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A New Class of Ng*B-Continuous and Ng*B-Irresolute Functions in Nts

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Abstract:-This paper's goal is to introduce a new class of function known as Ng*b-continuous function and determine how to characterize them in terms of Ng*b-interior, Ng*b-closure, and Ng*b-closed sets. Additionally, an attempt has been made to characterize Ng*b-irresolute function and define them in terms of Ng*b-closed sets, Ng*b-closure, and Ng*b-interior.

Keywords*:* NT, Ng*b -open sets, Ng*b- interior, Ng*b- closure, Ng*b- continuous functions, Ng*b –irresolute functions

1. Introduction

One of the fundamental ideas of topology is the continuity of functions. A continuous function is, in general, one for which slight variations in the input cause similarly tiny variations in the output. The notion of generalized closed sets in topological spaces was first presented by Levine [2] in 1970. Lellis Thivagar[3] introduced the idea of NT, which was defined as approximations and the boundary region of a subset of a universe using an equivalency relation on it. Nano closed sets, nano-interior, and nano-closure of a set were also defined. Nano generalized closed sets and their properties in NTSs were introduced and studied by Bhuvaneshwari et al. [5]. The properties of Nano gb closed sets in NTSs were introduced and studied by Mary A [1]. Few fundamental properties of Nano b open sets in NTSs are examined by Parimala et al. [6]. In NTSs, Sathishmohan et al. introduced the idea of Nano pre and Nano semi pre-neighborhoods and demonstrated several of its key findings [7, 8]. In topological space, Vidhya et al. introduced a new class of generalized closed sets called Ng*b-closed sets. They studied these fundamental characteristics and looked into some of their relationships with already-existing ones[9]. The concept of nano continuity was first introduced by Lellis Thivagar and Carmel Richard [4].

A new class of functions on NTSs, known as Ng*b-continuous functions, is presented in this paper. Their characterizations in terms of Ng*b-closed sets, Ng*b-closure, and Ng*b-interior are derived. Moreover, we have established Ng*b irresolute functions, and their representations in terms of Ng*b- closure and Ng*b -interior.

2. Preliminaries

Definition 2.1. [3] Let J be a non-empty finite set of objects called the universe and R be an equivalence relation on J named as indiscernibility relation. Then J is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another.

The pair (J, R) is said to be the approximation space. Let $X \subseteq J$. Then,

(i) The lower approximation of x with respect to R is the set of all objects, which can be for certain classified as X with respect to R and is denoted by $L_{R(X)}$. $L_{R(X)} = \cup_{x\in J} \{R(x):R(x) \subseteq X\}$ where R(x) denotes the equivalence class determined by $x \in J$.

(ii) The upper approximation of x with respect to R is the set of all objects which can be possibly classified as X with respect to R and is denoted by $J_R(X)$. $U_R(X)=\cup_{x\in J} \{R(x):R(x) \cap X \neq\varphi\}$.

(iii) The boundary region of x with respect to R is the set of all objects which can be classified neither as X nor as Not-X with respect to R and it is denoted by $B_R(X)$. $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2 If (J, R) is an approximation space and X, Y⊆J, then

(i) $L_R(X) \subseteq X \subseteq U_R(X)$

(ii) $L_R(\phi) = U_R(\phi) = \phi$

(iii) $L_R(U) = U_R(U) = U$

(iv) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$

(v) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$

(vi) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$

(vii) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$

(viii) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$

(ix) $U_R(X^C) = [L_R(X)]^C$ and $L_R(X^C) = [U_R(X)]^C$

(x) $U_R(U_R(X)) = L_R(U_R(X)) = U_R(X)$

(xi) $L_R(L_R(X)) = U_R(L_R(X)) = L_R(X)$

Definition 2.3. [3] Let J be the universe, R be an equivalence relation on J and $\tau_R = \{U,\varphi,L_R(X),U_R(X),B_R(X)\}$ where X ⊆J. Then τ_R satisfies the following axioms

(i) J and $\varphi \in \tau_R$.

(ii) The union of the elements of any sub-collection of τ_R is in τ_R.

(iii) The intersection of the elements of any finite sub collection of τ_R is in $\tau_R(X)$.

Then τ_R is a topology on J called the nano topology(NT) on J with respect to X. We call (J, τ_R) as NTS (NTS). The elements of $\tau_R(X)$ are called as nano open sets (NOSs). The complement of the nano open sets is called nano closed sets (NCSs).

Remark 2.4. If τ_R is the NT on J with respect to X, then the set B = { J, $L_R(X)$, $B_R(X)$} is the basis for τ_R.

Definition 2.5. [3] If (J, $\tau_R(X)$) is a NTS with respect to X where

X ⊆J and if A ⊆J,then

(i) The nano interior of A is defined as the union of all nano-open subsets of A and is denoted by Nint(A). That is, Nint(A) is the largest nano-open subset of A.

(ii) The nano closure of A is defined as the intersection of all nano-closed sets containing A and is denoted by Ncl(A). That is, Ncl(A) is the smallest nano-closed set containing A.

Definition 2.6. [9] A subset A of NTS (J, $\tau_R(X)$) is known as nano generalized star b-closed if Nbcl(A) ⊆ G, whenever A⊆ G where G is Ng-open.

3. Ng*b- continuous and Ng*b-irresolute

Definition 3.1: Let (I, $\tau_R(X)$) and (J, $\tau_R'(Y)$) be NTSs. Then a mapping f : (I, $\tau_R(X)$) ⟶ (J, $\tau_R'(Y)$) is Ng*b continuous on I if the inverse image of every Nano open set in J is Ng*b-open in I.

Example 3.2 : Let I = {1,2,3,4} with I/R= {{1, 4},{2},{3}} . Let X = {1, 2} ⊆ I. Then $\tau_R(X)$) ={I ,∅ ,{2},{1,2,4},{1,4}} . Let J = {x, y, z,w} with J/R = {{x},{y, z},{w}} and Y = {x, z} . Then $\tau_R'(Y)$ ={ J, ∅ ,{x},{x, y, z},{y, z}} . Define f: I ⟶ J f as f(1) = y, f(2) = w, f(3) = z, f(4)=x.Then Ng*b-closed ={U ,∅ ,{1},{2},{3},{4},{1,3},{1,4},{2,3},{3,4},{1,2,3},{1,3,4},{2,3,4}}and

Ng*bopen={I,∅,{1},{2},{3},{2,4},{2,3},{1,4},{1,2},{1,2,3},{1,3,4},{2,3,4},{1,2,4}}. That is, the inverse image of every nano-open set in J is Ng*b -open in I. Therefore, f is Ng*b -continuous.

Proposition 3.3: Let (I, τ_R(X)) and (V, τ_R' (Y)) be NTSs and let f: (I, τ_R(X)) → (J, τ_R' (Y)) be a map. Then f is Ng*b – continuous iff the inverse image of every nano closed subset of (J, τ_R' (Y)) is Ng*b -closed in (I, τ_R(X)).

Proof: Let F be nano closed subset in (J, τ_R' (Y)). Then J – F is open in (J, τ_R' (Y)). Since f is Ng*b – continuous, $f^{-1}(J-F)$ is Ng*b – open. But f^{-1}(J – F) = I – $f^{-1}(F)$. Therefore, f^{-1}(F) is Ng*b – closed in (I, τ_R(X)). Thus, the inverse image of every NOC in J is Ng*b closed in I, if f is Ng*b – continuous. Conversely let G be a nano open subset in (J, τ_R' (Y)). Then J – G is nano closed in (J, τ_R' (Y)). Since the inverse image of each closed subset in (J, τ_R' (Y)) is Ng*b – closed in (I, τ_R(X)). We have f^{-1}(J – G) is to be Ng*b – closed in (I, τ_R(X)). But f^{-1}(J – G) = I – f^{-1}(G). Thus f^{-1}(G) is Ng*b-open. Therefore f is Ng*b – continuous.

Definition 3.4 :A function f : (I, τ_R(X)) → (J, τ_R' (Y)) is called Ng*b-irresolute if f^{-1}(J) is Ng*b-closed in (I, τ_R(X)) for every Ng*b-closed set J in (J, τ_R' (Y))

Example 3.5: Let I = {*1*,*2*, *3*, *4*} with I/R= {{*1*, *4*},{*2*},{*3*}} . Let X = {*1*, *2*} ⊆ I. Then τ_R(X)) ={I ,∅ ,{*2*},{*1*,*2*,*4*},{*1*,*4*}} . Let J = {*x*, *y*, *z*,*w*} with J/*R* = {{*x*},{*y*, *z*},{*w*}} and *Y* = {*x*, *z*} . Then τ_R' (Y) ={ J, ∅ ,{*x*},{*x*, *y*, *z*},{*y*, *z*}} . Define f: I → J f as f(1) = y, f(2) = w, f(3) = z, f(4)=x. Therefore, f is Ng*b -irresolute.

Theorem 3.6: A function f : (I, τ_R(X)) → (J, τ_R' (Y)) is Ng*b-irresolute if and only if the inverse image of every Ng*b-closed set in (J, τ_R' (Y)) is Ng*b-closed set in (I, τ_R(X)).

Proof. Let f : (I, τ_R(X)) → (J, τ_R' (Y)) be a Ng*b-irresolute mapping and let A be a Ng*b-closed set in (J, τ_R' (Y)). Then J– A is Ng*b-open set in (J, τ_R' (Y)). Since f is Ng*b-irresolute,f^{-1}(J – A) is Ng*b-open in (I, τ_R(X)). i.e., I– f^{-1}(A) is Ng*b-open in (I, τ_R(X)) . Therefore f^{-1}(A) is Ng*b-closed in (I, τ_R(X)). Conversely, suppose that the inverse image of every Ng*b-closed set in (J, τ_R' (Y)) is Ng*b–closed set in ((I, τ_R(X)). Let B be a Ng*b- open set in (J, τ_R' (Y)). Then J – B is Ng*b– closed set in (J, τ_R' (Y)). Then f^{-1}(J – B) is Ng*b-closed in (I, τ_R(X)). i.e., I– f^{-1}(B) is Ng*b-closed in (I, τ_R(X)) .Therefore f^{-1}(B) is Ng*b-open in (I, τ_R(X)). Hence f is Ng*b-irresolute.

Definition 3.7: A topological space I is a Ng*b- space if every Ng*b- closed set is nano closed.

Proposition 3.8: Let (I, τ_R(X)) and (J, τ_R' (Y)) be NTSs and (J, τ_R' (Y)) be a Ng*b – space. If f: (I, τ_R(X)) → (J, τ_R' (Y)) and g: (J, τ_R' (Y)) →(W, τ_R'' (Z)) are Ng*b – continuous, then g o f is Ng*b – continuous.

Proof: Let H be nano open in ((W, τ_R'' (Z)). Since g is Ng*b – continuous, g^{-1}(H) is Ng*b – open in (J, τ_R' (Y)). But (J, τ_R' (Y)) is a Ng*b – space, hence g^{-1}(H) is nano open in (J, τ_R' (Y)). Thus $f^{-1}(g^{-1}(H))$ = (g o f) $^{-1}$(H) is Ng*b – open in (I, τ_R(X)). Therefore, g o f is Ng*b – continuous.

Proposition 3.9: Let (I, τ_R(X)), (J, τ_R' (Y)) and (W, τ_R'' (Z)) be NTSs. If f: (I, τ_R(X)) → (J, τ_R' (Y)) is Ng*b -continuous and g: (J, τ_R' (Y)) → (W, τ_R'' (Z)) is nano continuous then g o f is Ng*b –continuous.

Proof: Let H be a nano open subset of (W, τ_R'' (Z)). Since g is nano continuous, g^{-1}(H) is open in (J, τ_R' (Y)). Since f is Ng*b – continuous, f^{-1} $(g^{-1}(H))$ is Ng*b – open in (I, τ_R(X)). But $f^{-1}(g^{-1}(H))$ = (g o f) $^{-1}$(H). Therefore, g o f is Ng*b – continuous.

Proposition 3.10: Every Ng*b – irresolute map is Ng*b – continuous.

Proof: Assume that f is Ng*b – irresolute. Let H be a Nano closed set in J. Every Nano closed set is Ng*b – closed. That implies H be a Ng*b – closed set in J. Since f is Ng*b – irresolute, f^{-1}(H) is Ng*b – closed set in I. Thus f^{-1}(H) is Ng*b – closed set in I, for all Nano closed set H in J. That implies f is Ng*b – continuous.

Proposition 3.11: Let (I, τ_R(X)) and (J, τ_R' (Y)) be NTSs and f: (I, τ_R(X)) → (J, τ_R' (Y)) be a map. Then f is Ng*b – irresolute if and only if $f^{-1}(B)$ is Ng*b – closed in (I, τ_R(X)) whenever B is Ng*b – closed in (J, τ_R' (Y)).

Proof: Suppose B be a Ng*b -closed subset of (J, τ_R' (Y)). Then J – B is Ng*b - open in (J, τ_R' (Y)). Since f: (I, τ_R(X)) → (J, τ_R' (Y)) is Ng*b – irresolute, $f^{-1}(J-B)$ is Ng*b - open in (I, τ_R(X)). But, $f^{-1}(J-B) = I - f^{-1}(B)$, so that f^{-1}(B) is Ng*b - closed in (I, τ_R(X)). Conversely, Let A be a Ng*b - open subset in (J, τ_R' (Y)).Then J – A is Ng*b - closed in (J, τ_R' (Y)). By the assumption, $f^{-1}(J-A)$ is Ng*b - closed in (I, τ_R(X)). But $f^{-1}(J-A) = I - f^{-1}(A)$. Thus f^{-1}(A) is Ng*b - open in (I, τ_R(X)). Therefore, f is Ng*b - irresolute.

Proposition 3.12: Let (I, τ_R(X)), (J, τ_R' (Y)) and (W, τ_R'' (Z)) be NTSs. If f : (I, τ_R(X)) → (J, τ_R' (Y)) is a Ng*b–irresolute map and g: (J, τ_R' (Y)) → (W, τ_R'' (Z)) is a Ng*b- continuous map, then the composition g o f: (I, τ_R(X)) → (W, τ_R'' (Z)) is Ng*b - continuous.

Proof: Let G be a nano open subset of ((W, τ_R'' (Z)). Then g^{-1}(G) is a Ng*b - open in (J, τ_R' (Y)) as g is Ng*b – continuous. Hence,f^{-1} $(g^{-1}(G))$ is Ng*b – open in (I, τ_R(X)) because f is Ng*b – irresolute. Thus gof is Ng*b – continuous.

Proposition 3.13: Let (I, τ_R(X)), (J, τ_R' (Y)) and (W, τ_R'' (Z)) be NTSs. If f: (I, τ_R(X)) → (J, τ_R' (Y)) and g: (J, τ_R' (Y)) → (W, τ_R'' (Z)) are Ng*b - irresolute, then g o f: (I, τ_R(X)) → (W, τ_R'' (Z)) is Ng*b - irresolute.

Proof: Let F be Ng*b -open set in (W, τ_R'' (Z)). As g is Ng*b - irresolute, g^{-1}(F) is Ng*b -open in (J, τ_R' (Y)). Since, f is Ng*b - irresolute. $f^{-1}(g^{-1}(F))$ is Ng*b - open in (I, τ_R(X)). That implies (g o f) $^{-1}$F = (f $^{-1}g^{-1}(F)$) is Ng*b -open in (I, τ_R(X)). Hence gof is Ng*b - irresolute.

Proposition 3.14: Let (I, τ_R(X)) and (J, τ_R' (Y)) be NTSs and (J, τ_R' (Y)) be a Ng*b - space. If f: (I, τ_R(X)) → (J, τ_R' (Y)) be a Ng*b -continuous map and g: (J, τ_R' (Y)) → (W, τ_R'' (Z)) is a Ng*b - irresolute, then the composition g o f: (I, τ_R(X)) → (W, τ_R'' (Z)) is Ng*b – irresolute.

Proof: Let H be Ng*b – open in (W, τ_R'' (Z). Since g is Ng*b – irresolute, g^{-1}(H) is Ng*b – open in (J, τ_R' (Y)). As J is a Ng*b – space, g^{-1}(H) is nano open in (J, τ_R' (Y)). Since f is Ng*b -continuous,$f^{-1}(g^{-1}(H))$ is Ng*b – open in (I, τ_R(X)). Thus (g o f) $^{-1}$(H) is Ng*b – open in (I, τ_R(X)). Hence g o f is Ng*b - irresolute.

Characterization of Ng*b-continuous and Ng*b-irresolute functions in terms of Ng*b-closure

Theorem 4.1: A function f: (I, τ_R(X)) → (J, τ_R' (Y)) is Ng*b-continuous if and only if f($cl_{Ng^*b}(A)$) ⊆ $cl_{Ng^*b}(f(A))$ for every subset A of I.

Proof: Let f be Ng*b-continuous and $A \subseteq$ I. Then $f(A) \subseteq$J. cl_{Ng^*b} (f(A)) is nano closed in J. Since f is Ng*b-continuous, f^{-1}(cl_{Ng^*b} (f(A)))is Ng*b-closed in I. Since f(A) ⊆ cl_{Ng^*b} (f(A)), A ⊆ f^{-1}(cl_{Ng^*b} (f(A))) . Thus

f^{-1}(cl_{Ng^*b} (f(A)))is a Ng*b- closed set containing A. But, $cl_{Ng^*b}(A)$ is the smallest Ng*b- closed set containing A. Therefore $cl_{Ng^*b}(A)$ ⊆ f^{-1}($cl_{Ng^*b}(A)$). That is,$cl_{Ng^*b}(A)$ ⊆ f^{-1}($cl_{Ng^*b}(A)$. Conversely, let $cl_{Ng^*b}(A)$ ⊆ f^{-1}($cl_{Ng^*b}(A)$ for every subset A of I. If F is nano closed in J, since f^{-1}(F) ⊆ I, f(cl_{Ng^*b} (f^{-1}(F))) ⊆ cl_{Ng^*b} (f^{-1}(F)) ⊆ cl_{Ng^*b} (F).That is, cl_{Ng^*b} (f^{-1}(F)) ⊆ f^{-1}(cl_{Ng^*b} (F)) = f^{-1}(F)) , since F is nano closed. Thus cl_{Ng^*b} (f^{-1}(F)) ⊆ f^{-1}(F). But f^{-1}(F) ⊆ cl_{Ng^*b} (f^{-1}(F)) . Therefore, cl_{Ng^*b} (f^{-1}(F)) = f^{-1}(F). Therefore f^{-1}(F) is nano closed in I for every Nano closed set F in J . That is, f is Ng*b- continuous.

Theorem 4.2:A function f: (I, τ_R(X)) → (J, τ_R' (Y)) is Ng*b-continuous if and only if

$cl_{Ng^*b}(f^{-1}(B)) \subseteq f^{-1}\left(cl_{Ng^*b}(B)\right)$for every subset B of J .

Proof: If f is Ng*b- continuous and $B \subseteq$ J ,$cl_{Ng^*b}(B)$ is nano closed in J and hence $f^{-1}\left(cl_{Ng^*b}(B)\right)$is Ng*b-closed in I.Therefore, cl_{Ng^*b} [$f^{-1}\left(cl_{Ng^*b}(B)\right)$] = $f^{-1}\left(cl_{Ng^*b}(B)\right)$.

Since $B \subseteq cl_{Ng^*b}(B)$,$f^{-1}(B) f \subseteq^{-1}\left(cl_{Ng^*b}(B)\right)$.Therefore,$cl_{Ng^*b}(f^{-1}(B)) \subseteq cl_{Ng^*b}\left(f^{-1}\left(cl_{Ng^*b}(B)\right)\right) = f^{-1}\left(cl_{Ng^*b}(B)\right)$

That is, $cl_{Ng^*b}(f^{-1}(B)) \subseteq f^{-1}\left(cl_{Ng^*b}(B)\right)$. Conversely, let $cl_{Ng^*b}(f^{-1}(B)) \subseteq f^{-1}\left(cl_{Ng^*b}(B)\right)$ for every B $\subseteq$ J . Let B be nano closed in J . Then $cl_{Ng^*b}(B) = B$. By assumption, $cl_{Ng^*b}(f^{-1}(B)) \subseteq f^{-1}\left(cl_{Ng^*b}(B)\right) = f^{-1}(B)$. Thus, $cl_{Ng^*b}(f^{-1}(B)) \subseteq f^{-1}(B)$ .But $f^{-1}(B) \subseteq cl_{Ng^*b}(f^{-1}(B))$. Therefore, $cl_{Ng^*b}(f^{-1}(B)) = f^{-1}(B)$. That is, $f^{-1}(B)$ is Ng*b- closed in I for every Nano closed set B in J. Therefore, f is Ng*b- continuous on I.

Theorem 4.3:A function f: (I, τ_R(X)) → (J, τ_R' (Y)) is Ng*b-continuous on I if and only if

$f^{-1}(int_{Ng^*b}(B)) \subseteq int_{Ng^*b}(f^{-1}(B))$ for every subset B of J .

Proof: Let f be Ng*b- continuous and $B \subseteq J$. Then $int_{Ng^*b}(B)$ is nano-open in (J, τ_R' (Y)). Therefore $f^{-1}(int_{Ng^*b}(B))$ is Ng*b -open in (I, τ_R(X)). That is, $f^{-1}(int_{Ng^*b}(B)) \subseteq int_{Ng^*b}\left(f^{-1}\left(int_{Ng^*b}(B)\right)\right)$. Also, $int_{Ng^*b}(B) \subseteq B$ implies that $f^{-1}(int_{Ng^*b}(B)) \subseteq f^{-1}(B)$.

Therefore, $int_{Ng^*b}[f^{-1}(int_{Ng^*b}(B))] \subseteq int_{Ng^*b}(f^{-1}(B))$. That is, $f^{-1}(int_{Ng^*b}(B)) \subseteq int_{Ng^*b}(f^{-1}(B))$. Conversely, let $f^{-1}(int_{Ng^*b}(B)) \subseteq int_{Ng^*b}(f^{-1}(B))$ for every subset B of J . If B is nano-open in J , $int_{Ng^*b}(B) = B$. Also, $f^{-1}(int_{Ng^*b}(B)) \subseteq int_{Ng^*b}(f^{-1}(B))$. That is, $f^{-1}(B) \subseteq int_{Ng^*b}(f^{-1}(B))$. But $int_{Ng^*b}(f^{-1}(B)) \subseteq f^{-1}(B)$.Therefore, $f^{-1}(B) = int_{Ng^*b}(f^{-1}(B))$. Thus, $f^{-1}(B)$ is Ng*b- -open in I for every nano-open set B in J .Therefore, f is Ng*b- continuous.

Theorem 4.4: A mapping f: (I, τ_R(X)) → (J, τ_R' (Y)) is Ng*b- irresolute if and only if $cl_{Ng^*b}(f^{-1}(B)) \subseteq f^{-1}\left(cl_{Ng^*b}(B)\right)$

Proof. Let f: (I, τ_R(X)) → (J, τ_R' (Y)) be a Ng*b-irresolute mapping and $B \subseteq$ J, then $cl_{Ng^*b}(B)$ is Ng*b- closed in (J, τ_R' (Y)) and hence $f^{-1}\left(cl_{Ng^*b}(B)\right)$ is Ng*b-closed in (I, τ_R(X)). Therefore, $cl_{Ng^*b}\left(f^{-1}\left(cl_{Ng^*b}(B)\right)\right) = f^{-1}\left(cl_{Ng^*b}(B)\right)$.Since $B \subseteq cl_{Ng^*b}(B)$, f^{-1}(B)$\subseteq f^{-1}(cl_{Ng^*b}(B))$.

Therefore $cl_{Ng^*b}(f^{-1}(B)) \subseteq \left(f^{-1}\left(cl_{Ng^*b}(B)\right)\right) = f^{-1}\left(cl_{Ng^*b}(B)\right)$.Conversely, $cl_{Ng^*b}(f^{-1}(B)) \subseteq f^{-1}\left(cl_{Ng^*b}(B)\right)$ for every subset B $\subseteq$ J.Let B be a Ng*b-closed set in (J, τ_R' (Y)). Then $cl_{Ng^*b}(B) = B$. By assumption, , $cl_{Ng^*b}(f^{-1}(B)) \subseteq f^{-1}\left(cl_{Ng^*b}(B)\right) = f^{-1}$ (B). i.e., $cl_{Ng^*b}(f^{-1}(B)) \subseteq f^{-1}$ (B). But f^{-1} (B)$\subseteq cl_{Ng^*b}(f^{-1}(B))$ hence f^{-1} (B) = $cl_{Ng^*b}(f^{-1}(B))$, f^{-1} (B) is Ng*b-closed in (I, τ_R(X)) for every Ng*b-closed set B in J. Thus f is Ng*b-irresolute on (I, τ_R(X)).

Results

The continuous and irresolute functions of Ng*b have been defined, examined, and their characterizations in terms of Ng*b-closure and Ng*b-interior have also been derived in this paper.

Refrences

[1] A. D. A. Mary and I. Arockiarani, Properties of Nano GB-closed maps, *IOSR Journal of Mathematics*, 11(2)(II), 2015, 21–24.

[2] N. Levine, Generalized closed sets in topology, *Rend. circ. Math., Palermo*, 2(19), 1970, 89-96.

[3] L. Thivagar and C. Richard, On Nano forms of weekly open sets, *Int. J. Math. Stat. Invev*., 1(1), 2013, 31-37.

[4] L. Thivagar and C. Richard,On Nano Continuity,Mathematical Theory and Modeling,Vol.3, No.7, 2013,32-37.

[5] K. Bhuvaneshwari and K. Mythili Gnanapriya, Nano generalizesd closed sets, *International Journal of Scientific and Research Publications*, 4(5), 2014, 1–3.

[6] M. Parimala, C. Indirani and S. Jafari, On Nano b-open sets in NTSs, *Jordan Journal of Mathematics*, 9(3), 2016, 173–184.

[7] P. Sathishmohan. V. Rajendran, P. K. Dhanasekaran, C. Vignesh Kumar, More on pre neighbourhoods in Nano topological-spaces, *Journal of Applied Science and Computations*, 5(10), 2018, 899-907.

[8] P. Sathishmohan. V. Rajendran, C. Vignesh Kumar, P. K. Dhanasekara, On Nano semi pre neighbourhoods in NTSs, *Malaya Journal of Mathematics*, 6(1), 2018, 294-298.

[9] D. Vidhya, E. Subha, Nano g*b – closed sets in NTSs, *Journal of Mechanics of Continua and Mathematical Sciences*,7(2), 2020, 139-145.

[10] V. D. Vidhya, R. Parimelazhagan, g*b-closed sets in topological spaces, *Int.J. Contemp. Math. Sciences*, 7, 2012, 1305-1312.