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Thermal effect on vibration of clamped visco-elastic Elliptic plate with parabolic thickness variation in both directions

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Abstract

The analysis presented here is to study the effect of thermal gradient on the vibration of visco-elastic elliptical plate (having clamped boundary condition on all the four edges) of variable thickness whose thickness varies parabolically in both directions. The effect of linear temperature variation has been considered. A frequency equation of plate has been obtained by Rayleigh-Ritz technique with two terms of deflection function. The assumption of small deflection and linear visco-elastic properties of 'Kelvin' type are taken. Frequency corresponding to the first two modes of vibrations for clamped plate have been computed for various combinations of aspect ratio, thermal constants, and taper constants, non-homogeneity constant. Numerical computations have been performed for an alloy 'Duralium' and the results obtained are depicted graphically.

Keywords: Thermal effect, vibration, visco-elastic, elliptic plate, parabolically thickness, both directions.

1. Introduction

In the course of time, the study of vibration of plates has acquired great importance in the field of research, engineering and space technology. The visco-elastic behaviors of some materials invigorated scientists for modern designs and analysis techniques and their application to many practical problems. As technology develops new discoveries have intensified the need for solution of various problems of vibrations of plates with elastic or visco-elastic medium. Since new materials and alloys are in great use in the construction of technically designed structures therefore the application of visco-elasticity is the need of the hour. Tapered plates are generally used to model the structures. Plates with thickness variability are of great importance in a wide variety of engineering applications. Thermal induced vibrations of plates of variable thickness are very useful for research workers in nuclear and chemical engineering. The investigations have resulted in better designing of machine parts and structures subjected to cyclic loading such as gas turbines, jet engine, aircrafts and nuclear power plants. A study of the literature on vibration problems shows that the visco-elastic plates with thickness variation in two directions has received rather less attention than that of in one direction. Larrondo et al. [10] studied vibrations of simply supported rectangular plates with varying thickness and same aspect ratio cutouts. Free vibrations of rectangular plates of parabolically varying thickness have been investigated by Jain and Soni [9]. Bhatnagar and Gupta [3] discussed thermal effect on vibration of viscoelastic elliptic plate of variable thickness. Laura, Grossi, and Carneiro [11] studied transverse vibrations of rectangular plates with thickness varying in two directions and with edges elastically restrained against rotation. Singh and Saxena [18] worked on transverse vibration of rectangular plate with bi-directional thickness variation. Free vibration of visco-elastic orthotropic rectangular plates was studied by Sobotka [19]. Sharma [16] worked on some vibration problems of orthotropic plates and shells. The effect of thermal gradient on the frequencies of an orthotropic plate of linearly varying thickness has been discussed by Tomar and Gupta [22]. Taylor and Govindjee [20] gave solution of clamped rectangular plate problems. Vibration of rectangular plates by the Ritz method was given by Young [23]. Rossi [15] solved the problem of transverse vibrations of thin, orthotropic rectangular plates with rectangular cutouts with fixed boundaries. Vibration analysis of viscoelastic circular plate subjected to thermal gradient was given by Bhatnagar and Gupta [4]. Cox and Boxer [5] studied the vibration of rectangular plate of uniform thickness with all four-corner point supported. The fundamental frequency of transverse vibration of a clamped rectangular plate of cylindrical orthotropic has been investigated by Bombill and Laura [1]. Filipich et al. [6] considered the rectangular plates with two opposite edges simply supported and the other two with very general boundary conditions and studied the vibrations of such rectangular plates of variable thickness. Tomar and Gupta [21] discussed the effect of thermal gradient on frequencies of an orthotropic rectangular plate whose thickness varies in two directions. Nowacki [14] discussed thermo elasticity in his book. Leissa [12] given the detailed studied on the vibration of plates on his monogram. Nagaya [13] discussed the vibrations and dynamic response of visco-elastic plates on non-periodic elastic supports. Gupta and Khanna [8] solved the problem of vibration of visco-elastic rectangular plate with linearly thickness variations in both directions. Recently, Gupta and Khanna [7] analyzed the vibration of clamped visco-elastic rectangular plate with parabolic thickness variations. Singh and Chakarverty [17] solved the problem for vibration of skew plates using characteristic orthogonal polynomials. Bhat [2] analyses the natural frequencies of rectangular plates using characteristic orthogonal polynomials in Rayliegh- Ritz method.

The aim of present investigation is to study the effect of thermal gradient on the vibration of a clamped visco-elastic elliptic plate whose thickness varies parabolically in both directions. Here authors introduced the effect of thermal gradient on their work [7]. It is assumed that the plate is clamped on all the four edges and its temperature varies linearly. The basic differential equation of transverse motion of a visco-elastic elliptic plate of variable thickness is taken from Bhatnagar and Gupta [4]. To determine the frequency equation, Rayleigh-Ritz technique has been used. It is also assumed that deflection is small and linear visco-elastic properties are of 'Kelvin' type, having one spring and one dashpot in parallel. For various values of thermal gradients, aspect ratio and taper constants, time period and deflection at different points for the first two modes of vibration are evaluated. Results are displayed with graphs. All the material parameters used in numerical calculations have been taken for alloy 'Duralium'.

2. Equation of motion and constitutive relations

The equation of transverse motion of a visco-elastic plate of variable thickness, taken in Cartesian co-ordinates, governing density ρ, thickness of plate h, deflection w and time t is given by Bhatnagar and Gupta [4]:

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{yx}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = \rho h \frac{\partial^2 w}{\partial t^2} \tag{1}$$

The expressions for M_x, M_y, M_{yx} are given by

$$\left.\begin{aligned} M_x &= -\widetilde{D} D_1 \left(\frac{\partial^2 w}{\partial x^2} + \vartheta \frac{\partial^2 w}{\partial y^2}\right) \\ M_y &= -\widetilde{D} D_1 \left(\frac{\partial^2 w}{\partial y^2} + \vartheta \frac{\partial^2 w}{\partial x^2}\right) \\ M_{yx} &= -\widetilde{D} D_1 (1-\vartheta) \frac{\partial^2 w}{\partial x \partial y} \end{aligned}\right\} \tag{2}$$

where,
M_x = Bending moments along x-axis.
M_y =Bending moments along y-axis.
M_{yx} =Twisting moments.
On substitution the values M_x , M_y and M_{yx} from (2) in (1) and taking w, as a product of two function, equal to w(x,y,t) = W(x,y)T(t), (1) becomes:

$$[\,D_1\left(\frac{\partial^4 W}{\partial x^4}+2\frac{\partial^4 W}{\partial x^2\,\partial y^2}+\frac{\partial^4 W}{\partial y^4}\right)+2\frac{\partial D_1}{\partial x}\left(\frac{\partial^3 W}{\partial x^3}+\frac{\partial^3 W}{\partial x\,\partial y^2}\right)+2\frac{\partial D_1}{\partial y}\left(\frac{\partial^3 W}{\partial y^3}+\frac{\partial^3 W}{\partial x^2\,\partial y}\right)+\frac{\partial^2 D_1}{\partial x^2}\left(\frac{\partial^2 W}{\partial x^2}+\vartheta\frac{\partial^2 W}{\partial y^2}\right)+\frac{\partial^2 D_1}{\partial y^2}\left(\frac{\partial^2 W}{\partial y^2}+\vartheta\frac{\partial^2 W}{\partial x^2}\right)+2(1-\vartheta)\frac{\partial^2 D_1}{\partial x\,\partial y}\frac{\partial^2 W}{\partial x\,\partial y}]/\rho h w=-\frac{\ddot{T}}{\widetilde{D}T} \quad (3)$$

Here dot denote differentiation with respect to t, taking both sides of (2.3) are equal to a constant p^2, we have

$$[\,D_1\left(\frac{\partial^4 W}{\partial x^4}+2\frac{\partial^4 W}{\partial x^2\,\partial y^2}+\frac{\partial^4 W}{\partial y^4}\right)+2\frac{\partial D_1}{\partial x}\left(\frac{\partial^3 W}{\partial x^3}+\frac{\partial^3 W}{\partial x\,\partial y^2}\right)+2\frac{\partial D_1}{\partial y}\left(\frac{\partial^3 W}{\partial y^3}+\frac{\partial^3 W}{\partial x^2\,\partial y}\right)+\frac{\partial^2 D_1}{\partial x^2}\left(\frac{\partial^2 W}{\partial x^2}+\vartheta\frac{\partial^2 W}{\partial y^2}\right)+\frac{\partial^2 D_1}{\partial y^2}\left(\frac{\partial^2 W}{\partial y^2}+\vartheta\frac{\partial^2 W}{\partial x^2}\right)+2(1-\vartheta)\frac{\partial^2 D_1}{\partial x\,\partial y}\frac{\partial^2 W}{\partial x\,\partial y}-\rho h p^2 W=0 \quad (4)$$

Which is a differential equation of transverse motion for non-homogeneous plate of variable thickness and

$$\ddot{T}+p^2\,\widetilde{D}\,T=0 \quad (5)$$

Is a differential equation of time function of free vibration of visco-elastic plate.

3. Formulation of problem and its solution

Assuming that the elliptical plate be shown in figure 1.1 has a steady two dimensional temperature distribution vary linearly which is represented by

$$\tau=\tau_0\left(1-\frac{x}{a}-\frac{y}{b}\right) \quad (6)$$

where τ denotes the temperature excess above the reference temperature at any point on the diameter from the centre of the elliptic plate and τ_0 denotes the temperature at any point on the boundary of the elliptic plate i.e. $1-\frac{x^2}{a^2}-\frac{y^2}{b^2}=0$. And a, b are the length of semi-major and semi-minor axis.

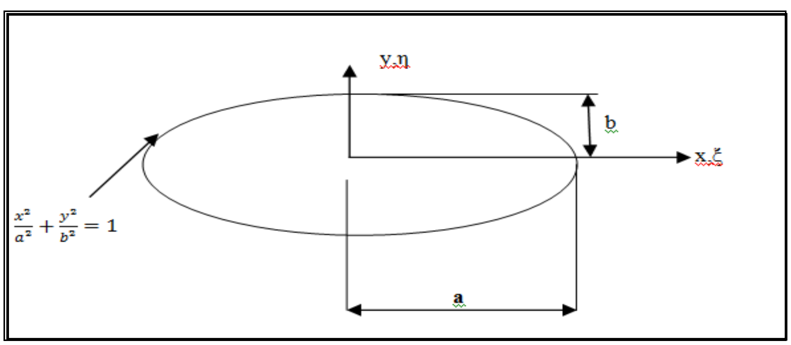

Fig. 1.1 Elliptical plate

The temperature dependent modulus of elasticity is taken as

$$E(\tau)=E_0\,(1-\gamma\,\tau) \quad (7)$$

where E_0 is the Young's modulus and γ is taken as slope variation.

From equation (6) and (7) we have,

$$E=E_0\left[\,1-\gamma\,\tau_0\left(1-\frac{x}{a}-\frac{y}{b}\right)\right]$$

$$E=E_0\left[\,1-\alpha\left(1-\frac{x}{a}-\frac{y}{b}\right)\right] \quad (8)$$

Where $\alpha=\gamma\,\tau_0$ ($0\leq\alpha<1$), is a parameter.

It is assumed that thickness varies parabolically along a diameter which is represented by

$$h = h_0 [\, 1 - \beta \left(\frac{x^2}{a^2} + \frac{y^2}{b^2} \right)] \tag{9}$$

Where β is known as tapering constant.

For non-homogeneous material, linear variation taken in density is

$$\rho = \rho_0 \, [\, 1 - c_1 \left(\frac{x}{a} + \frac{y}{b} \right)] \tag{10}$$

where c_1 ($0 \leq c_1 < 1$) is non − homogeneity constant.

The flexural rigidity of the plate is

$$D = \frac{E h^3}{12(1-\nu^2)} \tag{11}$$

Where ν is poisson's ratio.

Now using values of E and h from equations (8), (9) in 'D', we get

$$D = \frac{E_0 \, h_0{}^3 \left[1 - \alpha \left(1 - \frac{x}{a} - \frac{y}{b} \right) \right] [\, 1 - \beta \left(\frac{x^2}{a^2} + \frac{y^2}{b^2} \right)]^3}{12\,(\,1 - \nu^2\,)} \tag{12}$$

4. Boundary conditions and frequency equation

Boundary conditions for a non-homogeneous orthotropic (CCCC) elliptical plate are taken as

$$\left. \begin{aligned} W = W_x = 0 \; at \; x = 0, a \\ W = W_y = 0 \; at \; y = 0, b \end{aligned} \right\} \tag{13}$$

Deflection function W(x,y) of plate is assumed to be

$$W = A_1 \left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} \right)^2 + A_2 \left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} \right)^3 \tag{14}$$

where A_1 , A_2 are constants to satisfy boundary conditions.

Now, unitless variables having no dimension are using for our convince as

$$X = \frac{x}{a} , Y = \frac{y}{a} , \overline{W} = \frac{w}{a} , \overline{h} = \frac{h}{a} \tag{15}$$

The expressions for strain energy (P_E) and kinetic energy (K_E) are taken as

$$K_E = \frac{1}{2} \, p^2 \int_0^1 \int_0^{\sqrt{1-\xi^2}} \rho h \, W^2 dx \, dy \tag{16}$$

and

$$P_E = \frac{1}{2} \int_0^1 \int_0^{\sqrt{1-\xi^2}} D \left[\left(\frac{\partial^2 W}{\partial x^2} \right)^2 + \left(\frac{\partial^2 W}{\partial y^2} \right)^2 + 2\nu \frac{\partial^2 W}{\partial x^2} \frac{\partial^2 W}{\partial y^2} + 2(1-\nu) \left(\frac{\partial^2 W}{\partial x \, \partial y} \right)^2 \right] dx \, dy \tag{17}$$

5. Solution by Rayleigh-Ritz Method

Rayleigh – Ritz method is used to find an appropriate vibrational frequency. This method works on the phenomena that maximum strain energy (P_E) must equal to maximum kinetic energy (K_E). An equation in the following form is obtained as

$$\delta(P_E - K_E) = 0 \qquad (18)$$

Substitute the values from equations (9) , (10) , (12) , (14) and (15) in equation (16) and (17), then substituting the values of K_E, P_E in (18), we get

$$\delta(P_E^* - \lambda^2 K_E^*) = 0 \qquad (19)$$

Where,

$$K_E^* = \frac{1}{2}\, p^2 \rho_0\, h_0 \int_0^1 \int_0^{\sqrt{1-\xi^2}} \left(1 - c_1\left(X + Y\frac{a}{b}\right)\right)\left(1 - \beta\,\left(X + Y\frac{a}{b}\right)\right) W^2 dX\, dY \qquad (20)$$

$$P_E^* = \frac{1}{2}\int_0^1 \int_0^{\sqrt{1-\xi^2}} \left[\left[1 - \alpha\,\left(1 - X - Y\frac{a}{b}\right)\right]\left[1 + \beta\left(X + Y\frac{a}{b}\right)\right]^3 \left[\left(\frac{\partial^2 W}{\partial X^2}\right)^2 + \left(\frac{\partial^2 W}{\partial Y^2}\right)^2 + 2\nu \frac{\partial^2 W}{\partial X^2}\frac{\partial^2 W}{\partial y^2} + 2(1-\nu)\left(\frac{\partial^2 W}{\partial X\,\partial Y}\right)^2\right]\right] dX\, dY \qquad (21)$$

and frequency $\lambda^2 = \dfrac{12 p^2 \rho_0\, a^4 (1-\nu^2)}{E_0\, h_0^2}$

Now, on substituting the value of W, equation consist of two unknown constants i.e. A_1 & A_2 which is evaluate as follow:

$$\frac{\partial(P_E^* - \lambda^2 K_E^*)}{\partial A_n} = 0 \quad , \qquad \text{for } n = 1,2 \qquad (22)$$

On simplifying (22), we get

$$mn_1 A_1 + mn_2 A_2 = 0, \quad \text{for } n = 1, 2 \qquad (23)$$

Where mn_1, mn_2 (n = 1,2) involve parametric constant and the frequency parameter.
For non-trivial solution, the determinant of the co-efficient of equation (23) must be zero.
So, we get the frequency equation as

$$\begin{vmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{vmatrix} = 0 \qquad (24)$$

With the help of equation (24), we get quadratic equation in λ^2 from which the two values of λ^2 can be found. These two values of λ^2 represent the frequency vibration of two modes i.e. λ_1 (first mode) & λ_2 (second mode) for different values of taper constant and thermal gradient for a clamped plate.

6. Result and discussion

Frequency equation (24) is quadratic in λ^2, so it will give two roots. The frequency is derived for the first two modes of vibration for non – homogeneous elliptical plate having linearly varying thickness in both the directions, for the various values of taper constant (β), and thermal gradient (α), non – homogeneity constant (C_1). The value of passion ratio ν has been taken 0.345. All the results are calculated with the help of MAPLE software.
The results are shown in figures (1-5) for the first two modes of vibration for the elliptic plate.
Fig. 1, represents thermal gradient versus frequency with fixed value of Poisson ratio (ν = 0.345). It is clearly seen that as thermal gradient (α) increases from 0 to 1 results frequency increases. Figure 1 has shown the results for the following three cases:

i) $\beta = \xi = C_1 = 0.0$, a/b = 1.5, ii) $\beta = \xi = C_1 = 0.2$, a/b = 1.5 iii) $\beta = \xi = C_1 = 0.4$, a/b = 1.5

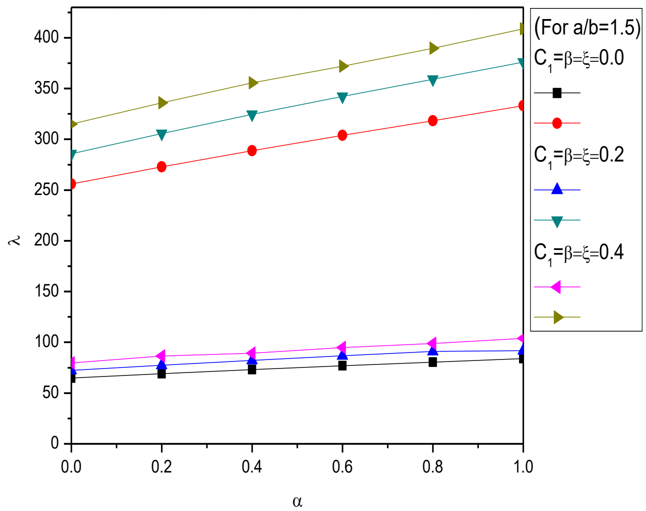

Fig. 1 Thermal gradient v/s Frequency

Fig. 2, represents taper constant versus frequency with fixed value of Poisson ratio (v = 0.345). It is clearly seen that as taper constant (β) increases from 0 to 1 results frequency decreases. Figure 2 has shown the results for the following three cases:

i) $\alpha = \xi = C_1 = 0.0$, a/b = 1.5, ii) $\alpha = \xi = C_1 = 0.2$, a/b = 1.5 iii) $\alpha = \xi = C_1 = 0.4$, a/b = 1.5

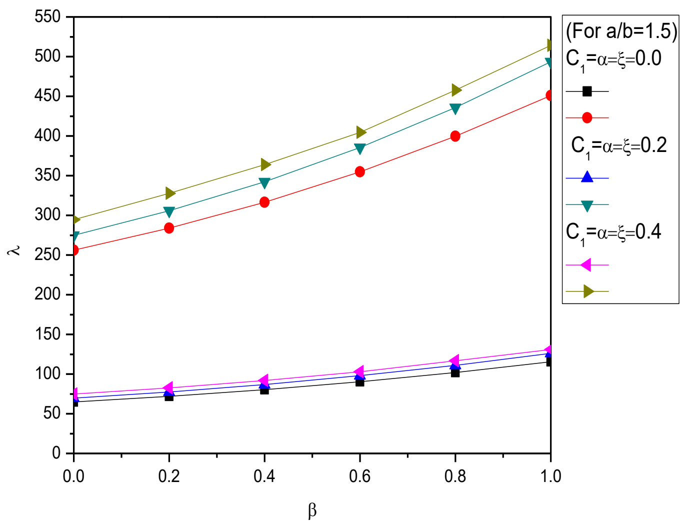

Fig. 2 Taper constant (β) vs frequency

Fig. 3, It is observed that for both modes of vibration, frequency parameter increases with the increase in non-homogeneity constant (C_1) 0 to 1. Figure 3 has shown the results for the following three cases:

i) $\alpha = \beta = \xi = 0.0$, a/b = 1.5, ii) $\alpha = \beta = \xi = 0.2$, a/b = 1.5 iii) $\alpha = \beta = \xi = 0.4$, a/b = 1.5

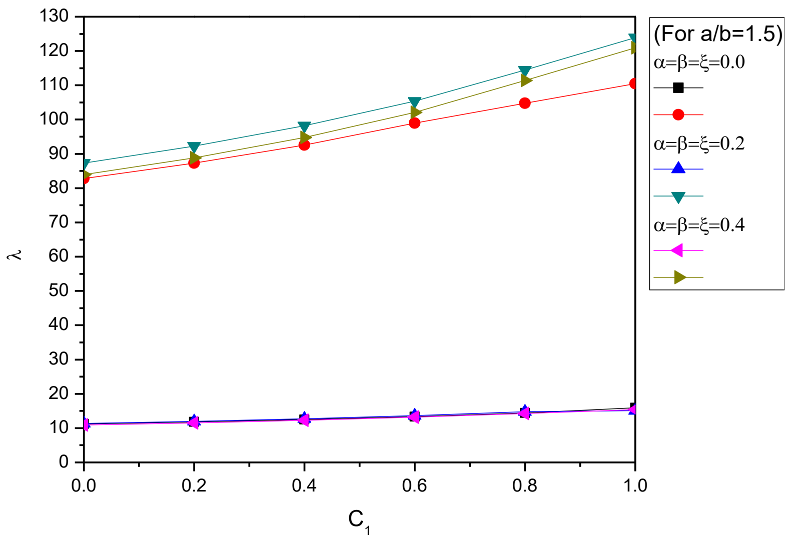

Fig. 3 Non-homogeneity vs Frequency (λ)

Fig. 4, It is observed that for both modes of vibration, frequency parameter increases with the increase in ξ 0 to 1. Figure 4 has shown the results for the following three cases:

i) $\alpha = \beta = C_1 = 0.0$, a/b = 1.5, ii) $\alpha = \beta = C_1 = 0.2$, a/b = 1.5 iii) $\alpha = \beta = C_1 = 0.4$, a/b = 1.5

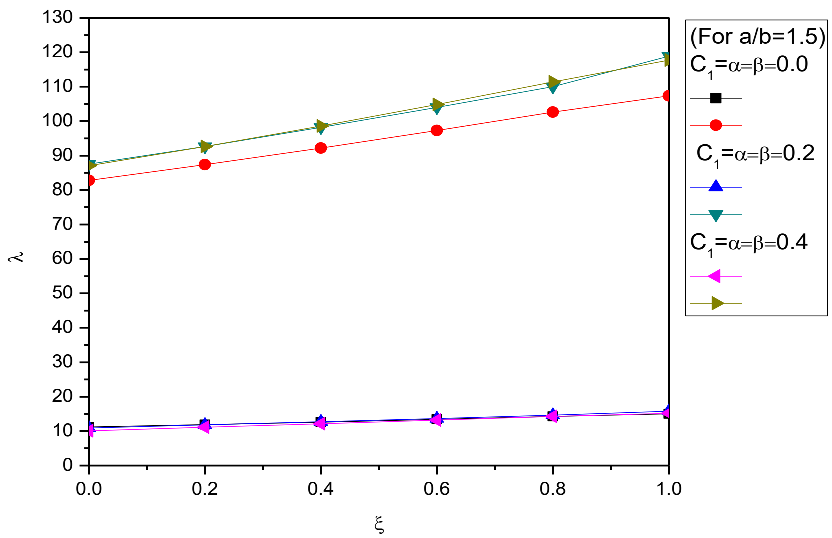

Fig. 4 ξ vs Frequency

Fig. 5, It is observed that for both modes of vibration, frequency parameter decreases with the increase in aspect ratio a/b 0.5 to 3. Figure 3 has shown the results for the following three cases:

i) $\alpha = \beta = \xi = C_1 = 0.0$, ii) $\alpha = \beta = \xi = C_1 = 0.2$ iii) $\alpha = \beta = \xi = C_1 = 0.4$

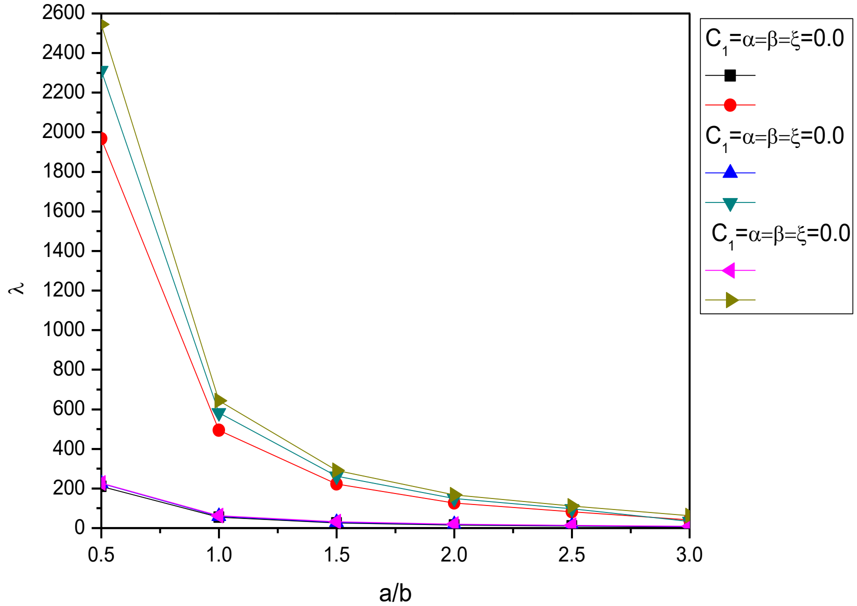

Fig. 5 Aspect ratio(a/b) vs Frequency (λ)

5. CONCLUSION

It can be clearly seen from the figures that frequency parameter increases with an increase in taper constant, thermal gradient and non-homogeneity constant. Also, frequency decreases with increase in the value of aspect ratio. Actually this is the need of the hour to develop more but authentic mathematical model for the help of mechanical engineers. Therefore mechanical engineers and technocrats are advised to study and get the practical importance of the present paper and to provide much better structure and machines with more safety and economy.

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