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Almost Perfectly ξ-Continuous Maps (APξCM)

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Abstract. In this paper the concepts of perfectly ξ-continuous maps and almost perfectly ξ-continuous maps have been introduced and various relationships of these maps with some other maps have also been discussed and established with the appropriate examples.

Keywords: ξ-continuous maps, almost completely ξ-continuous maps, totally ξ-continuous maps, strongly ξ-continuous maps

1 Introduction

Information systems are fundamental instruments for generating information understanding in any real-life sector, and topological information collection structures are appropriate mathematical models for both quantitative and qualitative information mathematics. In both the pure and applied domains, the importance of general topology is quickly increasing. In data mining, it plays a significant role [7, 20].In computer science and digital topology [8–10], computer topology for geometric and molecular design [16], quantum physics, high-energy physics, particle physics and superstring theory [12, 23], and one can observe the impact of particular topological spaces as well.

Continuity is most important concept in Mathematics and many different generalized forms of continuity have been studied and investigated. Levine [13] introduced weakly continuous functions and established some new results. Further, Son et al. [22] introduced weakly clopen and almost clopen functions. These authors [23] investigate that almost clopen functions are the generalized forms of perfectly continuous functions, regular set-connected functions and clopen functions.

The authors Arya,S. P., Gupta,R Anuradha, Baby Chacko and Singh D [1-2,21] introduced the concept of strongly continuous functions and almost perfectly continuous functions in topological spaces and established the various significant results. Benchalli S.S and Umadevi I Neeli Nour T.M [3, 19] studied the concept of totally semi-continuous functions and semi-totally continuous functions in topological spaces and verify the certain properties of the concept. Bhattacharya,S, [4] introduced and studied the concept of generalized regular closed sets and establish the various characterizations. Nithyanantha and Thangavelu [18] introduced the concept of binary topology between two sets and investigate some of the basic properties, where a binary topology from X to Y is a binary structure satisfying certain axioms that are analogous to the axioms of topology. Jamal M. Mustafa [6] studied binary generalized topological spaces and investigate the various relationships of the maps so discussed with some other maps.

1.1 Contribution

As outline, the concept of almost perfectly ξ-continuous maps, completely ξ-continuous maps, almost completely ξ-continuous maps, totally ξ-continuous maps, strongly ξ-continuous maps have been introduced and studied different properties of these maps in this paper. The significant of results have been shown by several counter examples.

1.2 Organization

The rest of the paper structured as follows: Some require basic definitions, concepts of ξ-topological and notations are discussed in Section 2. The section 3 has been headed by the concept of almost perfect ξ-continuous maps in ξ-topological spaces in which several other maps have also been are discussed and established the relationships. Section 4 concludes the paper. Throughout the paper $\wp(\Upsilon)$ denotes the power set of Υ.

2. Preliminaries

Some require and important definitions and concepts of ξ-topological space and notations have been given in this portion

Definition 2.1: Let Υ_1 and Υ_2 be any two non-void sets. Then ξ-topology (ξ_T) from Υ_1 to Υ_2 is a binary structure $\xi \subseteq \wp(\Upsilon_1) \times \wp(\Upsilon_2)$ satisfying the conditions i.e. $(\emptyset, \emptyset)$, $(\Upsilon_1, \Upsilon_2) \in \xi$ and If $\{(L_\alpha, M_\alpha) ; \alpha \in \Gamma\}$ is a family of elements of ξ, then $(\bigcup_{\alpha\in\Gamma} L_\alpha, \bigcup_{\alpha\in\Gamma} M_\alpha) \in \xi$. If ξ is ξ_T from Υ_1 to Υ_2, then $(\Upsilon_1, \Upsilon_2, \xi)$ is called a ξ-topological space $(\xi_T S)$ and the elements of ξ are called the ξ-open subsets of $(\Upsilon_1, \Upsilon_2, \xi)$. The elements of $\Upsilon_1 \times \Upsilon_2$ are called simply ξ-points.

Definition 2.2: Let Υ_1 and Υ_2 be any two non-void set and (L_1, M_1), (L_2, M_2) are the elements of $\wp(\Upsilon_1) \times \wp(\Upsilon_2)$. Then $(L_1, M_1) \subseteq (L_2, M_2)$ only if $L_1 \subseteq L_2$ and $M_1 \subseteq M_2$.

Remark 2.1: Let $\{T_\alpha ; \alpha \in \Lambda\}$ be the family of ξ_T from Υ_1 to Υ_2. Then, $\bigcap_{\alpha\in\Lambda} T_\alpha$ is also ξ_T from Υ_1 to Υ_2. Further $\bigcup_{\alpha\in\Lambda} T_\alpha$ need not be ξ_T.

Definition 2.3: Let $(\Upsilon_1, \Upsilon_2, \xi)$ be a $\xi_T S$ and $L \subseteq \Upsilon_1, M \subseteq \Upsilon_2$. Then (L, M) is called ξ-closed in $(\Upsilon_1, \Upsilon_2, \xi)$ if $(\Upsilon_1 \backslash L, \Upsilon_2 \backslash M) \in \xi$.

Proposition 2.1: Let$(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$. Then (Υ_1, Υ_2) and $(\emptyset, \emptyset)$ are ξ-closed sets. Similarly if $\{(L_\alpha, M_\alpha) : \alpha \in \Gamma\}$ is a family of ξ-closed sets, then $(\bigcap_{\alpha\in\Gamma} L_\alpha, \bigcap_{\alpha\in\Gamma} M_\alpha)$ is ξ-closed.

Definition 2.4: Let$(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$ and $(L, M) \subseteq (\Upsilon_1, \Upsilon_2)$. Let $(L, M)^{1^*}{}_\xi = \bigcap\{L_\alpha : (L_\alpha, M_\alpha)$ is ξ-closed set and $(L, M) \subseteq (L_\alpha, M_\alpha)\}$ and $(L, M)^{2^*}{}_\xi = \bigcap\{M_\alpha : (L_\alpha, M_\alpha)$ is ξ-closed set and $(L, M) \subseteq (L_\alpha, M_\alpha)\}$. Then $((L, M)^{1^*}{}_\xi, (L, M)^{2^*}{}_\xi)$ is ξ-closed set and $(L, M) \subseteq ((L, M)^{1^*}{}_\xi, (L, M)^{2^*}{}_\xi)$. The ordered pair $\left((L, M)^{1^*}{}_\xi, (L, M)^{2^*}{}_\xi\right)$ is called ξ-closure of (L, M) and is denoted $Cl_\xi(L, M)$ in $\xi_T S\ (X, Y, \mu)$ where $(L, M) \subseteq (\Upsilon_1, \Upsilon_2)$.

Proposition 2.2: Let$(L, M) \subseteq (\Upsilon_1, \Upsilon_2)$. Then (L, M) is ξ-open in $(\Upsilon_1, \Upsilon_2, \xi)$ iff $(L, M) = I_\xi(L, M)$ and (L, M) is ξ-closed in $(\Upsilon_1, \Upsilon_2, \xi)$ iff $(L, M) = Cl_\xi(L, M)$.

Proposition 2.3: Let $(L, M) \subseteq (N, P) \subseteq (\Upsilon_1, \Upsilon_2)$ and $(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$. Then $Cl_\xi(\emptyset, \emptyset) = (\emptyset, \emptyset)$, $Cl_\xi(X, Y) = (X, Y)$, $(L, M) \subseteq Cl_\xi(L, M)$, $(L, M)^{1^*}{}_\xi \subseteq (N, P)^{1^*}{}_\xi$, $(L, M)^{2^*}{}_\xi) \subseteq (N, P)^{2^*}{}_\xi$, $Cl_\xi(L, M) \subseteq Cl_\xi(N, P)$ and $Cl_\xi(Cl_\xi(L, M)) = Cl_\xi(L, M)$

Definition 2.5: Let$(\Upsilon_1, \Upsilon_2, \xi)$ is $\xi_T S$ and $(L, M) \subseteq (\Upsilon_1, \Upsilon_2)$. Let $(L, M)^{1^0}{}_\xi = \bigcup\{L_\alpha : (L_\alpha, M_\alpha)$is ξ-open set and $(L, M) \subseteq (L_\alpha, M_\alpha)\}$ and $(L, M)^{2^0}{}_\xi = \bigcup\{M_\alpha : (L_\alpha, M_\alpha)$ is ξ-open set and $(L, M) \subseteq (L_\alpha, M_\alpha)\}$. Then $((L, M)^{1^0}{}_\xi, (L, M)^{2^0}{}_\xi)$ is ξ-open set and $((L, M)^{1^0}{}_\xi, (L, M)^{2^0}{}_\xi) \subseteq (L, M)$. The ordered pair $\left((L, M)^{1^0}{}_\xi, (L, M)^{2^0}{}_\xi\right)$ is called ξ-interior of (L, M) and is denoted $I_\xi(L, M)$ in $\xi_T S\ (X, Y, \mu)$ where $(L, M) \subseteq (\Upsilon_1, \Upsilon_2)$.

Proposition 2.4: Let $(L, M) \subseteq (\Upsilon_1, \Upsilon_2)$. Then (L, M) is ξ-open set in $(\Upsilon_1, \Upsilon_2, \xi)$ iff $(L, M) = I_\xi(L, M)$.

Proposition 2.5: Let $(L,M) \subseteq (N,P) \subseteq (\Upsilon_1,\Upsilon_2)$ and $(\Upsilon_1,\Upsilon_2,\xi)$ is $\xi_T S$. Then $I_\xi(\emptyset,\emptyset) = (\emptyset,\emptyset)$, $I_\xi(X,Y) = (X,Y)$, ${(L,M)^{1^0}}_\xi \subseteq {(N,P)^{1^0}}_\xi$, ${(L,M)^{2^0}}_\xi \subseteq {(N,P)^{2^0}}_\xi$, $I_\xi(L,M) \subseteq I_\xi(N,P)$ and $I_\xi(I_\xi(L,M)) = I_\xi(L,M)$

Definition 2.6: Let $(\Upsilon_1,\Upsilon_2,\xi)$ is $\xi_T S$ and $(Z,\mathcal{T})$ be G_T. Then the map $\mathcal{F}:(Z,\mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is called ξ-continuous at $z \in Z$ if for any ξ-open set $(L,M) \in (\Upsilon_1,\Upsilon_2,\xi)$ with $\mathcal{F}(z) \in (L,M)$ then there exists $\mathcal{T}$-open G in $(Z,\mathcal{T})$ such that $z \in G$ and $\mathcal{F}(G) \subseteq (L,M)$. The mapping $\mathcal{F}$ is called ξ-continuous if it is ξ-continuous at each $z \in Z$.

Proposition 2.6: Let $(\Upsilon_1,\Upsilon_2,\xi)$ is $\xi_T S$ and $(Z,\mathcal{T})$ be G_T. Then the map $\mathcal{F}:(Z,\mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is called ξ-continuous if $\mathcal{F}^{-1}(L,M)$ is $\mathcal{T}$-open in $(Z,\mathcal{T})$ for every ξ-open set (L,M) in $(\Upsilon_1,\Upsilon_2,\xi)$.

3. Almost Perfectly ξ-Continuous Maps (APξCM)

In this section, the concept of almost perfectly ξ-continuous maps, completely ξ-continuous maps, almost completely ξ-continuous maps, totally ξ-continuous maps and strongly ξ-continuous maps in $\xi_T S$ have been introduced and established the relationships between these maps and some other maps by making the use of some counter examples.

Definition 3.1: Let $(\Upsilon_1,\Upsilon_2,\xi)$ is $\xi_T S$ and $(Z,\mathcal{T})$ be G_T. Then the map $\mathcal{F}:(Z,\mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is called almost perfectly ξ-continuous map (APξCM) if $\mathcal{F}^{-1}(L,M)$ is $\mathcal{T}$-clopen in $(Z,\mathcal{T})$ for every ξ-regular open set (L,M) in $(\Upsilon_1,\Upsilon_2,\xi)$.

Definition 3.2: Let $(\Upsilon_1,\Upsilon_2,\xi)$ is $\xi_T S$ and $(Z,\mathcal{T})$ be G_T. Then the map $\mathcal{F}:(Z,\mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is called completely ξ-continuous map (CξCM) if $\mathcal{F}^{-1}(L,M)$ is $\mathcal{T}$-regular open in $(Z,\mathcal{T})$ for every ξ-open set (L,M) in $(\Upsilon_1,\Upsilon_2,\xi)$.

Definition 3.3: Let $(\Upsilon_1,\Upsilon_2,\xi)$ is $\xi_T S$ and $(Z,\mathcal{T})$ be G_T. Then the map $\mathcal{F}:(Z,\mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is called almost completely ξ-continuous map (ACξCM) if $\mathcal{F}^{-1}(L,M)$ is $\mathcal{T}$-regular open in $(Z,\mathcal{T})$ for every ξ-regular open set (L,M) in $(\Upsilon_1,\Upsilon_2,\xi)$.

Definition 3.4: Let $(\Upsilon_1,\Upsilon_2,\xi)$ is $\xi_T S$ and $(Z,\mathcal{T})$ be G_T. Then the map $\mathcal{F}:(Z,\mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is called totally ξ-continuous map (TξCM) if $\mathcal{F}^{-1}(L,M)$ is $\mathcal{T}$-clopen in $(Z,\mathcal{T})$ for every ξ-open set (L,M) in $(\Upsilon_1,\Upsilon_2,\xi)$.

Definition 3.5: Let $(\Upsilon_1,\Upsilon_2,\xi)$ is $\xi_T S$ and $(Z,\mathcal{T})$ be G_T. Then the map $\mathcal{F}:(Z,\mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is called strongly ξ-continuous map (SξCM) if $\mathcal{F}^{-1}(L,M)$ is $\mathcal{T}$-clopen in $(Z,\mathcal{T})$ for every ξ-set (L,M) in $(\Upsilon_1,\Upsilon_2,\xi)$.

Proposition 3.1: Let $(\Upsilon_1,\Upsilon_2,\xi)$ is $\xi_T S$ and $(Z,\mathcal{T})$ be G_T. Then the map $\mathcal{F}:(Z,\mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is APξCM iff $\mathcal{F}^{-1}(L,M)$ is $\mathcal{T}$-clopen in $(Z,\mathcal{T})$ for every ξ-regular open set (L,M) in $(\Upsilon_1,\Upsilon_2,\xi)$.

Proof: Let $(\Upsilon_1,\Upsilon_2,\xi)$ is $\xi_T S$ and $(Z,\mathcal{T})$ be G_T and the map $\mathcal{F}:(Z,\mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is APξCM. Let (L,M) be ξ-regular closed set in $(\Upsilon_1,\Upsilon_2,\xi)$. Then $(X\backslash L, Y\backslash M)$ is ξ-regular open set in $(\Upsilon_1,\Upsilon_2,\xi)$. By definition $\mathcal{F}^{-1}(X\backslash L, Y\backslash M)$ is $\mathcal{T}$-clopen in $(Z,\mathcal{T})$. That is $(Z\backslash\mathcal{F}^{-1}(L,M))$ is $\mathcal{T}$-clopen in $(Z,\mathcal{T})$. This implies $\mathcal{F}^{-1}(L,M)$ is $\mathcal{T}$-clopen in $(Z,\mathcal{T})$.

Conversely, if (N,P) is ξ-regular open set in $(\Upsilon_1,\Upsilon_2,\xi)$. Then $(X\backslash N, Y\backslash P)$ is ξ-regular closed set in $(\Upsilon_1,\Upsilon_2,\xi)$. By hypothesis, $\mathcal{F}^{-1}(X\backslash N, Y\backslash P) = Z\backslash\mathcal{F}^{-1}(N,P)$ is $\mathcal{T}$-clopen in $(Z,\mathcal{T})$, which implies $\mathcal{F}^{-1}(N,P)$ is $\mathcal{T}$-clopen in $(Z,\mathcal{T})$. Thus, inverse image of every ξ-regular open set in $(\Upsilon_1,\Upsilon_2,\xi)$ is $\mathcal{T}$-clopen in $(Z,\mathcal{T})$. Therefore $\mathcal{F}:(Z,\mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is APξCM.

Proposition 3.2: CξCM $\Rightarrow\not\Leftarrow$ ACξCM

Proof: Let $(\Upsilon_1,\Upsilon_2,\xi)$ is $\xi_T S$ and $(Z,\mathcal{T})$ be G_T and the map $\mathcal{F}:(Z,\mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is CξCM. Let (L,M) be ξ-regular open set in $(\Upsilon_1,\Upsilon_2,\xi)$. Then (L,M) is ξ-open set in $(\Upsilon_1,\Upsilon_2,\xi)$. Therefore $\mathcal{F}^{-1}(L,M)$ is $\mathcal{T}$-regular open in $(Z,\mathcal{T})$ for every ξ-regular open set (L,M) in $(\Upsilon_1,\Upsilon_2,\xi)$. Hence $\mathcal{F}:(Z,\mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is ACξCM.

The converse can be illustrated in Example 3.1.

Example 3.1: Let $Z = \{m_1, m_2, m_3\}$, $\Upsilon_1 = \{m_1, m_2, m_3\}$ and $\Upsilon_2 = \{m_1, m_2, m_3\}$. Then $\mathcal{T} = \{\emptyset, \{m_1\}, \{m_1, m_2\}, \{m_1, m_3\}, \{m_2, m_3\}, Z\}$ and $\xi = \{(\emptyset,\emptyset), (\{m_2\},\{m_2\}), (\{\emptyset\},\{m_2\}), (\{m_1\},\{m_1,m_2\}), (\{m_1,m_2\},\{m_1,m_2\}), (\{\Upsilon_1\},\{m_1,m_3\}), (\Upsilon_1,\Upsilon_2)\}$. Clearly $\mathcal{T}$ is G_T on Z and ξ is ξ_T from Υ_1 to Υ_2. Then the ξ-

regular open sets in $(\Upsilon_1, \Upsilon_2, \xi)$ are $(\emptyset, \emptyset), (\{\emptyset\}, \{m_2\}), (\{\Upsilon_1\}, \{m_1, m_3\})$ and (Υ_1, Υ_2). Define $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ by $\mathcal{F}(m_i) = (m_i, m_i)$ where and $i = 1,2,3$. Now $\mathcal{F}^{-1}(\emptyset,\ \emptyset) = \emptyset$, $\mathcal{F}^{-1}(\{\emptyset\}, \{m_2\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{\Upsilon_1\}, \{m_1, m_3\}) = \{\emptyset\}$ and $\mathcal{F}^{-1}(\Upsilon_1,\ \Upsilon_2) = Z$. This shows that the inverse image of every ξ-regular open set in $(\Upsilon_1, \Upsilon_2, \xi)$ is $\mathcal{T}$-regular open in $(Z, \mathcal{T})$. Hence $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is ACξCM but not CξCM because $\mathcal{F}^{-1}(\{m_2\}, \{m_2\}) = \{m_2\}$ and $\mathcal{F}^{-1}(\{m_3\}, \{m_3\}) = \{m_3\}$, where $\{m_2\}$ and $\{m_3\}$ are not $\mathcal{T}$-regular open sets in $(Z, \mathcal{T})$.

Proposition 3.3: SξCM $\Rightarrow\not\Leftarrow$ APξCM

Proof: Let $(\Upsilon_1, \Upsilon_2, \xi)$ is ξ_TS and $(Z, \mathcal{T})$ be G_T and the map $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is SξCM. Let (L, M) be ξ-regular open set in $(\Upsilon_1, \Upsilon_2, \xi)$. Since $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is SξCM, therefore $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-clopen in $(Z, \mathcal{T})$ for every ξ-regular open set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$. Hence $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is APξCM.

The converse can be illustrated in Example 3.2.

Example 3.2: In Example 3.1, $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is APξCM but not SξCM because $\mathcal{F}^{-1}(\{m_2\}, \{m_2\}) = \{m_2\}$ and $\mathcal{F}^{-1}(\{m_3\}, \{m_3\}) = \{m_3\}$, where $\{m_2\}$ and $\{m_3\}$ are not $\mathcal{T}$-clopen sets in $(Z, \mathcal{T})$.

Proposition 3.4: APξCM $\Rightarrow\not\Leftarrow$ ACξCM

Proof: Let $(\Upsilon_1, \Upsilon_2, \xi)$ is ξ_TS and $(Z, \mathcal{T})$ be G_T and the map $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is APξCM. Let (L, M) be ξ-regular open set in $(\Upsilon_1, \Upsilon_2, \xi)$. Since every $\mathcal{T}$-clopen is $\mathcal{T}$-regular open in $(Z, \mathcal{T})$, therefore $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-regular open in $(Z, \mathcal{T})$ for every ξ-regular open set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$. Hence $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is ACξCM.

The converse can be illustrated in Example 3.3.

Example 3.3: Let $Z = \{m_1, m_2, m_3, m_4\}$, $\Upsilon_1 = \{m_1, m_2, m_3, m_4\}$ and $\Upsilon_2 = \{m_1, m_2, m_3, m_4\}$. Then $\mathcal{T} = \{\emptyset, \{m_1\}, \{m_2\}, \{m_3\}, \{m_1, m_2\}, \{m_2, m_3\}, \{m_1, m_2, m_3\}, \{m_1, m_2, m_4\}, \{m_1, m_3, m_4\}, Z\}$ and $\xi = \{(\emptyset, \emptyset), (\{m_1\}, \{m_1\}), (\{m_2\}, \{m_2\}), (\{\emptyset\}, \{m_2\}), (\{m_1\}, \{m_1, m_2\}), (\{m_1, m_2\}, \{m_1, m_2\}), (\{\Upsilon_1\}, \{m_1, m_3, m_4\}), (\Upsilon_1, \Upsilon_2)\}$. Clearly $\mathcal{T}$ is G_T on Z and ξ is ξ_T from Υ_1 to Υ_2. Then the ξ-regular open sets in $(\Upsilon_1, \Upsilon_2, \xi)$ are $(\emptyset, \emptyset), (\{\emptyset\}, \{m_2\}), (\{m_1\}, \{m_1\}), (\{m_2\}, \{m_2\}), (\{\Upsilon_1\}, \{m_1, m_3, m_4\})$ and (Υ_1, Υ_2). Further $\mathcal{T}$-clopen sets in $(Z, \mathcal{T})$ are $\emptyset, \{m_2\}, \{m_3\}, \{m_1, m_2, m_4\}$, $\{m_1, m_3, m_4\}$ and Z. Define $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ by $\mathcal{F}(m_i) = (m_i, m_i)$ where and $i = 1,2,3,4$. Now $\mathcal{F}^{-1}(\emptyset,\ \emptyset) = \emptyset$, $\mathcal{F}^{-1}(\emptyset,\ m_2) = \emptyset$, $\mathcal{F}^{-1}(\{m_1\}, \{m_1\}) = \{m_1\}$, $\mathcal{F}^{-1}(\{m_2\}, \{m_2\}) = \{m_2\}$, $\mathcal{F}^{-1}(\{\Upsilon_1\}, \{m_1, m_3, m_4\}) = \emptyset$ and $\mathcal{F}^{-1}(\Upsilon_1,\ \Upsilon_2) = Z$. This shows that the inverse image of every ξ-regular open set in $(\Upsilon_1, \Upsilon_2, \xi)$ is $\mathcal{T}$-regular open in $(Z, \mathcal{T})$. Hence $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is ACξCM but not APξCM, because $\mathcal{F}^{-1}(\{m_1\}, \{m_1\}) = \{m_1\}$, where $\{m_1\}$is $\mathcal{T}$-regular open set but not $\mathcal{T}$-clopen set in $(Z, \mathcal{T})$.

Proposition 3.5: SξCM $\Rightarrow\not\Leftarrow$ ACξCM

Proof: Let $(\Upsilon_1, \Upsilon_2, \xi)$ is ξ_TS and $(Z, \mathcal{T})$ be G_T and the map $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is SξCM. Let (L, M) be ξ-regular open set in $(\Upsilon_1, \Upsilon_2, \xi)$. Since every $\mathcal{T}$-clopen is $\mathcal{T}$-regular open in $(Z, \mathcal{T})$, therefore $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-regular open in $(Z, \mathcal{T})$ for every ξ-regular open set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$. Hence $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is ACξCM.

The converse can be illustrated in Example 3.4.

Example 3.4: In Example 3.3, $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is ACξCM but not SξCM, because $\mathcal{F}^{-1}(\{m_1\}, \{m_1\}) = \{m_1\}$, where $\{m_1\}$is $\mathcal{T}$-regular open set but not $\mathcal{T}$-clopen set in $(Z, \mathcal{T})$.

Proposition 3.6: APξCM $\Rightarrow\not\Leftarrow$ TξCM

Proof: Let $(\Upsilon_1, \Upsilon_2, \xi)$ is ξ_TS and $(Z, \mathcal{T})$ be G_T and the map $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is APξCM. Let (L, M) be ξ-regular open set in $(\Upsilon_1, \Upsilon_2, \xi)$. Since every ξ-regular open set in $(\Upsilon_1, \Upsilon_2, \xi)$ is ξ-open set, therefore $\mathcal{F}^{-1}(L, M)$ is $\mathcal{T}$-clopen in $(Z, \mathcal{T})$ for every ξ-open set (L, M) in $(\Upsilon_1, \Upsilon_2, \xi)$. Hence $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is TξCM.

The converse can be illustrated in Example 3.5.

Example 3.5: Let $Z = \{m_1, m_2, m_3, m_4\}$, $\Upsilon_1 = \{m_1, m_2, m_3, m_4\}$ and $\Upsilon_2 = \{m_1, m_2, m_3, m_4\}$. Then $\mathcal{T} = \{\emptyset, \{m_2\}, \{m_3\}, \{m_1, m_2\}, \{m_2, m_3\}, \{m_1, m_2, m_3\}, \{m_1, m_2, m_4\}, \{m_1, m_3, m_4\}, Z\}$ and $\xi = \{(\emptyset, \emptyset), (\{m_2\}, \{m_2\}), (\{\emptyset\}, \{m_2\}), (\{m_1\}, \{m_1, m_2\}), (\{m_1, m_2\}, \{m_1, m_2\}), (\{\Upsilon_1\}, \{m_1, m_3, m_4\}), (\Upsilon_1, \Upsilon_2)\}$. Clearly $\mathcal{T}$ is G_T on Z and ξ is ξ_T from Υ_1 to Υ_2. Then the $\mathcal{T}$-clopen sets in $(Z, \mathcal{T})$ are $\emptyset, \{m_2\}, \{m_3\}, \{m_1, m_2, m_4\}, \{m_1, m_3, m_4\}$ and Z. Define $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ by $\mathcal{F}(m_i) = (m_i, m_i)$ where and $i = 1,2,3,4$. Now $\mathcal{F}^{-1}(\emptyset, \emptyset) = \emptyset$, $\mathcal{F}^{-1}(\{m_2\}, \{m_2\}) = \{m_2\}$, $\mathcal{F}^{-1}(\{\emptyset\}, \{m_2\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{m_1\}, \{m_1, m_2\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{m_1, m_2\}, \{m_1, m_2\}) = \{\emptyset\}$, $\mathcal{F}^{-1}(\{\Upsilon_1\}, \{m_1, m_3, m_4\}) = \{\emptyset\}$ and $\mathcal{F}^{-1}(\Upsilon_1, \Upsilon_2) = Z$. This shows that the inverse image of every ξ-open set in $(\Upsilon_1, \Upsilon_2, \xi)$ is $\mathcal{T}$-clopen in $(Z, \mathcal{T})$. Hence $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is TξCM but not APξCM, because $\mathcal{F}^{-1}(\{m_1\}, \{m_1, m_2\}) = \{\emptyset\}$ and $\mathcal{F}^{-1}(\{m_1, m_2\}, \{m_1, m_2\}) = \{\emptyset\}$, where $(\{m_1\}, \{m_1, m_2\})$ and $(\{m_1, m_2\}, \{m_1, m_2\})$ are ξ-open sets but not ξ-regular open sets in $(\Upsilon_1, \Upsilon_2, \xi)$.

Proposition 3.7: ACξCM $\not\Leftrightarrow$ TξCM

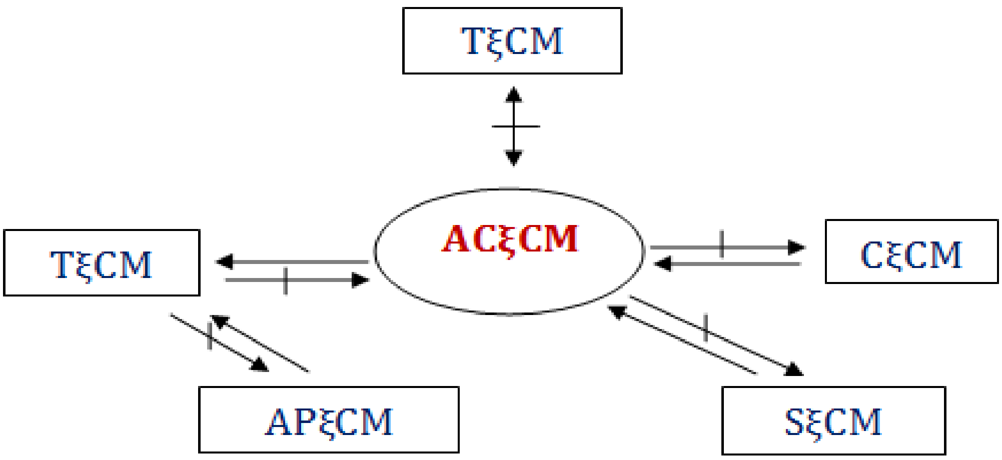
The result can be illustrated in Example 3.6 and Example 3.7.

Example 3.6: Let $Z = \{m_1, m_2, m_3, m_4\}$, $\Upsilon_1 = \{m_1, m_2, m_3, m_4\}$ and $\Upsilon_2 = \{m_1, m_2, m_3, m_4\}$. Then $\mathcal{T} = \{\emptyset, \{m_1\}, \{m_2\}, \{m_3\}, \{m_1, m_2\}, \{m_2, m_3\}, \{m_1, m_2, m_3\}, \{m_1, m_2, m_4\}, \{m_1, m_3, m_4\}, Z\}$ and $\xi = \{(\emptyset, \emptyset), (\{m_1\}, \{m_1\}), (\{m_2\}, \{m_2\}), (\{\emptyset\}, \{m_2\}), (\{m_1\}, \{m_1, m_2\}), (\{m_1, m_2\}, \{m_1, m_2\}), (\{\Upsilon_1\}, \{m_1, m_3, m_4\}), (\Upsilon_1, \Upsilon_2)\}$. Clearly $\mathcal{T}$ is G_T on Z and ξ is ξ_T from Υ_1 to Υ_2. Then the ξ-regular open sets in $(\Upsilon_1, \Upsilon_2, \xi)$ are $(\emptyset, \emptyset), (\{\emptyset\}, \{m_2\}), (\{m_1\}, \{m_1\}), (\{m_2\}, \{m_2\}), (\{\Upsilon_1\}, \{m_1, m_3, m_4\})$ and (Υ_1, Υ_2). Further $\mathcal{T}$-regular open sets in $(Z, \mathcal{T})$ are $\emptyset, \{m_1\}, \{m_2\}, \{m_3\}, \{m_1, m_2, m_4\}$, $\{m_1, m_3, m_4\}$ and Z. Define $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ by $\mathcal{F}(m_i) = (m_i, m_i)$ where and $i = 1,2,3,4$. Now $\mathcal{F}^{-1}(\emptyset, \emptyset) = \emptyset$, $\mathcal{F}^{-1}(\emptyset, m_2) = \emptyset$, $\mathcal{F}^{-1}(\{m_1\}, \{m_1\}) = \{m_1\}$, $\mathcal{F}^{-1}(\{m_2\}, \{m_2\}) = \{m_2\}$, $\mathcal{F}^{-1}(\{\Upsilon_1\}, \{m_1, m_3, m_4\}) = \emptyset$ and $\mathcal{F}^{-1}(\Upsilon_1, \Upsilon_2) = Z$. This shows that the inverse image of every ξ-regular open set in $(\Upsilon_1, \Upsilon_2, \xi)$ is $\mathcal{T}$-regular open in $(Z, \mathcal{T})$. Hence $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is ACξCM but not TξCM, because $\mathcal{F}^{-1}(\{m_1\}, \{m_1\}) = \{m_1\}$, where $\{m_1\}$is $\mathcal{T}$-regular open set in $(Z, \mathcal{T})$ but not $\mathcal{T}$-clopen set in $(Z, \mathcal{T})$.

Example 3.7: In Example 3.5, $\mathcal{F}: (Z, \mathcal{T}) \to \Upsilon_1 \times \Upsilon_2$ is TξCM but not APξCM, because $\mathcal{F}^{-1}(\{m_1\}, \{m_1, m_2\}) = \{\emptyset\}$ and $\mathcal{F}^{-1}(\{m_1, m_2\}, \{m_1, m_2\}) = \{\emptyset\}$, where $(\{m_1\}, \{m_1, m_2\})$ and $(\{m_1, m_2\}, \{m_1, m_2\})$ are ξ-open sets but not ξ-regular open sets in $(\Upsilon_1, \Upsilon_2, \xi)$.

4. Conclusion

In this paper, a very useful concept of almost perfectly ξ-continuous maps, completely ξ-continuous maps, almost completely ξ-continuous maps, totally ξ-continuous maps, strongly ξ-continuous maps have been introduced and established the relationships between these maps and some other maps. The significant of results have been shown by several counter examples. In the present direction, we have categorised maps in generalized binary topological spaces and investigated the behaviour of presented maps with the help of appropriate examples. Further conclusion is illustrated in following figure



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