

Analyse of the Free Transverse Vibrations of a Carbon Nanotube Resting on a Winkler Foundation Exposed to a Thermal Load and an Axial Magnetic Field

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Abstract: In this paper, we study the free transverse vibrational behaviour of a nanotube resting on an elastic Winkler foundation when subjected to an axial magnetic field and/or a thermal field. The equation of transverse motion of the nanotube is determined on the basis of non-local Eringen elasticity and Euler-Bernoulli beam theory. By exploiting this equation and the boundary conditions, we determine a homogeneous system whose eigenvalues have been determined using the Newton-Raphson algorithm. In this study, the effects of the non-local parameter, the elastic foundation, the axial magnetic field and the thermal load at high temperature on the linear natural frequencies were examined and identified numerically and/or graphically. At the end of this work, the magnetic field is used to eliminate the decrease in the natural vibration frequencies of a nanotube undergoing a change in temperature. In the course of this work, all the results are compared with those of the existing literature and are in good agreement. Magnetic field strengths, temperature variations, elastic foundation stiffness and nonlocal parameter values are taken from existing literature.

Keywords: Single-Walled Carbon Nanotube, nanobeam, Winkler foundation, Eringen's nonlocal elasticity, magnetic field, thermal loading, vibration, natural Frequencies.

1. Introduction

Thanks to their small size and exceptional mechanical, electrical and thermal properties, nanotubes are the focus of a wide range of scientific research. In the field of vibrations, numerical, theoretical and practical research has been carried out into the various cases (longitudinal vibrations, transverse vibrations, torsional vibrations) with or without the action of physical fields.

Sardar and Reza [1] studied the effects of thermal stress and a magnetic field on the nonlinear transverse vibration of a nanotube resting in an elastic medium. Their results for linear vibration indicate that low temperatures increase natural frequencies, while high temperatures decrease them. Burak E. Yapanmiş and Şevki Akkoca [2] examined the effects of two elastic foundations (linear and nonlinear) and the magnetic field on nonlinear vibration characteristics for two cases: a simply supported nanobeam and an embedded nanobeam. With regard to linear vibrations, they concluded that increasing the elastic foundation coefficient or the magnetic field intensity increases the values of linear natural frequencies. The same study states that increasing the nonlocal parameter leads to a decrease in natural frequencies. A. Benzair and L. Boumia [3] analyzed the

thermal effect on the vibration of a carbon nanotube using the Timoshenko beam model. They deduced that natural frequencies without thermal effect are smaller than those at low ambient temperatures and larger than those at high ambient temperatures. Ismail Esen and A. Eltaher [4] they presented the effects of magnetic and thermal fields on the vibration of a cracked FG microbeam. They found that increasing the temperature decreases the natural frequencies and that this effect is more significant in the presence of cracks. On the other hand the effect of the magnetic field is opposite to that of the temperature. Subrat Kamar and Mohammad Sedighi [5] proposed the results of different theories used to study the vibration of a carbon nanotube exposed to a magnetic field, temperature distributions and variation in humidity concentration. For increasing moisture concentration, they concluded that this reduced the frequency parameters for the different elasticity theories used. Vibrations of double and single walled carbon nanotubes under the influence of thermal and magnetic field are proposed by P. Ponnusamy and A. Amuthalakshmi [6]. They reported graphically the influence of these two physical fields on natural frequencies for different modes and different nonlocal parameters. The effects of magnetic and thermal fields on the vibration of a cracked nanobeam reacting in an elastic medium are also examined by Danilo Karlicic and Milan Cajic [7]. Their study is carried out on clamped-clamped nanobeams and simply supported nanobeams. They discovered that the natural frequencies of the first case are greater and less sensitive to changes in the magnetic and thermal fields than those of the second case. Jingnong Jiang and Lifeng Wang [8] studied the effect of thermal stress and nonlocal parameter on the vibration of a nanobeam with different boundary conditions. M.G. Sobamowo [9] studied the influence of the nonlocal parameter on the linear frequencies of a piezoelectric nanobeam in a low temperature and high temperature environment. He also investigated the combined effect of nonlocal parameter variation and elastic foundation parameter on the nonlinear frequencies of this piezoelectric nanobeam in low temperature and high temperature.

2. Objectives

The aim of this work is to study the variations in the natural vibration frequencies of a nanotube resting on a Winkler elastic foundation in a thermal environment subjected to a magnetic field. By considering the effects of temperature change, the nonlocal coefficient, the stiffness of the elastic foundation and the magnetic field strength as control parameters, this study seeks to give an insight into the behaviour of the linear vibrational frequencies of the nanotube following the variation of one or more of the above-mentioned parameters. At the end of this study, we used the magnetic field to suppress the variations in the natural frequencies of a nanotube suffered following a change in temperature.

3. Methods

Consider a clamped-clamped single-walled carbon nanotube (SWCNT) resting on a Winkler elastic foundation subjected to a thermal load and a magnetic field, as shown in Figure 1.

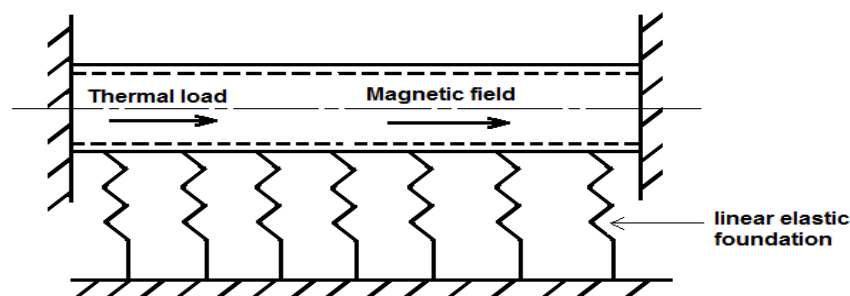


Fig.1. Schematic of a clamped-clamped nanotube resting on a Winkler foundation subjected to thermal loading and magnetic field.

According to the theory of nonlocal elasticity founded by Eringen, the deformation at a point is a function of the stress field at any point of the body. For one-dimensional bodies, the nonlocal equation for axial bending stress is expressed as follows [10]:

$$\sigma_{xx} - (e_0 a)^2 \frac{\partial^2 \sigma_{xx}}{\partial x^2} = E \varepsilon_{xx} \quad (1)$$

Where σ_{xx} is the axial stress, ε_{xx} is the axial strain and E is Young's modulus of the carbon nanotube, e_0 is material constant experimentally determined, a is a characteristic internal length and $(e_0 a)^2$ is the nonlocal parameter.

The relationship between strain and displacement given by Von Karman for a straight beam, assuming small strains, moderate rotations and large transverse displacements, as follows [11]:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} - z \frac{\partial^2 w}{\partial x^2} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \quad (2)$$

Where z -axis is vertical axis and $w(x,t)$ denotes the nanotube transverse deflection. Substituting (2) into (1), multiplying with zdA on both side of Eq. (1) and integrating over the area A of cross-section of the carbon nanotube and we use these following relationships:

$$\int zdA = 0; I = \int z^2 dA; M = \int z \sigma_{xx} dA \quad (3)$$

We obtain:

$$M - (e_0 a)^2 \frac{\partial^2 M}{\partial x^2} = -EI \frac{\partial^2 w}{\partial x^2} \quad (4)$$

Where I is the moment of inertia and M is the bending moment.

The equation of transverse motion of this nanotube can be written as follows [11]:

$$\frac{\partial Q}{\partial x} = \rho A \frac{\partial^2 w}{\partial t^2} + K_w w - F_m \quad (5)$$

Where ρ is the mass density, A is the cross-sectional area, t denotes time, Q is the transverse shear force, K_w is the elastic foundation stiffness and F_m is the magnetic force per unit length.

According to the Euler-Bernoulli beam theory, the relationship linking the shear force Q , the bending moment M and the axial force N is written as follows [11,12]:

$$\frac{\partial Q}{\partial x} = \frac{\partial^2 M}{\partial x^2} + N \frac{\partial^2 w}{\partial x^2} \quad (6)$$

Where N is the axial force exerted on the nanotube. It is given by the following relationship [11]:

$$N = N_t + \frac{EA}{2L} \int_0^L \left(\frac{\partial w}{\partial x} \right)^2 dx \quad (7)$$

Where N_t is the axial force caused by the change in temperature. It is given by the following relation [13]:

$$N_t = -EA \frac{\alpha_x \Delta T}{1 - 2\nu} \quad (8)$$

Where α_x is the coefficient of thermal expansion ν is the Poisson's ratio and ΔT is temperature change.

Substituting Eq. (6) into Eq. (4), we obtain:

$$M = (e_0 a)^2 \left(\frac{\partial Q}{\partial x} - N \frac{\partial^2 w}{\partial x^2} \right) - EI \frac{\partial^2 w}{\partial x^2} \quad (9)$$

Using the expression for M from Eq. (9) in Eq. (4), the dynamic equation for the carbon nanotube will be written as follows:

$$EI \frac{\partial^4 w}{\partial x^4} + \frac{\partial Q}{\partial x} - N \frac{\partial^2 w}{\partial x^2} = (e_0 a)^2 \left(\frac{\partial^3 Q}{\partial x^3} - N \frac{\partial^4 w}{\partial x^4} \right) \quad (20)$$

The introduction of equation (5) into equation (10) with the elimination of non-linear terms, gives us the final expression of the equation governing the transverse vibration movement of the nanotube:

$$\rho A \frac{\partial^2 w}{\partial t^2} + EI \frac{\partial^4 w}{\partial x^4} + k_w w - F_m + N_t \frac{\partial^2 w}{\partial x^2} - (e_0 a)^2 \left(\rho A \frac{\partial^2 w}{\partial t^2 \partial x^2} + k_w \frac{\partial^2 w}{\partial x^2} - F_m \frac{\partial^2 w}{\partial x^2} + N_t \frac{\partial^4 w}{\partial x^4} \right) = 0 \quad (31)$$

The magnetic force per unit length is expressed as follows [14]:

$$F_m = \eta A H_x^2 \frac{\partial^2 w}{\partial x^2} \quad (12)$$

Where η is the magnetic field permeability and H_x is the longitudinal magnetic field.

With the following non-dimensional parameters:

$$X = \frac{x}{L}; W = \frac{w}{L}; T = \frac{t}{L^2} \sqrt{\frac{EI}{\rho A}}; \mu = \frac{e_0 a}{L}; K_w = \frac{k_w L^4}{EI}; N_{temp} = \frac{N_t L^2}{EI}; H_m = \frac{F_m L^2}{EI} \quad (13)$$

We obtain the non-dimensional equation of motion:

$$\frac{\partial^2 W}{\partial T^2} + \frac{\partial^4 W}{\partial X^4} + K_w W + N_{temp} \frac{\partial^2 W}{\partial X^2} - H_m \frac{\partial^2 W}{\partial X^2} - \mu^2 \left(\frac{\partial^4 W}{\partial T^2 \partial X^2} + K_w \frac{\partial^2 W}{\partial X^2} + N_{temp} \frac{\partial^4 W}{\partial X^4} - H_m \frac{\partial^4 W}{\partial X^4} \right) = 0 \quad (14)$$

Solving this equation requires the use of the separation of variables method, which leads us to write:

$$W(X, T) = W(X) e^{i\omega T} \quad (15)$$

Introducing equation (15) into equation (14), we obtain the transverse displacement of the nanotube, which can be expressed as follows [15, 16]:

$$\frac{\partial^4 W(X)}{\partial X^4} + \eta \frac{\partial^2 W(X)}{\partial X^2} - \gamma^4 W(X) = 0 \quad (16)$$

Where:

$$\gamma^4 = \frac{\omega^2 - K_w}{1 + (H_m - N_{temp})\mu^2}, \quad \eta = \frac{\mu^2(\omega^2 - K_w) + N_{temp} - H_m}{1 + (H_m - N_{temp})\mu^2} \quad (17)$$

Solving equation (16) leads to a linear modal form, which can be expressed as [17, 18]:

$$W_i(X) = C_1 \sin\left(\sqrt{\left(\frac{1}{2}\eta + \frac{1}{2}\sqrt{\eta^2 + 4\gamma_i^4}\right)}(X)\right) + C_2 \cos\left(\sqrt{\left(\frac{1}{2}\eta + \frac{1}{2}\sqrt{\eta^2 + 4\gamma_i^4}\right)}(X)\right) + C_3 \sinh\left(\sqrt{\left(-\frac{1}{2}\eta + \frac{1}{2}\sqrt{\eta^2 + 4\gamma_i^4}\right)}(X)\right) + C_4 \cosh\left(\sqrt{\left(-\frac{1}{2}\eta + \frac{1}{2}\sqrt{\eta^2 + 4\gamma_i^4}\right)}(X)\right) \quad (18)$$

For i varies from 1 to mode number N .

We study a clamped-clamped nanotube. The boundary conditions give the following relationships:

$$\begin{aligned} W(0) &= 0, & W'(0) &= 0 \\ W(1) &= 0, & W'(1) &= 0 \end{aligned} \quad (19)$$

The system of eq. (18) offers the possibility of calculating the natural frequencies, by looking for a non-trivial solution to find the solutions where the determinant of the matrix of the system of eq. (20) cancels, namely:

$$\det(Y) = 0 \quad (20)$$

Where Y is the matrix of the system found after injecting the boundary conditions (19) into equation (18).

4. Results and discussion

In order to compare the results obtained in this work with those obtained by other researchers such as Farzad Ebrahimi and Fatemeh Mahmoodi [21], we carried out a parametric study using the following values for the dimensionless parameters: $E = 1$, $I = 1$, $\rho = 1$, $L = 1$ and $\nu = 0.3$. The results of the non-dimensional frequencies from this study are listed below and compared with those from other studies. These comparisons show that our results are in good agreement with previous studies.

In this work, we began by analysing the effects of varying the nonlocal parameter on the natural frequencies, then we studied the effects of varying the other parameters (elastic foundation, magnetic field and thermal load) with a fixed or variable nonlocal parameter, and finally we combined the effects of the magnetic field and the thermal load by fixing the other two parameters. The results obtained are reported numerically in the form of tables or graphically by curves. The coefficient of thermal expansion is $\alpha_x = 1.1 \times 10^{-6}$ for high temperature [1, 21].

Effect of the nonlocal parameter

In Table 1, we studied the effect of the variation of the nonlocal parameter on the natural frequencies of transverse vibration of a clamped-clamped carbon nanotube without magnetic field or thermal charge. We find that the increase in the non-local parameter decreases the natural frequencies. This means that the non-local effect reduces the stiffness of the material.

Table 1. Dimensionless natural frequencies for different dimensionless nonlocal parameters (μ).

μ	Mode			
	source	1	2	3
0	Present	22.373285	61.672822	120.903391
	Ref.[21]	22.3733	61.6728	120.903
0.1	Present	21.109034	50.983166	85.716433
	Ref.[21]	-----	-----	-----
0.2	Present	18.289416	36.423927	54.523961
	Ref.[20]	18.2894	36.4239	54.5240
0.3	Present	15.353604	27.001656	38.833960
	Ref.[20]	-----	-----	-----
0.4	Present	12.904730	21.139827	29.9624261
	Ref.[20]	12.9047	21.1398	29.9624

0.5	Present	10.991367	17.272969	24.332388
	Ref.[21]	10.9910	17.2730	24.3320
0.6	Present	9.509194	14.565570	20.458870
	Ref.[20]	9.5092	14.5656	20.4589
0.7	Present	8.348283	12.575842	17.636946
	Ref.[20]	-----	-----	-----
0.8	Present	7.423595	11.056481	15.492388
	Ref.[20]	7.4236	11.0565	15.4924
0.9	Present	6.674087	9.860423	13.808899
	Ref.[20]	-----	-----	-----
1	Present	6.056555	8.895434	12.453027
	Ref.[21]	6.05656	8.89543	12.453

Effect of the rigidity of the elastic foundation

To analyse the effect of elastic foundation stiffness, we ignored the effects of the magnetic field and thermal load. The first three dimensionless natural frequencies are given and compared with previous studies in table 2 for different dimensional foundation stiffnesses and values of the nonlocal parameter ranging from 0 to 0.5. This allowed us to conclude that increasing K_w leads to an increase in natural frequencies. This conclusion can be quickly drawn by a simple interpretation of Figure 2, which shows the variation of the first natural frequency as a function of the nonlocal parameter and the rigidity of the Winkler foundation. This figure also shows that increasing the nonlocal parameter decreases the first natural frequency of the nanotube studied.

Table 2. The first three dimensionless natural frequencies of a clamped-clamped nanotube, for different dimensionless foundation stiffness and different nonlocal parameters.

Kw	μ	source	Mode		
			1	2	3
0	0	Present	22.373285	61.672822	120.903391
		Ref.[20]	22.3733	61.6728	120.9034
	0.1	Present	21.109034	50.983166	85.716433
		Ref.[20]	-----	-----	-----
	0.2	Present	18.289416	36.423927	54.523961
		Ref.[20]	18.2894	36.4239	54.5240
	0.3	Present	15.353604	27.001656	38.833960
		Ref.[20]	-----	-----	-----
	0.4	Present	12.904730	21.139827	29.9624261
		Ref.[20]	12.9047	21.1398	29.9624
	0.5	Present	10.991367	17.272969	24.332388

	Ref.[20]	-----	-----	-----
1	0	Present	22.395622	61.680929
		Ref.[19]	22.3956	61.6809
	0.1	Present	21.132707	50.992972
		Ref.[19]	21.1327	50.993
	0.2	Present	18.316734	36.437652
		Ref.[19]	18.3167	36.4377
	0.3	Present	15.386136	27.020167
		Ref.[19]	15.3861	27.0202
	0.4	Present	12.943418	21.163466
		Ref.[19]	12.9434	21.1635
10	0.5	Present	11.036763	17.301892
		Ref.[19]	11.0368	17.3019
	0	Present	22.595661	61.753842
		Ref.[19]	22.5957	61.7538
	0.1	Present	21.344585	51.081143
		Ref.[19]	21.3446	51.0811
	0.2	Present	18.560785	36.560942
		Ref.[19]	18.5608	36.5609
	0.3	Present	15.675879	27.186199
		Ref.[19]	15.6759	27.1862
50	0.4	Present	13.286537	21.375039
		Ref.[19]	13.2865	21.375
	0.5	Present	11.437226	17.560053
		Ref.[19]	11.4372	17.5601
	0	Present	23.464098	62.076864
		Ref.[19]	23.4641	62.0769
	0.1	Present	22.261880	51.471188
		Ref.[19]	22.2619	51.4712
	0.2	Present	19.608741	37.103941
		Ref.[19]	19.6087	37.1039
	0.3	Present	16.903644	27.912174
		Ref.[19]	16.9036	27.9122
	0.4	Present	14.715028	22.291081

	0.5	Ref.[19]	14.715	22.2911	30.7855
		Present	13.069436	18.664283	25.339003
		Ref.[19]	13.0694	18.6643	25.339
100	0	Present	24.506405	62.478292	121.316240
		Ref.[19]	24.5064	62.4783	121.316
	0.1	Present	23.357896	51.954626	86.297780
		Ref.[19]	23.3579	51.9546	86.2978
	0.2	Present	20.844729	37.771715	55.433405
		Ref.[19]	20.8447	37.7717	55.4334
	0.3	Present	18.323023	28.793913	40.100828
		Ref.[19]	18.323	28.7939	40.1008
	0.4	Present	16.325810	23.385728	31.587133
		Ref.[19]	16.3258	23.3857	31.5871
	0.5	Present	14.859682	19.958844	26.307130
		Ref.[19]	14.8597	19.9588	26.3071
200	0	Present	26.468167	63.273510	121.727688
		Ref.[19]	26.4682	63.2735	121.7278
	0.1	Present	25.408489	52.908253	86.875237
		Ref.[19]	25.408489	52.9083	86.8752
	0.2	Present	23.119315	39.073040	56.328167
		Ref.[19]	23.1193	39.073	56.3282
	0.3	Present	20.874222	30.480968	41.328881
		Ref.[19]	20.8742	30.481	41.3289
	0.4	Present	19.145027	25.434077	33.132264
		Ref.[19]	19.145	25.4341	33.1323
	0.5	Present	17.911174	22.323876	28.143651
		Ref.[19]	17.9112	22.3239	28.1437
500	0	Present	31.631691	65.601349	122.953772
		Ref.[19]	31.6317	65.6013	122.954
	0.1	Present	30.750468	55.671206	88.5850264
		Ref.[19]	30.7505	55.6712	88.585
	0.2	Present	28.887761	42.739940	58.930996
		Ref.[19]	28.8878	42.7399	58.931
	0.3	Present	27.124402	35.058372	44.811566

	Ref.[19]	27.1244	35.0584	44.8116
0.4	Present	25.817282	30.771615	37.386454
	Ref.[19]	25.8173	30.7716	37.3865
0.5	Present	24.916062	28.255185	33.046408
	Ref.[19]	24.9161	28.2552	33.0464

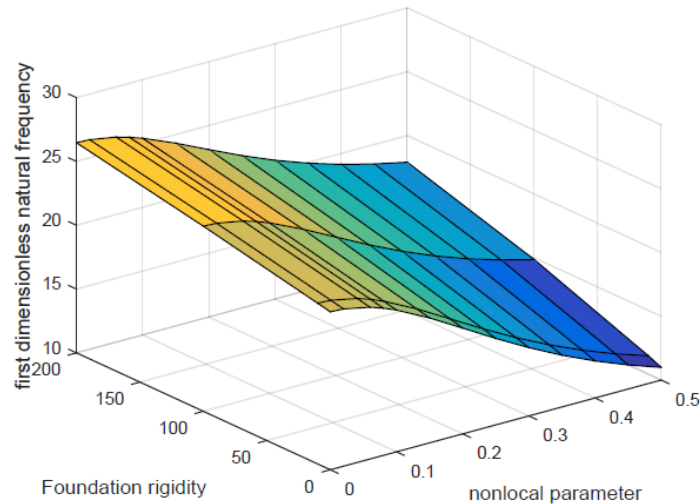


Fig.2. Representation of the variation in the first fundamental frequency of the clamped-clamped carbon nanotube as a function of the variations of the elastic foundation stiffness and the nonlocal parameter when $H_m = 0$ and $\Delta T = 0$

Effect of the thermal load:

In table 3, we have studied the effects of the variation of the thermal load at high temperature on the natural frequencies of the clamped-clamped nanotube without magnetic load and with or without elastic foundation for three values of the nonlocal parameter (0, 0.5 and 1). The results obtained show that a positive temperature variation at high temperatures decreases the natural frequencies of the nanotube. This means that high temperatures reduce the rigidity of the nanotube.

Table 3. The first three dimensionless natural frequencies of a clamped-clamped nanotube resting on Winkler foundation and exposed to variable thermal field at high temperature without magnetic fields ($H_m = 0$).

Kw	ΔT	μ	source	Mode		
				1	2	3
0	0	0	Present	22.373285	61.672822	120.903391
			Ref.[21]	22.3733	61.6728	120.903
	0.5	0.5	Present	10.991367	17.272969	24.332388
			Ref.[21]	10.9914	17.273	24.3324
	1	1	Present	6.056555	8.895434	12.453027
			Ref.[21]	6.05656	8.89543	12.453

100	0	Present	22.373209	61.672720	120.903279
		Ref.[21]	32.3730	61.6724	120.903
	0.5	Present	10.990951	17.272346	24.331530
		Ref.[21]	10.9897	17.2705	24.3290
	1	Present	6.055701	8.894196	12.451304
		Ref.[21]	6.05318	8.89054	12.4462
200	0	Present	22.373134	61.672617	120.903166
		Ref.[21]	22.3727	61.6720	120.903
	0.5	Present	10.990535	17.271723	24.330672
		Ref.[21]	10.9881	17.268	24.3256
	1	Present	6.054847	8.892957	12.449580
		Ref.[21]	6.0498	8.88564	12.4394
300	0	Present	22.373058	61.672514	120.903054
		Ref.[21]	-----	-----	-----
	0.5	Present	10.990119	17.271100	24.329814
		Ref.[21]	-----	-----	-----
	1	Present	6.053993	8.891719	12.447856
		Ref.[21]	-----	-----	-----
10	0	Present	22.595661	61.753842	120.944739
		Ref.[19]	22.5957	61.7538	120.945
	0.5	Present	11.437226	17.560053	24.537015
		Ref.[19]	11.4372	17.5601	24.537
	1	Present	6.832412	9.440802	12.848264
		Ref.[19]	-----	-----	-----
100	0	Present	22.595586	61.753740	120.944627
	0.5	Present	11.436827	17.559440	24.536164
	1	Present	6.831655	9.439635	12.846593
200	0	Present	22.595511	61.753637	120.944515
	0.5	Present	11.436427	17.558827	24.535313
	1	Present	6.830898	9.438469	12.844923
300	0	Present	22.595436	61.753535	120.944402
	0.5	Present	11.436027	17.558214	24.534463
	1	Present	6.830141	9.437301	12.843252

The figure 3 shows the effect of an increase in temperature on the dynamic behaviour of the nanotube studied. It is clear that this decreases the first natural frequency of vibration of the nanotube. The figure also shows that this law of variation is linear. This result is in good agreement with the result of Sardar and Reza [1].

The figure 4 shows that the conclusions drawn for the first mode are valid for the other two modes of vibration. A law that can be generalised to all the modes of vibration of the nanotube studied in this work.

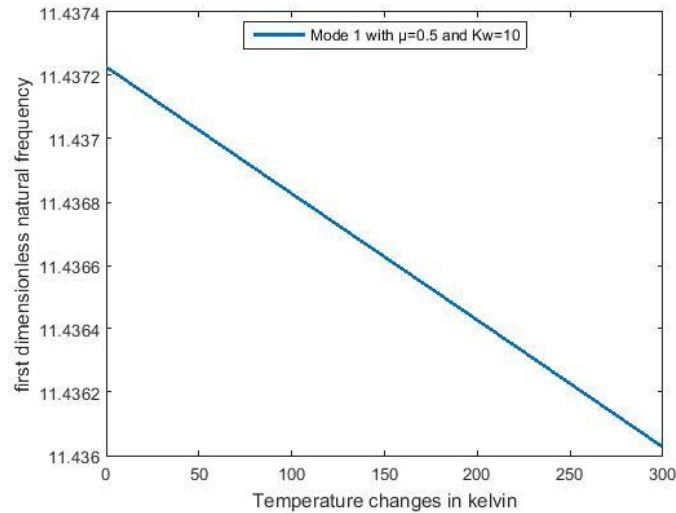


Fig.3. First dimensionless linear frequency of clamped-clamped nanotube as a function of temperature variations for $\mu = 0.5$ and $K_w = 10$

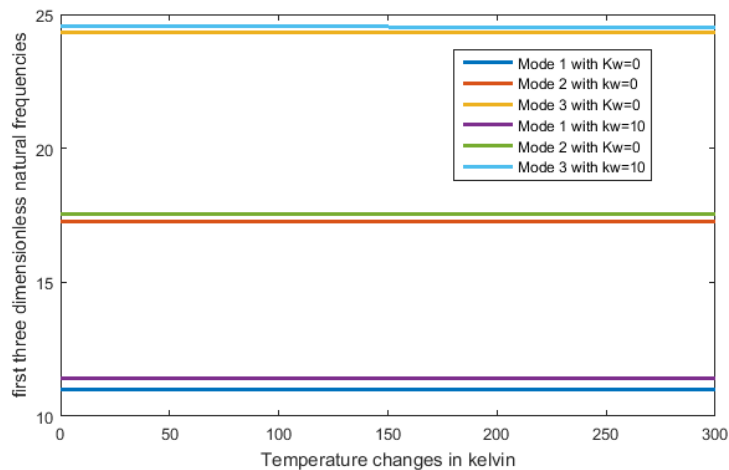


Fig.4. First three dimensional linear frequencies of the clamped-clamped nanotube as a function of temperature variations for $\mu = 0.5$ with or without elastic foundation

Effect of the magnetic load

In table 4, we have studied the effect of varying the magnetic field on the natural frequencies of transverse vibration of the clamped-clamped carbon nanotube at room temperature without or with an elastic foundation of dimensionless rigidity $K_w = 10$ for three values of the nonlocal parameter ($\mu = 0$, $\mu = 0.5$ and $\mu = 1$).

We find that the natural frequencies of the nanotube increase with increasing longitudinal magnetic field intensity. We can therefore conclude that increasing the intensity of the magnetic field increases the rigidity of

our nanotube. This result is consistent with those of other authors such as Burak E. Yapanmis and Sevki Akkoca [2].

Table 4. The first three dimensionless natural frequencies of a clamped-clamped carbon nanotube on a Winkler foundation ($K_w = 10$) subjected to the effect of a magnetic field without thermal loading for three values of the the nonlocal parameter ($\mu = 0$, $\mu = 0.5$ and $\mu = 1$).

Kw	Hm	μ	source	Mode		
				1	2	3
10	0	0	Present	22.595661	61.753842	120.944739
			Ref.[19]	22.5957	61.7538	120.945
		0.5	Present	11.437226	17.560053	24.537015
			Ref.[19]	11.4372	17.5601	24.537
		1	Present	6.832412	9.440802	12.848264
			Ref.[19]	-----	-----	-----
	1	0	Present	22.866110	62.125495	121.352918
			Ref.[2]	-----	-----	-----
		0.5	Present	12.808811	19.663107	27.456435
			Ref.[2]	12.8088	19.663	27.456
		1	Present	9.181104	13.009122	17.920336
			Ref.[2]	-----	-----	-----
15	0	0	Present	26.334949	67.097868	126.926138
			Ref.[2]	-----	-----	-----
		0.5	Present	25.092562	38.504413	53.655669
			Ref.[2]	25.092	38.504	53.655
		1	Present	24.715258	35.927098	50.059829
			Ref.[2]	-----	-----	-----
	30	0	Present	29.559490	72.018748	132.630207
			Ref.[2]	-----	-----	-----
		0.5	Present	33.592578	51.544389	71.803885
			Ref.[2]	33.592	51.544	71.803
		1	Present	34.278362	49.923781	69.619646
			Ref.[2]	-----	-----	-----

In Figure 5, we show the effect of varying the nonlocal parameter on the nanotube natural frequency by plotting the natural frequency (ω) as a function of this parameter (μ) for five magnetic field strengths. Over a range of variation in μ from 0 to 0.5, we find that increasing the nonlocal parameter decreases the first natural frequency, but that with increasing magnetic field strength, this decreasing variation in natural frequencies can be reversed

to give an increasing frequency evolution. The figure also shows that the increase in the magnetic field effect increases the first natural frequency.

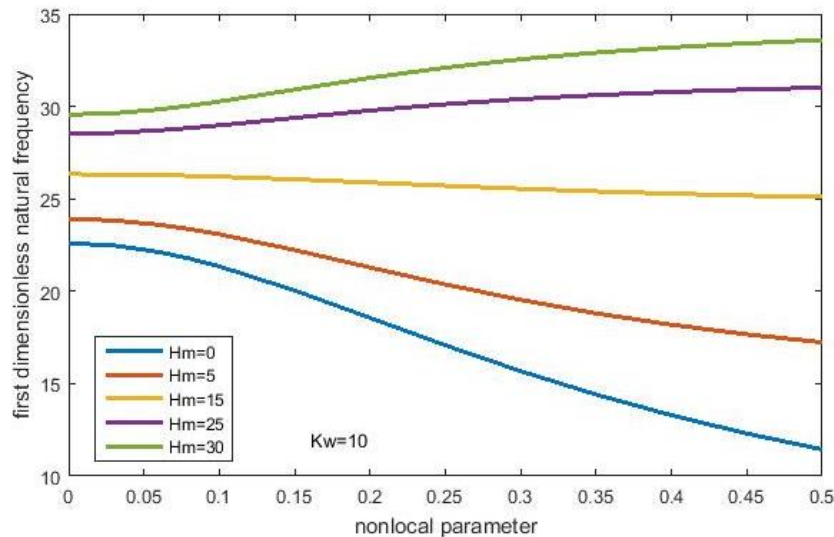


Fig.5. The effect of the magnetic load H_m on the first natural frequency of the nanotube for different values of the nonlocal parameter and for $K_w = 10$

Vibratory behaviour of the clamped-clamped carbon nanotube under the combined effects of a thermal load and a magnetic field

In the figure 6, we have plotted the variation of the first fundamental frequency of the clamped carbon nanotube as a function of the variation of the dimensionless magnetic field intensity and the change in temperature for a non-local parameter $\mu = 0.5$ and a dimensionless stiffness $K_w = 10$ of the elastic foundation.

Analysis of this figure shows that an increase in the magnetic field leads to an increase in the natural frequency and that a positive change of 300 K in temperature compared with ambient temperature leads to a slight decrease in this frequency. We also noticed that at a fixed magnetic field, the frequency varies linearly with the change in temperature. This result is illustrated in figure 7. The same figure we have shown through a plot that the magnetic field can be used to keep the natural frequencies constant despite the change in temperature. this comes down to the fact that the normal force caused by the thermal charge and that caused by the magnetic field have opposite signs in the equation governing the movement (equation 14).

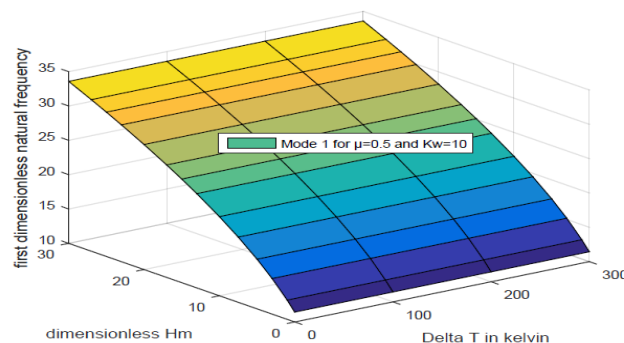


Fig.6. Representation of the variation in the first fundamental frequency of the clamped-clamped carbon nanotube as a function of the variations of the dimensionless magnetic field effect (H_m) and the temperature (Delta T) change for $\mu = 0.5$ and $K_w = 10$

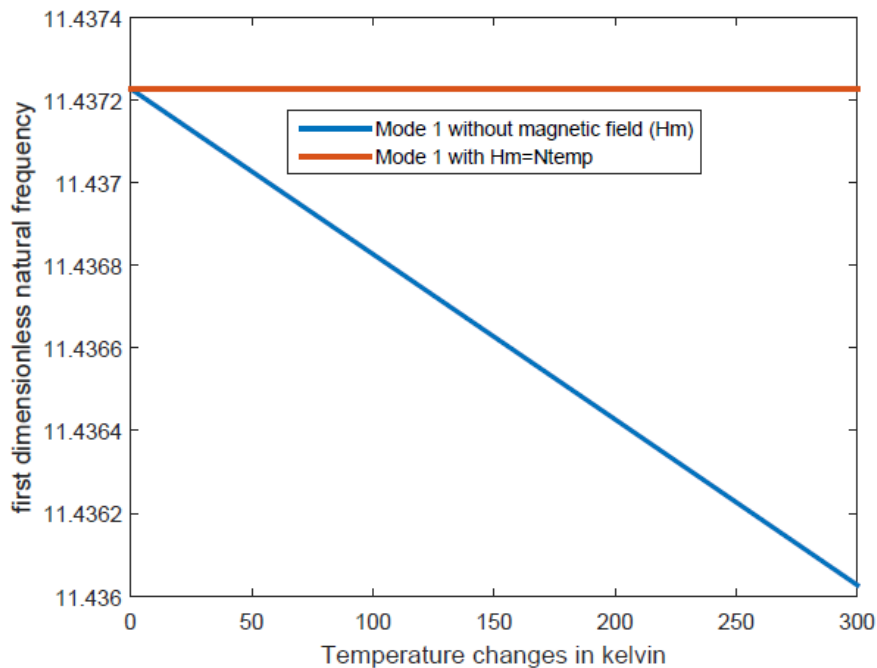


Fig.7. Representation of the variation in the first fundamental frequency of the clamped-clamped carbon nanotube as a function of the variation of the temperature for $H_m = 0$ and $H_m = N_{temp}$

5. Conclusion

In this paper, we work on a clamped-clamped nanotube resting in an elastic Winkler foundation. This nanotube is considered exposed to a magnetic field and/or a thermal loading. The aim is to study the effects of the magnetic field, the temperature change (at high temperature), the rigidity of the foundation elastic and the change of the nonlocal parameter, on the variations in natural frequencies. The equation of motion of our studied nanotube was obtained by applying nonlocal elasticity and Euler-Bernoulli beam theory. The transverse motion is assumed to be harmonic and boundary conditions of the proposed model have been introduced to find a homogeneous system whose determinant produces a nonlinear equation solved numerically by the Newton-Raphson iteration method.

The results obtained in our work demonstrated an excellent level of concordance with those obtained from the literature, confirming the reliability of our resolution process.

Based on this work, we can draw the following conclusions:

- An increase in the strength of the magnetic field leads to an increase in the natural frequencies.
- In the high-temperature range, the increase in temperature leads to a reduction in the natural frequencies of the nanotube under study.
- The negative effect of an increase in temperature on the natural vibration frequencies of the nanotube can be eliminated by applying a magnetic field.
- For nanotubes resting on an elastic foundation, the nonlocal effect reduces the rigidity of the material and that of the foundation increases the rigidity of the structure.
- The nonlocal parameter can have a considerable effect on the variation of the natural frequency. It is therefore important to use the theory of nonlocal elasticity to study the vibrations of nanotubes (nanobeams).

In summary, this work allowed us to deepen our knowledge of the behavior of nanotubes and their physical properties. Through this modest work we wish to contribute to the enrichment of the field of nanostructure mechanics.

References

- [1] Abdullah Sardar S., Shahrokh Hosseini-Hashemi, Nazhad A. Hussein and Reza Nazemnezhad: Thermal Stress and Magnetic Effects on Nonlinear Vibration of Nanobeams Embedded in Nonlinear Elastic Medium. *Journal of Thermal Stresses* 43(10): 1316–1332 (2020).
- [2] Burak E. Yapanmis, Necla Togun, Suleyman M. Bagdatli and Sevki Akkoca: Magnetic Field Effect on Nonlinear Vibration of Nonlocal Nanobeam Embedded in Nonlinear Elastic Foundation. *Structural Engineering and Mechanics* 79 (6): 723–35 (2021).
- [3] A. Benzair, A. Tounsi, A. Besseghier, H. Heireche, N. Moulay and L. Boumia: The Thermal Effect on Vibration of Single-Walled Carbon Nanotubes Using Nonlocal Timoshenko Beam Theory. *Journal of Physics D: Applied Physics* 41 (22): 225404 (2008).
- [4] Ismail Esen, Cevat Ozarpa and Mohamed A. Eltaher: Free Vibration of a Cracked FG Microbeam Embedded in an Elastic Matrix and Exposed to Magnetic Field in a Thermal Environment. *Composite Structures* 261 (April): 113552 (2021).
- [5] Subrat Kumar Jena, S. Chakraverty, Mohammad Malikan and Hamid Mohammad-Sedighi: Hygro-Magnetic Vibration of the Single-Walled Carbon Nanotube with Nonlinear Temperature Distribution Based on a Modified Beam Theory and Nonlocal Strain Gradient Model. *International Journal of Applied Mechanics* 12 (05): 2050054 (2020).
- [6] P. Ponnusamy and A. Amuthalakshmi: Influence of Thermal and Magnetic Field on Vibration of Double Walled Carbon Nanotubes Using Nonlocal Timoshenko Beam Theory. *Procedia Materials Science*, 2nd International Conference on Nanomaterials and Technologies (CNT 2014), 10 (January): 243–53 (2015).
- [7] Danilo Karličić, Dragan Jovanović, Predrag Kozić and Milan Cajić: Thermal and Magnetic Effects on the Vibration of a Cracked Nanobeam Embedded in an Elastic Medium. *Journal of Mechanics of Materials and Structures* 10 (February): 43–62 (2025).
- [8] Jiang Jingnong and Lifeng Wang: Analytical Solutions for Thermal Vibration of Nanobeams with Elastic Boundary Conditions. *Acta Mechanica Solida Sinica* 30 (5): 474–83 (2017).
- [9] M. G. Sobamowo: Analysis of Nonlinear Vibration of Piezoelectric Nanobeam Embedded in Multiple Layers Elastic Media in a Thermo-Magnetic Environment Using Iteration Perturbation Method. *Journal of Solid Mechanics* 14 (2): 221–51 (2022).
- [10] Valipour P., Ghasemi S. E., Mohammad R. K. and Ganji D. D.: Theoretical analysis on nonlinear vibration of fluid flow in single walled carbon nanotube. *Journal of Theoretical and Applied physics* 10(3): 211-218 (2016).
- [11] Zheng Lyu, Yaowen Yang and Hu Liu: High-Accuracy Hull Iteration Method for Uncertainty Propagation in Fluid-Conveying Carbon Nanotube System under Multi-Physical Fields. *Applied Mathematical Modelling* 79 (March), 362-380 (2020).
- [12] Soltani P. And A. Farshidianfar.: Periodic solution for nonlinear vibration of a fluid-conveying carbon nanotube, based on the nonlocal continuum theory by energy balance method. *Applied Mathematical Modelling* 36 (8): 3712-24 (2012).
- [13] T. Murmu and S. C. Pradhan: Thermo-mechanical vibration of a single-walled carbon nanotube embedded in an elastic medium based on nonlocal elasticity theory. *Computational Materials Science* 46(4), 854–859 (2009).
- [14] T. Murmu, M. A. McCarthy and S. Adhikari: Nonlocal elasticity based magnetic field affected vibration response of double single-walled carbon nanotube Systems. *Journal of Applied Physics* 111 (11): 113511 (2012).
- [15] El khoudar Y., Adri A., Outassafte O., El hantati I., Rifai S., and Benamar R.: Analysis of geometrically nonlinear free vibrations of functional graded beams in a thermal environment. *Ecmmas Proceedings COMPDYN 2021*, pp. 5191-5200. Athens, Greece (2021).
- [16] El khoudar Y., Adri A., Outassafte O., Rifai S. and Benamar R.: An analytical approach to geometrically nonlinear free and forced vibration of piezoelectric functional gradient beams resting on elastic foundations in thermal environments. *Mechanics of advanced Materials and Structures* 30(1), 131-143 (2023).

- [17] El Khouddar Yassine, Ahmed Adri, Omar Outassafte, Issam El Hantati, Said Rifai and Rhali Benamar: Influence of Hygro-Thermal Effects on the Geometrically Nonlinear Free and Forced Vibrations of Piezoelectric Functional Gradient Beams with Arbitrary Number of Concentrated Masses. *Archive of Applied Mechanics* 92 (9): 2767–84 (2022).
- [18] El Khouddar Y., A. Adri, O. Outassafte, S. Rifai, and R. Benamer.: Non-Linear Forced Vibration Analysis of Piezoelectric Functionally Graded Beams in Thermal Environment. *International Journal of Engineering* 34 (11): 2387-2397 (2021).
- [19] Togun Necla and Süleyman Murat Bağdatlı.: Nonlinear Vibration of a Nanobeam on a Pasternak Elastic Foundation Based on Non-Local Euler-Bernoulli Beam Theory. *Mathematical and Computational Applications* 21 (1), (2016).
- [20] Suleyman M. Bagdatli: Non-Linear Transverse Vibrations of Tensioned Nanobeams Using Nonlocal Beam Theory. *Structural Engineering & Mechanics* 53 (July): 281, (2015).
- [21] Ebrahimi Farzad and Fatemeh Mahmoodi: Vibration Analysis of Carbon Nanotubes with Multiple Cracks in Thermal Environment. *Advances in Nano Research* 6 (1): 57–80, (2018).