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Somewhat G_F-Continuous Maps

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Abstract: Topologists have introduced and investigated many different generalizations of continuous maps in fuzzy topology which proved to be very useful for the development of modern technology. The field of the mathematical science which goes under the name of topology especially in fuzzy topology is concerned with all questions directly or indirectly related to continuity. In this paper we defined and characterized the concept of somewhat $G_{\mathcal{F}}$ − continuous maps and obtained some significant results in this context with help of various supporting examples.

Keywords: Fuzzy open set, fuzzy topological space, $G_{\mathcal{F}}$ − topological space

1. Introduction

Fuzzy topology has found applications in upcoming fields such as digital topology and image processing. It was in the year 1968 that C. L. Chang [5] grafted the notion of a fuzzy set into general topology and for such spaces he attempted to develop the basic topological concepts like open set, closed set, neighborhood, interior of a set, continuity, compactness etc. Azad [1] has introduced the concept of fuzzy semi-open sets in fuzzy topological spaces. Beceren [2] introduced and studied the concept of strongly α-continuous functions, strong semi-continuity and fuzzy pre-continuity and investigate various characterizations. Further the author verified that fuzzy strongly α-continuous map is the stronger form of fuzzy α-continuous map. Csaszar [6] introduced the notions of generalized topological spaces. He also introduced the notions of continuous functions and associated interior and closure operators on generalized neighbourhood systems and generalized topological spaces. In 1963, Norman Levine [10] has presented the thought of semi-open sets and semi-continuous maps in topological spaces. Mashhour et. al. [8-9] introduced and studied pre-open sets, α-open sets, semi-open sets, semi-pre-open sets, simply open sets, regular semi open sets and locally closed sets respectively. Palaniappan and Rao [13] introduced and investigated regular generalized open sets, generalized pre-open sets. In 1974, Arya and Gupta [16] introduced the concept of complete continuity which is an intermediate between strong continuity and continuity. Palani Cheety [11] introduced the concept of generalized fuzzy topology and investigates various properties. In this paper, we have introduced the concept of somewhat

$G_{\mathcal{F}}$ −continuous maps and verify the results with the help of some counter examples.

2. Organization

Some required basic definitions, concepts of $G_{\mathcal{F}}$ − topological space and notations are discussed in Section 3. In section 4, we have introduced and studied the concept of somewhat $G_{\mathcal{F}}$ − continuous map. Finally, Section 5 concludes the paper.

3. Preliminaries

Definition 3.1: Let X be a non-empty universal crisp set. A **fuzzy topology** on X is a non-empty collection τ of fuzzy sets on X satisfying the following conditions

i) Fuzzy sets 0 and 1 belong to τ
ii) Any arbitrary union of members of τ is in τ
iii) A finite intersection of members of τ is in τ

Here 0 and 1 represent the Zero Fuzzy Set and the Whole Fuzzy set on X, defined as, $0(x)=0, \forall x \in X$ $1(x)=1, \forall x \in X$ and the pair (X, τ) is called Fuzzy Topological Space on X. For Convenience, we shall denote the fuzzy topological space simply as X.

Definition 3.2: Let X be a crisp set and let "$\mathcal{T}$" be a collection of fuzzy sets on X. Then "$\mathcal{T}$" is called $G_{\mathcal{F}} -$ topologyon X if it satisfies following conditions

i) The fuzzy sets 0 and 1 are in $'\mathcal{T}'$ where $0,1: X \to I$ are defined as $0(x) = 0$ and $1(x) = 1$ for all $x \in X$
ii) If $\{\lambda_j\}$, $j \in J$ is any family of fuzzy sets on X where $\lambda_j \in \mathcal{T}$ then $\cup_{j\in J} \lambda_j \in \mathcal{T}$

The pair $(X, \mathcal{T})$ is called $G_{\mathcal{F}} -$ topological space

Definition 3.3: Let $(X, \mathcal{T})$ be $G_{\mathcal{F}}$ −topological space. The members of the collection $'\mathcal{T}'$ are called $G_{\mathcal{F}}$ −open sets in $G_{\mathcal{F}}$ −topological space. The complement of $G_{\mathcal{F}}$ −open set in X is called $G_{\mathcal{F}} -$ close set.

Definition 3.4: Let $(X, \mathcal{T})$ be $G_{\mathcal{F}}$ −topological space. For a fuzzy set A in X the Closure of A is defined as $Cl_{\mathcal{T}}(A) = \inf\{K : A \subseteq K, K^C \in \mathcal{T}\}$. Thus $Cl_{\mathcal{T}}(A)$ is the smallest closed set in X containing the fuzzy $G_{\mathcal{F}}$ −open set A. From the definition, if follows that $Cl_{\mathcal{T}}(A)$ is the intersection of all $G_{\mathcal{F}}$ −closed sets in X containing A.

Definition 3.5: Let $(X, \mathcal{T})$ be $G_{\mathcal{F}}$ −topological space. For a fuzzy set A in X, the interior of A, is defined as $I_{\mathcal{T}}(A) = \mathrm{Sup}\{Q : Q \subseteq A, Q \in \mathcal{T}\}$. Thus $I_{\mathcal{T}}(A)$ is the largest $G_{\mathcal{F}}$ −open set in X contained in the fuzzy set A. From the definition, if follows that $I_{\mathcal{T}}(A)$ is the union of all $G_{\mathcal{F}}$ −open set in X contained in A.

Proposition 3.1: Let $(X, \mathcal{T})$ be$G_{\mathcal{F}}$ −topological space. Then:

i) 0 and 1 are fuzzy $G_{\mathcal{F}}$ −closed sets in X.
ii) Arbitrary intersection of $G_{\mathcal{F}}$ −closed sets in X is $G_{\mathcal{F}}$ −closed set in X.

Definition 3.6: Let $(X, \mathcal{T})$ be $G_{\mathcal{F}}$ −topological space. Then a fuzzy set A in X is called $G_{\mathcal{F}} - \alpha$ −open set if $A \subseteq I_{\mathcal{T}}(Cl_{\mu}(I_{\mathcal{T}}(A)))$. The fuzzy set λ in X is called $G_{\mathcal{F}} - \alpha$ −closed set if λ^c is $G_{\mathcal{F}} - \alpha$ −open set in X.

Definition 3.7: Let $(X, \mathcal{T})$ be $G_{\mathcal{F}} -$ topological space. Then a fuzzy set λ in X is called $G_{\mathcal{F}}$ −semi open set if $\lambda \subseteq Cl_{\mathcal{T}}(I_{\mathcal{T}}(\lambda))$. The fuzzy set λ in X is called $G_{\mathcal{F}}$ −semi closed set if λ^c is $G_{\mathcal{F}}$ −semi open set in X.

Definition 3.8: Let $(X, \mathcal{T})$ be $G_{\mathcal{F}} -$ topological space. Then a fuzzy $\mathcal{S}$et λ in X is called $G_{\mathcal{F}}$ −pre-open set if $\lambda \subseteq I_{\mathcal{T}}(Cl_{\mathcal{T}}(\lambda))$. The fuzzy set λ in X is called $G_{\mathcal{F}} - \beta$ –open set if $\lambda \subseteq Cl_{\mathcal{T}}(I_{\mathcal{T}}(Cl_{\mathcal{T}}(\lambda)))$. 1

Definition 3.9: Let$(X, \mathcal{T}_1)$ and $(Y, \mathcal{T}_2)$ be two $G_{\mathcal{F}}$ −topological spaces and let $\mathcal{F}: X \to Y$ be a mapping from set X to set Y. Then f is called $G_{\mathcal{F}}$ −continuous mapping if $f^{-1}(\lambda)$ is $G_{\mathcal{F}}$ −open in $(X, \mathcal{T}_1)$ for each $G_{\mathcal{F}}$ −open set λ in $(Y, \mathcal{T}_2)$.

Proposition 3.2: Let $(X, \mathcal{T}_1)$ and $(Y, \mathcal{T}_2)$ be two $G_{\mathcal{F}}$ −topological spaces and let $\mathcal{F}: X \to Y$ be a mapping from set X to set Y. Then following statements are all equivalent:

i) $\mathcal{F}$ is fuzzy continuous.
ii) For each fuzzy closed set λ in Y, $\mathcal{F}^{-1}(\lambda)$ is fuzzy closed set in X.

Proposition 3.3: Let $(X, \mathcal{T}_1)$ and $(Y, \mathcal{T}_2)$ be two $G_{\mathcal{F}}$ −topological spaces and let $\mathcal{F}: X \to Y$ be a mapping from set X to set Y. Let B be a basis for $\mathcal{T}_2$ if for each $G_{\mathcal{F}} -$ basic open set μ in B, $\mathcal{F}^{-1}(\mu)$ is fuzzy open in X, then $\mathcal{F}: X \to Y$ is $G_{\mathcal{F}}$ −continuous.

Proposition 3.4: Let $(X, \mathcal{T}_1)$ and $(Y, \mathcal{T}_2)$ and $(Z, \mathcal{T}_3)$ be $G_{\mathcal{F}}$ −topological spaces and $\mathcal{F}: X \to Y$, $\mathcal{G}: Y \to Z$, be maps. If '$\mathcal{F}$' and '$\mathcal{G}$' are $G_{\mathcal{F}}$ −continuous maps, then $\mathcal{G}o\mathcal{F}: X \to Y$ is also $G_{\mathcal{F}}$ −continuous map.

4. SOMEWHAT $G_{\mathcal{F}} -$ CONTINUOUS MAPS

Definition 4.1: A mapping $\mathcal{F}: (X, \mathcal{T}_1) \to (Y, \mathcal{T}_2)$ is said to be somewhat $G_{\mathcal{F}} -$ *continuous maps* if $A \neq 0_Y$ is$G_{\mathcal{F}} - \mathit{open\ set}$ on Y and $\mathcal{F}^{-1}(A) \neq 0_X$ then there is $G_{\mathcal{F}} - \mathit{open\ set}$ $B \neq 0_X$ on X such that $B \subseteq \mathcal{F}^{-1}(A)$.

Definition 4.2: A mapping $\mathcal{F}: (X, \mathcal{T}_1) \to (Y, \mathcal{T}_2)$ is said to be somewhat $G_{\mathcal{F}} - \mathit{pre}$ −continuous map if $A \neq 0_Y$ is $G_{\mathcal{F}} - \mathit{open\ set}$ on Y and $\mathcal{F}^{-1}(A) \neq 0_X$ then there is $G_{\mathcal{F}} - \mathit{pre} - \mathit{open\ set}$ $B \neq 0_X$ on X such that $B \subseteq \mathcal{F}^{-1}(A)$.

Remark 4.1: In a $G_{\mathcal{F}}-$ topological space $(X,\mathcal{T}_1,\mathcal{T}_2)$ every $G_{\mathcal{F}}-$ continuous map **is** pairwise $G_{\mathcal{F}}-$ pre $-$ continuous map but not converse in general which is shown in Example 4.1

Example 4.1: Let $X=\{x_1,x_2\}$ and $Y=\{y_1,y_2\}$ .Consider fuzzy sets $A=\{(x_1,0.3),(x_2,0.6)\}$, $B=\{(x_1,0.5),(x_2,0.4)\}$, $C=\{(x_1,0.5),(x_2,0.6)\}$ on X. Again $P=\{(y_1,0.2),(y_2,0.5)\}$, $Q=\{(y_1,0.6),(y_2,0.3)\}$, $R=\{(y_1,0.6),(y_2,0.6)\}$and $S=\{(y_1,0.6),(y_2,0.6)\}$ on Y. Let $\mathcal{T}_1=\{0,A,B,C,1\}$, $\mathcal{T}_2=\{0,P,Q,R,S,1\}$. Then we define a mapping $\mathcal{F}(X,\mathcal{T}_1)\to(Y,\mathcal{T}_2)$ such that $\mathcal{F}(x_1)=y_1$ and $\mathcal{F}(x_2)=y_2$. Now we observe that set $S=\{(y_1,0.6),(y_2,0.6)\}\neq 0_Y$ is $G_{\mathcal{F}}-open\ set$ on Y and $\mathcal{F}^{-1}(S)\neq 0_X$ and there is $G_{\mathcal{F}}-pre-open\ set$ $B=\{(x_1,0.5),(x_2,0.4)\}\neq 0_X$ on X such that $B\subseteq\mathcal{F}^{-1}(S)$. Therefore $\mathcal{F}(X,\mathcal{T}_1)\to(Y,\mathcal{T}_2)$ is somewhat $G_{\mathcal{F}}-pre-continuous\ map$ but not $G_{\mathcal{F}}-continuous\ map$ because $S=\{(y_1,0.6),(y_2,0.6)$ is not $G_{\mathcal{F}}-open\ set$ in Y.

Definition 4.3: A mapping $\mathcal{F}:(X,\mathcal{T}_1)\to(Y,\mathcal{T}_2)$ is said to be somewhat $G_{\mathcal{F}}-semi-continuous\ maps$ if $A\neq 0_Y$ is $G_{\mathcal{F}}-open\ set$ on Y and $\mathcal{F}^{-1}(A)\neq 0_X$ then there is $G_{\mathcal{F}}-semi-open\ set$ $B\neq 0_X$ on X such that $B\subseteq\mathcal{F}^{-1}(A)$.

Remark 4.2: In a $G_{\mathcal{F}}-$topological space $(X,\mathcal{T}_1,\mathcal{T}_2)$ every $G_{\mathcal{F}}-$ continuous map **is** pairwise $G_{\mathcal{F}}-$ semi $-$ continuous map but not converse in general which is shown in Example 4.2

Example 4.2: In Example 4.1, the mapping $\mathcal{F}(X,\mathcal{T}_1)\to(Y,\mathcal{T}_2)$ is somewhat $G_{\mathcal{F}}-semi-continuous\ map$ but not $G_{\mathcal{F}}-continuous\ map$ because of the same reason as in Example 4.1

Definition 4.4: A mapping $\mathcal{F}:(X,\mathcal{T}_1)\to(Y,\mathcal{T}_2)$ is said to be somewhat $G_{\mathcal{F}}-\beta-continuous\ maps$ if $A\neq 0_Y$ is $G_{\mathcal{F}}-open\ set$ on Y and $\mathcal{F}^{-1}(A)\neq 0_X$ then there is $G_{\mathcal{F}}-\beta-open\ set$ $B\neq 0_X$ on X such that $B\subseteq\mathcal{F}^{-1}(A)$.

Remark 4.3: In a $G_{\mathcal{F}}-$ topological space $(X,\mathcal{T}_1,\mathcal{T}_2)$ every $G_{\mathcal{F}}-$ continuous map **is** pairwise $G_{\mathcal{F}}-\beta-$ continuous map but not converse in general which is shown in Example 4.3.

Example 4.3: In Example 4.1, the mapping $\mathcal{F}(X,\mathcal{T}_1)\to(Y,\mathcal{T}_2)$ is somewhat $G_{\mathcal{F}}-\beta-continuous\ map$ but not $G_{\mathcal{F}}-continuous\ map$ because of the same reason as in Example 4.1.

Definition 4.5: A mapping $\mathcal{F}:(X,\mathcal{T}_1)\to(Y,\mathcal{T}_2)$ is said to be somewhat $G_{\mathcal{F}}-\alpha-continuous\ maps$ if $A\neq 0_Y$ is $G_{\mathcal{F}}-open\ set$ on Y and $\mathcal{F}^{-1}(A)\neq 0_X$ then there is $G_{\mathcal{F}}-\alpha-open\ set$ $B\neq 0_X$ on X such that $B\subseteq\mathcal{F}^{-1}(A)$.

Remark 4.4: In a $G_{\mathcal{F}}-$ topological space $(X,\mathcal{T}_1,\mathcal{T}_2)$ every $G_{\mathcal{F}}-$ continuous map **is** pairwise $G_{\mathcal{F}}-\alpha-$ continuous map but not converse in general which is shown in Example 4.4.

Example 4.4: In Example 4.1, the mapping $\mathcal{F}(X,\mathcal{T}_1)\to(Y,\mathcal{T}_2)$ is somewhat $G_{\mathcal{F}}-\alpha-continuous\ map$ but not $G_{\mathcal{F}}-continuous\ map$ because of the same reason as in Example 4.1

5. Conclusion

In this Paper we have studied a new concept of somewhat $G_{\mathcal{F}}-$ continuous maps and have established several relationships with the help of some counter examples, which are summarized as below in figure-1

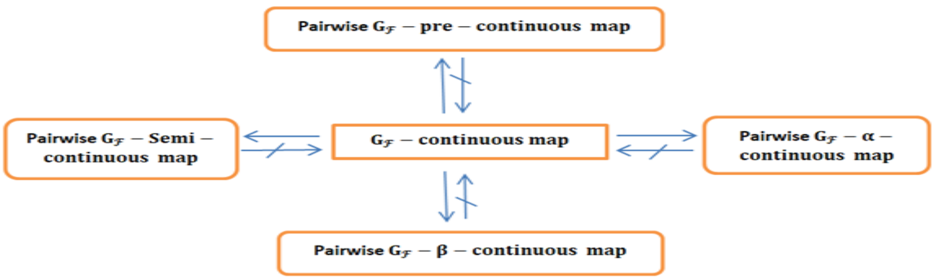

Figure-1

References

[1] Azad, K.K., On fuzzy semi continuity, fuzzy almost continuity and fuzzy weak continuity, J. Math. Anal. Appl. 82 14-32, (1981).

[2] Beceren, Y., On strongly α-continuous functions, Far East J. Math. Sci. (FJMS), Special Volume, Part I-12, 51-58, (2000).

[3] Bin Shahana, A.S., On fuzzy strong semi-continuity and fuzzy precontinuity, Fuzzy Sets and Systems 44 303-308, (1991).

[4] Bin Shahana, A.S., Mappings in fuzzy topological spaces, Fuzzy Sets and Systems 61, 209-213, (1994).

[5] Chang, C.L., Fuzzy topological spaces, J.Math. Anal. Appl.24, 182-190, (1968).

[6] Csaszar, A., Generalized open sets in generalized topologies, Acta MathematicaHungaria 96, 351-357, (2002).

[7] Donchev J., Contra Continuous Functions and Strongly S-Closed Spaces, Internat. J. Math. & Math. Sci. 19(2), 303-310, (1996)

[8] Mashhour, A.S., M.E., Abd-el-Monsef, and El-Deeb, S,N., On pre-continuous and weak pre-continuous mappings, Proc. Math. Phys. Soc. Egypt, 53 (1982), 47-53.

[9] Mashhour A. S., Hasanein I. A., EL-Deeb S. N., α-continuous and α-open mappings, Acta Math. Hungar., 41 (1983), 213 -218.

[10] Norman Levine, Semi open sets and semi continuity in Topological spaces, Amer.Math.Monthly.70, (1963), 36-41.

[11] Palani Cheety G. Generalizaed Fuzzy Topology, Italian J. Pure Appl. Math., 24,91-96, (2008)

[12] Palaniappan N. Fuzzy Topology, Narosa Publishing House, New Delhi. (2002)

[13] Palaniappan ,N. and Rao, K.C, Regular Generalized Closed Sets, Kyungpook Math. J., 33 (2) (1993), 211-219.

[14] Roy B Sen R; On a type decomposition of continuity African Mathematical Union and Springer Verlag Berlin Heidelberg (2013)

[15] Shrivastava, M., Maitra, J.K., and Shukla, M., A note on fuzzy α-continuous maps, Vikram Mathematical Journal 29, (2008).

[16] Sujeet Chaturvedi, J.K.Maitra and Nazir Ahmad Ahengar, Fuzzy Strongly α-Continuous Maps on generalized Topological Spaces, Journal of Emerging Technologies and Inovative Research (JETIR), 5(8), 312-315, 2018.

[17] Thakur, S.S. and Singh, S., On fuzzy semi-preopen sets and fuzzy semi-precontinuity, Fuzzy Sets and System, 98, 383-392, (1998).

[18] Zadeh, L.A., Fuzzy sets, Inform. and Control 8, 338-353, (1965).