0.7],[0.8,0.9],[0.2,0.6]}, \frac{b}{[0.2,0.4],[0.5,0.7],[0.6,0.7],[0.6,0.8]}, \frac{c}{[0.1,0.3],[0.1,0.3],[0.6,0.9],[0.7,0.9]}\right\}$.
Then $\tau = \{0_{QNV}, G, 1_{QNV}\}$ is QNVT on X.

Then QNVS A=$\left\{x, \frac{a}{[0.2,0.6],[0.2,0.8],[0.1,0.2],[0.4,0.8]}, \frac{b}{[0.6,0.8],[0.1,0.4],[0.3,0.4],[0.2,0.4]}, \frac{c}{[0.7,0.9],[0.2,0.3],[0.1,0.4],[0.1,0.3]}\right\}$ is QNVCS in X but not QNVRCS in X.

Theorem 4.13. Every QNV αCS is QNVSCS but not conversely.

Proof. Let A be QNV αCS in X. Then QNV cl (QNV int (QNvcl (A))) ⊆ A. Since A ⊆ QNV cl (A), so QNV int (QNV cl (A)) ⊆ A. Hence, A is quadripartitioned neutrosophic vague semi closed set in X.

Example 4.14.

Let X = {a, b,c} and $G_1=\left\{x, \frac{a}{[0.3,0.6],[0.6,0.7],[0.8,0.9],[0.4,0.7]}, \frac{b}{[0.2,0.4],[0.5,0.7],[0.6,0.9],[0.6,0.8]}, \frac{c}{[0.1,0.5],[0.1,0.3],[0.7,0.8],[0.5,0.9]}\right\}$, $G_2=\left\{x, \frac{a}{[0.5,0.7],[0.8,0.9],[0.1,0.2],[0.3,0.5]}, \frac{b}{[0.7,0.9],[0.6,0.8],[0.2,0.5],[0.1,0.3]}, \frac{c}{[0.6,0.8],[0.3,0.4],[0.3,0.4],[0.2,0.4]}\right\}$.

Then $\tau = \{0_{QNV}, G_1, G_2, 1_{QNV}\}$ is QNVT on X.

The QNVS $A=\left\{x, \frac{a}{[0.4,0.7],[0.2,0.8],[0.8,0.9],[0.3,0.6]}, \frac{b}{[0.5,0.7],[0.1,0.4],[0.1,0.9],[0.3,0.5]}, \frac{c}{[0.4,0.9],[0.2,0.3],[0.7,0.8],[0.1,0.6]}\right\}$ is QNVSCS in X but not QNVαCS in X.

Theorem 4.15. Every QNVPCS is QNVSPCS but not conversely.

Proof. Let A be QNVPCS in X. By hypothesis QNV cl (QNV int (A)) ⊆ A. Therefore QNV int (QNV cl (QNV int (A))) ⊆QNV int (A) ⊆ A. Therefore QNV int (QNV cl (QNV int (A))) ⊆A. Hence A is QNVSPCS in X.

Example 4.16 Let X = {a, b} and $G_1=\left\{x, \frac{a}{[0.9,1],[0.6,0.7],[0.2,0.3],[0,0.1]}, \frac{b}{[0.7,0.9],[0.5,0.7],[0.4,0.5],[0.1,0.3]}\right\}$, $G_2=\left\{x, \frac{a}{[0.1,0.5],[0.4,0.6],[0.7,0.9],[0.5,0.9]}, \frac{b}{[0.4,0.6],[0.4,0.7],[0.8,0.9],[0.4,0.6]}\right\}$. Then $\tau = \{0_{QNV}, G_1, G_2, 1_{QNV}\}$ is QNVT on X.

The QNVS $A=\left\{x, \frac{a}{[0.7,0.8],[0.2,0.8],[0.5,0.6],[0.2,0.3]}, \frac{b}{[0.6,0.8],[0.1,0.4],[0.7,0.9],[0.2,0.4]}\right\}$ is QNVSPCS in X but not QNVPCS in X.

Theorem 4.17. Every QNVCS is QNVGPCS but not conversely.

Proof. Let A be QNVCS in X and let A ⊆ U and U be QNVOS in X. Since QNV pcl (A) ⊆Q NV cl (A) and A is QNVCS in X, QNV pcl (A) ⊆ QNV cl (A) = A ⊆ U. Therefore A is QNVGPCS in X.

Example 4.18. Let X = {a, b} and $G=\left\{x, \frac{a}{[0.4,0.7],[0.6,0.7],[0.6,0.8],[0.3,0.6]}, \frac{b}{[0.3,0.5],[0.5,0.7],[0.4,0.7],[0.5,0.7]}\right\}$.

Then $\tau = \{0_{QNV}, G, 1_{QNV}\}$ is QNVT on X.

The QNVS $A=\left\{x, \frac{a}{[0.2,0.6],[0.2,0.8],[0.4,0.7],[0.4,0.8]}, \frac{b}{[0.1,0.4],[0.1,0.4],[0.4,0.7],[0.6,0.9]}\right\}$ is QNVGPCS in X but not QNVCS in X.

Theorem 4.19. Every QNVGCS is QNVGPCS but not conversely.

Proof. Let A be QNVGCS in X and let A ⊆ U and U is QNVOS in (X,τ). Since QNV pcl (A) ⊆ QNV cl (A) and by hypothesis, QNV pcl (A) ⊆ U. Therefore A is QNVGPCS in X.

Example 4.20. Let X = {a, b} and $G_1=\left\{x, \frac{a}{[0.1,0.4],[0.5,0.6],[0.6,0.7],[0.6,0.9]}, \frac{b}{[0.2,0.5],[0.5,0.7],[0.7,0.9],[0.5,0.8]}\right\}$, $G_2=\left\{x, \frac{a}{[0.7,0.9],[0.6,0.7],[0.2,0.6],[0.1,0.3]}, \frac{b}{[0.8,0.9],[0.7,0.8],[0.4,0.5],[0.1,0.2]}\right\}$. Then $\tau = \{0_{QNV}, G_1, G_2, 1_{QNV}\}$ is QNVT on X.

The QNVS $A=\left\{x, \frac{a}{[0.8,0.9],[0.2,0.8],[0.2,0.3],[0.1,0.2]}, \frac{b}{[0.6,0.8],[0.1,0.4],[0.1,0.2],[0.2,0.4]}\right\}$ is QNVGPCS in X but not QNVGCS in X.

Theorem 4.21. Every QNV αCS is QNVGPCS but not conversely.

Proof. Let A be QNV αCS in X and let A ⊆ U and U be QNVOS in X. By hypothesis, QNV cl (QNV int (QNV cl (A))) ⊆A. Since A ⊆ QNV cl (A) ;QNV cl (QNV int (A)) ⊆ QNV cl (QNV int (QNV cl (A))) ⊆ A. Hence QNV pcl (A) ⊆ A ⊆ U. Therefore A is QNVGPCS in X.

Example 4.22. Let X = {a, b} and $G_1=\left\{x, \frac{a}{[0.6,0.9],[0.6,0.7],[0.3,0.5],[0.1,0.4]}, \frac{b}{[0.7,0.8],[0.5,0.7],[0.2,0.4],[0.2,0.3]}\right\}$, $G_2=\left\{x, \frac{a}{[0.1,0.3],[0.4,0.6],[0.5,0.8],[0.7,0.9]}, \frac{b}{[0.4,0.5],[0.4,0.7],[0.3,0.7],[0.5,0.6]}\right\}$. Then $\tau = \{0_{QNV}, G_1, G_2, 1_{QNV}\}$ is QNVT on X.

The QNVS A=$\left\{x, \frac{a}{[0.8,0.9],[0.2,0.8],[0.1,0.4],[0.1,0.2]}, \frac{b}{[0.7,0.9],[0.1,0.4],[0.2,0.3],[0.1,0.3]}\right\}$ is QNVPCS in X but not QNVαCS in X.

Theorem 4.23. Every QNVRCS is QNVGPCS but not conversely.

Proof. Let A be a QNVRCS in X. By Definition 3.10, A = QNV cl (QNV int (A)). This implies QNV cl (A) = QNV cl (QNV int (A)).Therefore QNV cl (A) = A. That is A is QNVCS in X. By Theorem 4.17, A is QNVGPCS in X.

Example 4.24. Let X = {a, b} and G$_1$=$\left\{x, \frac{a}{[0.1,0.4],[0.6,0.7],[0.5,0.7],[0.6,0.9]}, \frac{b}{[0.2,0.4],[0.5,0.7],[0.6,0.8],[0.6,0.8]}\right\}$, G$_2$=$\left\{x, \frac{a}{[0.6,0.9],[0.7,0.8],[0.5,0.8],[0.1,0.4]}, \frac{b}{[0.7,0.8],[0.6,0.7],[0.2,0.4],[0.2,0.3]}\right\}$. Then $\tau = \{0_{QNV}, G_1, G_2, 1_{QNV}\}$ is QNVT on X.

The QNVS A=$\left\{x, \frac{a}{[0.7,0.9],[0.2,0.8],[0.3,0.5],[0.1,0.3]}, \frac{b}{[0.6,0.9],[0.1,0.4],[0.2,0.4],[0.1,0.4]}\right\}$ is QNVGPCS in X but not QNVRCS in X.

Theorem 4.25. Every QNVPCS is QNVGPCS but not conversely.

Proof. Let A be QNVPCS in X and let A ⊆ U and U is QNVOS in X. By Definition 3.10,Q NV cl (QNV int (A)) ⊆ A. This implies QNV pcl (A) = A ∪ QNV cl (NV int (A)) ⊆ A. ThereforeQ NV pcl (A) ⊆ U. Hence A is QNVGPCS in X.

Example 4.26. Let X = {a, b,c} and

G=$\left\{x, \frac{a}{[0.5,0.7],[0.6,0.7],[0.3,0.6],[0.3,0.5]}, \frac{b}{[0.4,0.8],[0.5,0.7],[0.2,0.5],[0.2,0.6]}, \frac{c}{[0.2,0.6],[0.1,0.3],[0.4,0.5],[0.4,0.8]}\right\}$.

Then $\tau = \{0_{QNV}, G, 1_{QNV}\}$ is QNVT on X.

The QNVS A=$\left\{x, \frac{a}{[0.6,0.7],[0.2,0.8],[0.2,0.5],[0.3,0.4]}, \frac{b}{[0.7,0.8],[0.7,0.9],[0.1,0.4],[0.2,0.3]}, \frac{c}{[0.5,0.8],[0.2,0.3],[0.2,0.4],[0.2,0.5]}\right\}$ is QNVGPCS in X but not QNVPCS in X.

Theorem 4.27. Every QNVαGCS is QNVGPCS but not conversely.

Proof. Let A be QNVαGCS in X and let A ⊆ U and U is QNVOS in (X, τ). By Result 3.14, A∪ QNV cl (QNV int (QNV cl (A))) ⊆ U. This implies QNV cl (QNV int (QNV cl (A))) ⊆U and QNV cl (QNV int (A)) ⊆ U. Thus QNV pcl (A) = A ∪ QNV cl (QNV int (A)) ⊆ U. Hence A is QNVGPCS in X.

Example 4.28. Let X = {a, b} and G$_1$=$\left\{x, \frac{a}{[0.6,0.8],[0.6,0.7],[0.1,0.2],[0.2,0.4]}, \frac{b}{[0.7,0.9],[0.5,0.7],[0.2,0.5],[0.1,0.3]}\right\}$, G$_2$=$\left\{x, \frac{a}{[0.2,0.4],[0.4,0.6],[0.5,0.6],[0.6,0.8]}, \frac{b}{[0.3,0.6],[0.4,0.7],[0.7,0.8],[0.4,0.7]}\right\}$. Then $\tau = \{0_{QNV}, G_1, G_2, 1_{QNV}\}$ is QNVT on X.

The QNVS A=$\left\{x, \frac{a}{[0.8,0.9],[0.2,0.8],[0.2,0.7],[0.1,0.2]}, \frac{b}{[0.5,0.7],[0.1,0.4],[0.3,0.4],[0.3,0.5]}\right\}$ is QNVGPCS in X but not QNVαCS in X.

Theorem 4.29. Every QNVGPCS is QNVSPCS but not conversely.

Proof. Let A be QNVGPCS in X , this implies QNV pcl (A) ⊆ U whenever A ⊆ U and U is QNVOS in X. By hypothesis QNV cl (QNV int (A)) ⊆ A. Therefore QNV int (QNV cl (QNV int (A))) ⊆ QNV int (A) ⊆ A. Therefore QNV int (QNV cl (QNV int (A))) ⊆ A. Hence A is QNVSPCS in X.

Example 4.30. Let X = {a, b, c} and

G=$\left\{x, \frac{a}{[0.1,0.3],[0.3,0.5],[0.4,0.6],[0.7,0.9]}, \frac{b}{[0.2,0.4],[0.5,0.7],[0.7,0.9],[0.6,0.8]}, \frac{c}{[0.3,0.4],[0.1,0.3],[0.8,0.9],[0.6,0.7]}\right\}$.

Then $\tau = \{0_{QNV}, G, 1_{QNV}\}$ is QNVT on X.

The QNVS A= G is QNVSPCS in X but not QNVGPCS in X.

Theorem 4.31. Every QNVGPCS is QNVGSPCS but not conversely.

Proof. Let A be QNVGPCS in X and let A ⊆ U and U is QNVOS in X. By hypothesis QNV cl (QNV int (A)) ⊆ A ⊆ U.Therefore QNV int (QNV cl (QNV int (A))) ⊆QNV int (U) ⊆ U. This implies QNV spcl (A) ⊆ U whenever A ⊆ U and U is QNVOS in X. Therefore A is QNVGSPCS in X.

Example 4.32. Let X = {a, b,c} and

$G=\left\{x, \frac{a}{[0.2,0.4],[0.6,0.7],[0.6,0.8],[0.6,0.8]}, \frac{b}{[0.1,0.3],[0.5,0.7],[0.1,0.3],[0.7,0.9]}, \frac{c}{[0.4,0.5],[0.1,0.3],[0.2,0.6],[0.5,0.6]}\right\}$.

Then $\tau = \{0_{QNV}, G, 1_{QNV}\}$ is QNVT on X.

The QNVS A= G is QNVGSPCS in X but not QNVGPCS in X.

Proposition 4.33. QNVSCS and QNVGPCS are independent to each other.

Example 4.34. Let X = {a, b, c} and

$G=\left\{x, \frac{a}{[0.2,0.6],[0.6,0.7],[0.7,0.8],[0.4,0.8]}, \frac{b}{[0.1,0.4],[0.5,0.7],[0.5,0.6],[0.6,0.9]}, \frac{c}{[0.3,0.5],[0.1,0.3],[0.7,0.8],[0.5,0.7]}\right\}$.

Then $\tau = \{0_{QNV}, G, 1_{QNV}\}$ is QNVT on X.

The QNVS A= G is QNVSCS in X but not QNVGPCS in X.

Example 4.35. Let X = {a, b, c} and

$G=\left\{x, \frac{a}{[0.4,0.7],[0.6,0.7],[0.1,0.2],[0.3,0.6]}, \frac{b}{[0.5,0.6],[0.5,0.7],[0.2,0.6],[0.4,0.5]}, \frac{c}{[0.6,0.8],[0.1,0.3],[0.4,0.5],[0.2,0.4]}\right\}$.

Then $\tau = \{0_{QNV}, G, 1_{QNV}\}$ is QNVT on X.

The QNVS $A=\left\{x, \frac{a}{[0.2,0.5],[0.2,0.8],[0.6,0.7],[0.5,0.8]}, \frac{b}{[0.3,0.4],[0.1,0.4],[0.4,0.6],[0.6,0.7]}, \frac{c}{[0.5,0.7],[0.2,0.3],[0.6,0.7],[0.3,0.5]}\right\}$ is QNVGPCS in X but not QNVSCS in X.

Proposition 4.36. QNVGSCS and QNVGPCS are independent to each other.

Example 4.37. Let X = {a, b,c} and

$G=\left\{x, \frac{a}{[0.4,0.5],[0.6,0.7],[0.1,0.2],[0.5,0.6]}, \frac{b}{[0.3,0.6],[0.5,0.7],[0.2,0.7],[0.4,0.7]}, \frac{c}{[0.2,0.7],[0.1,0.3],[0.2,0.6],[0.3,0.8]}\right\}$.

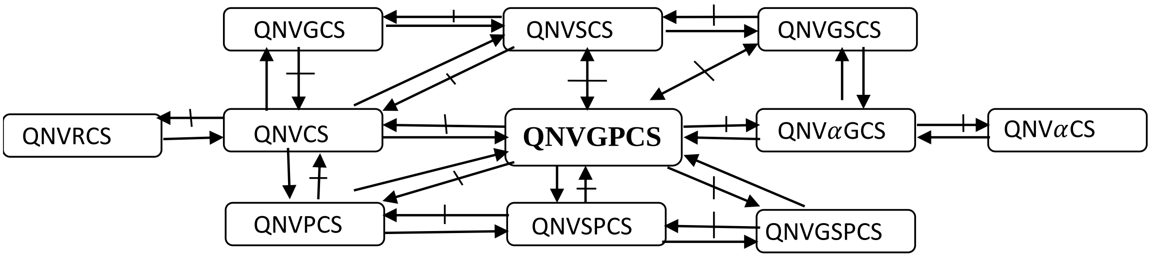
Then $\tau = \{0_{QNV}, G, 1_{QNV}\}$ is QNVT on X.

The QNVS A= G is QNVGSCS in X but not QNVGPCS in X.

Example 4.38. Let X = {a, b} and $G_1=\left\{x, \frac{a}{[0.3,0.5],[0.6,0.7],[0.7,0.8],[0.5,0.7]}, \frac{b}{[0.2,0.4],[0.5,0.7],[0.6,0.7],[0.6,0.8]}\right\}$, $G_2=\left\{x, \frac{a}{[0.7,0.8],[0.7,0.9],[0.1,0.4],[0.2,0.3]}, \frac{b}{[0.8,0.9],[0.6,0.8],[0.2,0.5],[0.1,0.2]}\right\}$. Then $\tau = \{0_{QNV}, G_1, G_2, 1_{QNV}\}$ is QNVT on X.

The QNVS $A=\left\{x, \frac{a}{[0.2,0.5],[0.2,0.8],[0.8,0.9],[0.5,0.8]}, \frac{b}{[0.1,0.3],[0.1,0.4],[0.7,0.8],[0.7,0.9]}\right\}$ is QNVGPCS in X but not QNVGSCS in X.

Remark 4.39. We have the following implications by summing up the above theorems.



In this diagram by A → B we mean A implies B but not conversely and A ↮ B means A and B are independent of each other. None of them is reversible.

Remark 4.40. The union of any two QNVGPCSs is not QNVGPCS in general as seen in the following example.

Example 4.41. Let X = {a, b,c} and

G=$\left\{x, \frac{a}{[0.4,0.7],[0.6,0.7],[0.2,0.3],[0.3,0.6]}, \frac{b}{[0.5,0.6],[0.5,0.7],[0.3,0.4],[0.4,0.5]}, \frac{c}{[0.6,0.7],[0.1,0.3],[0.3,0.6],[0.3,0.4]}\right\}$.

Then $\tau = \{0_{QNV}, G, 1_{QNV}\}$ is QNVT on X and the QNVSs

A=$\left\{x, \frac{a}{[0.6,0.9],[0.2,0.8],[0.6,0.7],[0.1,0.4]}, \frac{b}{[0.3,0.4],[0.1,0.4],[0.5,0.6],[0.6,0.7]}, \frac{c}{[0.4,0.5],[0.2,0.3],[0.6,0.7],[0.5,0.6]}\right\}$,
B=$\left\{x, \frac{a}{[0.7,0.8],[0.2,0.8],[0.1,0.2],[0.2,0.3]}, \frac{b}{[0.6,0.8],[0.1,0.4],[0.3,0.4],[0.2,0.4]}, \frac{c}{[0.6,0.9],[0.2,0.3],[0.1,0.3],[0.1,0.4]}\right\}$ are QNVGPCSs in X but A∪B is not QNVGPCS in X.

5. Quadripartitioned Neutrosophic Vague Generalized Pre-open Set

In this section we introduce quadripartitioned neutrosophic vague generalized pre-open set and their properties are deliberated.

Definition 5.1. A QNVS A is said to be quadripartitioned neutrosophic vague generalized pre-open set (QNVGPOS in short) in (X, τ) if the complement A^c is QNVGPCS in (X, τ). The family of all QNVGPOSs of QNVTS (X, τ) is denoted by QNVGPO (X).

Example 5.2. Let X = {a, b,c} and let $\tau = \{0_{QNV}, G, 1_{QNV}\}$ is QNVT on X, where

G=$\left\{x, \frac{a}{[0.6,0.8],[0.1,0.2],[0.2,0.4],[0.2,0.4]}, \frac{b}{[0.7,0.8],[0.1,0.2],[0.2,0.4],[0.2,0.3]}, \frac{c}{[0.6,0.9],[0.1,0.3],[0.3,0.4],[0.1,0.4]}\right\}$.

Then the QNVS

A=$\left\{x, \frac{a}{[0.3,0.5],[0.2,0.3],[0.2,0.4],[0.5,0.7]}, \frac{b}{[0.6,0.7],[0.2,0.5],[0.4,0.6],[0.3,0.4]}, \frac{c}{[0.6,0.8],[0.1,0.2],[0.2,0.4],[0.2,0.4]}\right\}$ is QNVGPOS in X.

Theorem 5.3. For any QNVTS (X, τ) , we have the following results.

(1). Every QNVOS is QNVGPOS but not conversely.

(2). Every QNVROS is QNVGPOS but not conversely.

(3). Every QNV α OS is QNVGPOS but not conversely.

(4). Every QNVPOS is QNVGPOS but not conversely.

The converse of the above theorem need not be true which can be seen from the following examples.

Example 5.4. Let X = {a, b} and G₁=$\left\{x, \frac{a}{[0.2,0.5],[0.6,0.7],[0.2,0.4],[0.5,0.8]}, \frac{b}{[0.3,0.6],[0.5,0.7],[0.3,0.4],[0.4,0.7]}\right\}$, G_1=$\left\{x, \frac{a}{[0.6,0.7],[0.7,0.8],[0.1,0.2],[0.3,0.4]}, \frac{b}{[0.8,0.9],[0.5,0.7],[0.3,0.4],[0.1,0.2]}\right\}$. Then $\tau = \{0_{QNV}, G_1, G_2, 1_{QNV}\}$ is QNVT on X.

The QNVS A=$\left\{x, \frac{a}{[0.4,0.7],[0.2,0.8],[0.5,0.6],[0.3,0.6]}, \frac{b}{[0.2,0.5],[0.1,0.4],[0.5,0.8],[0.5,0.8]}\right\}$ is QNVGPOS in X but not QNVOS in X.

Example 5.5. Let X = {a, b} and G_1=$\left\{x, \frac{a}{[0.7,0.9],[0.2,0.3],[0.1,0.2],[0.1,0.3]}, \frac{b}{[0.8,0.9],[0.5,0.7],[0.2,0.5],[0.1,0.2]}\right\}$, G_1=$\left\{x, \frac{a}{[0.2,0.3],[0.1,0.2],[0.5,0.6],[0.7,0.8]}, \frac{b}{[0.4,0.6],[0.4,0.6],[0.3,0.6],[0.4,0.6]}\right\}$. Then $\tau = \{0_{QNV}, G_1, G_2, 1_{QNV}\}$ is QNVT on X.

The QNVS A=$\left\{x, \frac{a}{[0.5,0.6],[0.4,0.7],[0.5,0.6],[0.4,0.5]}, \frac{b}{[0.2,0.5],[0.1,0.4],[0.5,0.8],[0.5,0.8]}\right\}$ is QNVPSOS in X but not QNVROS in X.

Example 5.6.

Let X = {a, b,c} and G_1=$\left\{x, \frac{a}{[0.2,0.4],[0.7,0.8],[0.3,0.4],[0.6,0.8]}, \frac{b}{[0.3,0.5],[0.1,0.3],[0.6,0.9],[0.5,0.7]}, \frac{c}{[0.4,0.5],[0.7,0.9],[0.6,0.7],[0.5,0.6]}\right\}$., G_1=$\left\{x, \frac{a}{[0.6,0.8],[0.8,0.9],[0.2,0.4],[0.2,0.4]}, \frac{b}{[0.7,0.9],[0.2,0.4],[0.5,0.6],[0.1,0.3]}, \frac{c}{[0.7,0.8],[0.8,0.9],[0.5,0.6],[0.2,0.3]}\right\}$.

Then $\tau = \{0_{QNV}, G_1, G_2, 1_{QNV}\}$ is QNVT on X.

The QNVS A=$\left\{x, \frac{a}{[0.4,0.5],[0.1,0.2],[0.5,0.6],[0.5,0.6]}, \frac{b}{[0.3,0.6],[0.5,0.6],[0.5,0.8],[0.4,0.7]}, \frac{c}{[0.4,0.5],[0.2,0.3],[0.3,0.4],[0.5,0.6]}\right\}$ is QNVGPOS in X but not NVα OS in X.

Example 5.7.

Let X = {a, b,c} and G=$\left\{x, \frac{a}{[0.7,0.8],[0.1,0.2],[0.2,0.5],[0.2,0.3]}, \frac{b}{[0.6,0.8],[0.1,0.4],[0.2,0.4],[0.2,0.4]}, \frac{c}{[0.7,0.9],[0.2,0.3],[0.3,0.4],[0.1,0.3]}\right\}$. Then $\tau = \{0_{QNV}, G, 1_{QNV}\}$ is QNVT on X.

The QNVS A=$\left\{x, \frac{a}{[0.1,0.2],[0.6,0.8],[0.5,0.6],[0.8,0.9]}, \frac{b}{[0.2,0.3],[0.8,0.9],[0.5,0.8],[0.7,0.8]}, \frac{c}{[0.1,0.2],[0.7,0.9],[0.3,0.4],[0.8,0.9]}\right\}$ is QNVGPOS in X but not QNVPOS in X.

Remark 5.8. The intersection of any two QNVGPOSs is not QNVGPOS in general and it is shown in the following example.

Example 5.9.

Let X = {a, b,c} and G=$\left\{x, \frac{a}{[0.3,0.5],[0.6,0.7],[0.2,0.4],[0.5,0.7]}, \frac{b}{[0.4,0.5],[0.6,0.8],[0.2,0.4],[0.2,0.7]}, \frac{c}{[0.2,0.4],[0.7,0.9],[0.3,0.4],[0.6,0.8]}\right\}$. Then $\tau = \{0_{QNV}, G, 1_{QNV}\}$ is QNVT on X.

Let A=$\left\{x, \frac{a}{[0.4,0.6],[0.5,0.7],[0.2,0.4],[0.4,0.6]}, \frac{b}{[0.4,0.7],[0.1,0.4],[0.4,0.6],[0.3,0.6]}, \frac{c}{[0.5,0.9],[0.7,0.9],[0.3,0.4],[0.1,0.5]}\right\}$,

B = $\left\{x, \frac{a}{[0.4,0.7],[0.4,0.8],[0.2,0.4],[0.3,0.6]}, \frac{b}{[0.3,0.5],[0.4,0.5],[0.4,0.6],[0.5,0.7]}, \frac{c}{[0.2,0.6],[0.7,0.9],[0.3,0.4],[0.4,0.8]}\right\}$ are QNVGPOSs in X but A ∩ B is not QNVGPOS in X.

Theorem 5.10. Let (X,) be QNVTS. If A ∈ QNVGPO(X) then V ⊆ QNV int (QNV cl (A)) whenever V ⊆ A and V is QNVCS in X.

Proof. Let A ∈ QNVGPO(X). Then A^c is QNVGPCS in X. Therefore QNV pcl (A^c) ⊆ U whenever A^c ⊆ U and U is QNVOS in X. That is QNV cl (QNV int (A^c)) ⊆ U. This implies U^c ⊆ QNV int (QNV cl (A)) whenever U^c ⊆A and U^c is QNVCS in X. Replacing U^c by V, we get V ⊆ NV int (QNV cl (A)) whenever V ⊆ A and V is QNVCS in X.

Theorem 5.11. Let (X, τ) be QNVTS. Then for every A ∈ QNVGPO(X) and for every B ∈ QNVS(X), NV pint (A) ⊆ B ⊆ A implies B ∈ QNVGPO(X).

Proof. By hypothesis $A^c \subseteq B^c \subseteq$ (QNV pint (A))^c. Let $B^c \subseteq$ U and U be QNVOS. Since $A^c \subseteq B^c$, $A^c \subseteq$ U. But A^c is QNVGPCS, QNV pcl (A^c) ⊆ U. Also $B^c \subseteq$ (QNV pint (A))^c = QNV pcl (A^c). Therefore QNV pcl (B^c) ⊆ QNV pcl (A^c) ⊆ U. Hence B^c is QNVGPCS. Which implies B is QNVGPOS of X.

Theorem 5.12. A QNVS A of QNVTS (X, τ) is QNVGPOS if and only if F ⊆ QNV pint (A) whenever F is QNVCS and F ⊆ A.

Proof. Necessity: Suppose A is QNVGPOS in X. Let F be QNVCS and F ⊆ A. Then F^c is QNVOS in X such that $A^c \subseteq F^c$. Since A^c is QNVGPCS, we have QNV pcl (A^c) ⊆ F^c. Hence (QNV pint (A))^c ⊆ Fc. Therefore F ⊆ QNV pint (A).

Sufficiency: Let A be QNVS of X and let F ⊆ QNV pint (A) whenever F is QNVCS and F ⊆ A. Then $A^c \subseteq F^c$ and F^c is QNVOS. By hypothesis, (QNV pint (A))^c ⊆ F^c. Which implies QNV pcl (A^c) ⊆ F^c. Therefore A^c is QNVGPCS of X. Hence A is QNVGPOS of X.

Corollary 5.13. A QNVS A of a QNVTS (X, τ) is QNVGPOS if and only if F ⊆ QNV int (QNV cl (A)) whenever F is QNVCS and F ⊆ A.

Proof. Necessity: Suppose A is QNVGPOS in X. Let F be QNVCS and F ⊆ A. Then F^c is QNVOS in X such that $A^c \subseteq F^c$. Since A^c is QNVGPCS, we have QNV pcl (A^c) ⊆ Fc. Therefore QNV cl (QNV int (A^c)) ⊆ F^c. Hence (QNV int (QNV cl (A)))^c ⊆ F^c. This implies F ⊆ QNV int (QNV cl (A)).

Sufficiency: Let A be QNVS of X and let F ⊆ QNV int (QNV cl (A)) whenever F is QNVCS and F ⊆ A. Then $A^c \subseteq F^c$ and F^c is QNVOS. By hypothesis, (QNV int (QNV cl (A)))^c ⊆ F^c. Hence QNV cl (QNV int (A^c)) ⊆ F^c, which implies QNV pcl (A^c) ⊆ F^c. Hence A is QNVGPOS of X.

Theorem 5.14. For a QNVS A, A is QNVOS and QNVGPCS in X if and only if A is QNVROS in X.

Proof. Necessity: Let A be QNVOS and QNVGPCS in X. Then QNV pcl (A) ⊆ A. This implies QNV cl (QNV int (A)) ⊆ A. Since A is QNVOS, it is QNVPOS. Hence A ⊆ QNV int (QNV cl (A)). Therefore A = QNV int (QNV cl (A)). Hence A is QNVROS in X.

Sufficiency: Let A be QNVROS in X. Therefore A = QNV int (QNV cl (A)). Let A ⊆ U and U is QNVOS in X. This implies QNV pcl (A) ⊆ A. Hence A is QNVGPCS in X.

6. Applications of Quadripartitioned Neutrosophic Vague Generalized Pre-closed Sets

In this section we provide some applications of quadripartitioned neutrosophic vague generalized pre-closed sets.

Definition 6.1. A QNVTS (X, τ) is said to be quadripartitioned neutrosophic vague $T_{1/2}$ space (QNV $T_{1/2}$ in short) if every NVGCS in X is NVCS in X.

Definition 6.2. A QNVTS (X, τ) is said to be quadripartitioned neutrosophic vague p $T_{1/2}$ space (QNVp $T_{1/2}$in short) if every QNVPCS in X is NVCS in X.

Definition 6.3. A QNVTS (X, τ) is said to be quadripartitioned neutrosophic vague gp$T_{1/2}$ space (QNVgp $T_{1/2}$ in short) if every QNVGPCS in X is QNVCS in X.

Definition 6.4. A QNVTS (X, τ) is said to be a quadripartitioned neutrosophic vague gpT_p space (QNV gpT_p in short) if every QNVGPCS in X is QNVPCS in X.

Theorem 6.5. Every QNV $T_{1/2}$ space is QNV gpT_p space. But the converse is not true in general.

Proof. Let X be QNV $T_{1/2}$ space and let A be QNVGCS in X, we know that every QNVGCS is QNVGPCS, hence A is QNVGPCS in X. By hypothesis A is QNVCS in X. Since every QNVCS is QNVPCS, A is QNVPCS in X. Hence X is QNV gpT_p space.

Example 6.6. Let X = {a, b} and G=$\left\{x, \frac{a}{[0.6,0.7],[0.1,0.2],[0.1,0.3],[0.3,0.4]}, \frac{b}{[0.8,0.9],[0.1,0.2],[0.2,0.4],[0.1,0.2]}\right\}$. Then $\tau = \{0_{QNV}, G, 1_{QNV}\}$ is QNVT on X.

Let A=$\left\{x, \frac{a}{[0.7,0.8],[0.2,0.3],[0.2,0.4],[0.2,0.3]}, \frac{b}{[0.3,0.5],[0.6,0.7],[0.4,0.6],[0.5,0.7]}\right\}$. Then (X,$\tau$) is QNV gp$T_p$ space. But it is not NV $T_{1/2}$ space since A is QNVGCS but not QNVCS in X.

Theorem 6.7. Every QNV gp$T_{1/2}$ space is QNV gpT_p space. But the converse is not true in general.

Proof. Let X be QNV gp$T_{1/2}$ space and let A be QNVGPCS in X. By hypothesis A is QNVCS in X. Since every QNVCS is QNVPCS, A is QNVPCS in X. Hence X is QNV gpT_p space.

Example 6.8. Let X = {a, b} and G=$\left\{x, \frac{a}{[0.5,0.7],[0.1,0.2],[0.2,0.4],[0.3,0.5]}, \frac{b}{[0.3,0.8],[0.1,0.3],[0.2,0.4],[0.2,0.7]}\right\}$. Then $\tau = \{0_{QNV}, G, 1_{QNV}\}$ is QNVT on X.

Let A=$\left\{x, \frac{a}{[0.4,0.8],[0.2,0.8],[0.2,0.4],[0.2,0.6]}, \frac{b}{[0.2,0.5],[0.1,0.4],[0.4,0.6],[0.5,0.8]}\right\}$. Then (X,$\tau$) is QNV gp$T_{1/2}$ space. But it is not QNV gpT_p space since A is QNVGPCS but not QNVCS in X.

Theorem 6.9. Let (X,τ) be NVTS and X is NV gp$T_{1/2}$ space then,

(1). Any union of QNVGPCSs is QNVGPCS

(2). Any intersection of QNVGPOSs is QNVGPOS.

Proof.

(1). Let $\{A_i\}_{i\in J}$ is a collection of QNVGPCSs in QNV gp$T_{1/2}$ space (X,τ). (Therefore every QNVGPCS is QNVCS. But the union of QNVCS is QNVCS. Hence the union of QNVGPCS is QNVGPCS in X.

(2). It can be proved by taking complement of (1).

Theorem 6.10. A QNVTS X is QNV gp$T_{1/2}$ space if and only if QNVGPO(X) = QNVPO(X).

Proof. Necessity: Let A be QNVGPOS in X, then A^c is QNVGPCS in X. By hypothesis A^c is QNVGPCS in X. Therefore A is QNVPOS in X. Hence QNVGPO(X) = QNVPO(X).

Sufficiency: Let A be QNVGPCS in X. Then A^c is QNVGPOS in X. By hypothesis A^c is QNVGPOS in X. Therefore A is QNVPCS in X. Hence X is QNV gpT_p space.

References:

[1] D.Andrijevic, Semi-pre open sets, Mat. Vesnik, 38(1986), 24-32.
[2] S.P.Arya and T.M.Nour, Characterizations of s-normal spaces, Indian J Pure Appl. Math., 21(1990), 717-719.
[3] Arokiarani, I., Dhavaseelan, R., Jafari, S., & Parimala, M. (2017). On some new notations and functions in neutrosophic topological spaces. Neutrosophic Sets and Systems, 16, 16-19.
[4] Atanassov, K. (1986). Intuitionistic fuzzy sets. Fuzzy Sets and Systems, 20, 87-96.
[5] Chatterjee, R., Majumdar, P., & Samanta, S.K. (2016). On some similarity measures and entropy on quadripartitioned single valued neutrosophic sets. Journal of Intelligent and Fuzzy Systems, $30(4), 2475 - 2485$
[6] Das, S., & Pramanik, S. (2020). Generalized neutrosophic b-open sets in neutrosophic topological space. Neutrosophic Sets and Systems, 35, 522-530.
[7] J.Dontchev, On generalizing semi-pre open sets, Mem. Fac. Sci. Kochi Univ. Ser. A. Math., 16(1995), 35-48.

[8] Ebenanjar, E., Immaculate, J., & Wilfred, C.B. (2018). On neutrosophic b-open sets in neutrosophic topological space. Journal of Physics Conference Series, 1139(1),012062.
[9] Gau, W.L.; Buehrer, D.J. Vague sets. IEEE Trans. Systems Man and Cybernet, 1993; 23(2), pp. 610–614.
[10] Iswarya, P., & Bageerathi, K. (2016). On neutrosophic semi-open sets in neutrosophic topological spaces. International Journal of Mathematical Trends and Technology, 37 (3), 214-223.
[11] N.Levine, Generalized closed sets in topological spaces, Rend. Circ. Mat. Palermo, 19(1970), 89-96.
[12] N.Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 70(1963), 36-41.
[13] H.Maki, K.Balachandran and R.Devi, Associated topologies of generalized -closed sets and -generalized closed sets, Mem. Fac. Sci. Kochi Univ. Ser. A Math., 15(1994), 51-63.
[14] H.Maki, J.Umehara and T.Noiri, Every topological space is pre-T1=2, Mem. Fac. Sci.Kochi. Univ. Ser. A. Mah., 17(1996), 33-42.
[15] A.S.Mashour, M.E.Abd. EI- Monsef and S.N.EI-Deeb, On pre-continuous mappings and weak pre-continuous mappings, Proc. Math. Phys. Soc. Egypt., 53(1982), 47-53.
[16] T. Madhumathi and F.Nirmala Irudayam. Neutrosophic pre-open set in simple extended neutrosophic topology. International Journal of Research in Advent Technology, Vol.7, No.4, April 2019,383-388.
[17] Maheswari, C., Sathyabama, M., & Chandrasekar, S. (2018). Neutrosophic generalized b-closed sets in neutrosophic topological spaces. Journal of Physics Conference Series, 1139 (1), 012065.
[18] A.Mary Margaret and M.Trinita Pricilla, neutrosophic vague generalized pre-closed sets in neutrosophic vague topological spaces, International Journal of Mathematics And its Applications,Volume 5, Issue 4-E (2017), 747-759.
[19] Mohammed Ali Jaffer, I., & Ramesh, K. (2019). Neutrosophic generalized pre regular closed sets. Neutrosophic Sets and System, 30, 171-181.
[20] Pushpalatha, A., & Nandhini, T. (2019). Generalized closed sets via neutrosophic topological spaces. Malaya Journal of Matematik, $7(1),50-54$.
[21] O.Njastad, On some classes of nearly open sets, Paci_c J Math., 15(1965), 961-870.
[22] Ramesh, K. (2020). Ngpr homomorphism in neutrosophic topological spaces. Neutrosophic Sets and Systems, 32, 25-37.
[23] Rao, V.V., & Srinivasa, R. (2017). Neutrosophic pre-open sets and pre-closed sets in neutrosophic topology. International Journal of Chem Tech Research, $10(10),449-458$.
[24] Salama, A.A., & Alblowi, S.A. (2012). Neutrosophic set and neutrosophic topological space. ISOR Journal of Mathematics, 3 (4), 31-35.
[25] Salama, A.A., & Alblowi, S.A. (2012). Generalized neutrosophic set and generalized neutrosophic topological space. Computer Science and Engineering, 2 (7), 129-132.
[26] Shawkat Alkhazaleh. Neutrosophic vague set theory. Critical Review. 2015; 10, pp. 29–39.
[27] Smarandache, F. (2005). Neutrosophic set: a generalization of the intuitionistic fuzzy sets. International Journal of Pure and Applied Mathematics, 24, 287-297.
[28] Suman Das, Rakhal Das, and Carlos Granados. (2021).Topology on Quadripartitioned Neutrosophic Sets. Neutrosophic Sets and Systems, Vol. 45, 54-61.
[29] M.Stone, Application of the theory of Boolean rings to general topology, Trans. Amer. Math. Soc., 41(1937), 374-481.
[30] Zadeh, L. A. (1965). Fuzzy sets. Information and Control, 8 (3), 338-353.