

Mechanical Vibration of Trapezoidal Visco Elastic Plate With Thermal Effect Along With Thickness Variation

Ashish Kumar Sharma¹ and Tamanna^{*2}

¹Associate Professor, Department of Mathematics, IEC University, Baddi, H.P, India.

^{*2} Research Scholar, Department of Mathematics, IEC University, Baddi, H.P, India.

Email: ashishk482@gmail.com¹, tamannasharma3232@gmail.com^{*2}

Abstract

This work aims to examine the influence of bi-parabolic thickness temperature change on the vibrations of bi-linear temperature non-homogeneous trapezoidal plates. The plate has boundary conditions on all four edges that are simply supported (SSSS). The modulus of elasticity is not uniform because of temperature fluctuation. Different numerical values of the tapering constant, non-homogeneity, aspect ratio and temperature gradient are derived to determine the frequency of the 1st and 2nd modes of vibration.

Keywords: Frequencies, Density, Thickness, Non-Homogeneity, Aspect Ratio, Thermal Gradient, Taper Constants, Trapezoidal Plate, Vibration.

1. Introduction

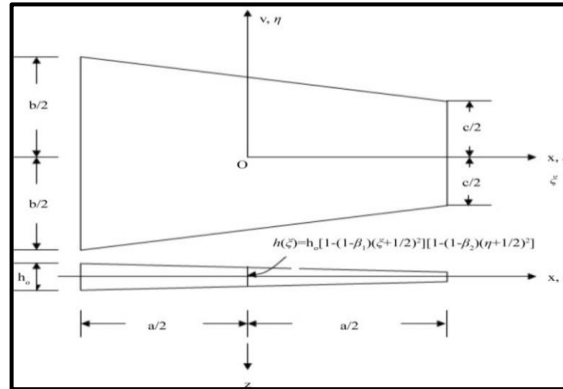
The majority of structures and machines are controlled by high temperatures. This causes certain vibrations in the system. System strength, durability, and efficiency are all impacted by vibrations. Vibration studies aim to minimize vibration by designing machinery and buildings correctly and precisely. Therefore, before finishing the design of structures, design engineers as well as researchers must possess pre-knowledge about the vibrational properties of systems. Plate geometry affects the vibrational analysis of those plates. Nowadays, engineering applications employ plates in a variety of forms, including parallelograms, ellipticals, circulars, and rectangles. Plates with diverse geometries, boundary conditions on edges, as well as complicating impacts have frequently been observed in a variety of structures, including those used in nuclear reactor technology, aerospace, machine design, telecommunications, naval structures, as well as earthquake-resistant constructions.

According to published studies numerous scholars have been motivated to study in this area by vibration analysis. Gupta and Sharma [1] examined how a linear temperature gradient affected the vibrations of trapezoidal plates with parabolically variable thicknesses. Sharma and Gupta [2] investigated how a non-homogeneous trapezoidal plate along with parabolically variable thickness was affected by linear temperature behaviour. In his monograph, Leissa [3] offered a comprehensive collection of research articles on the plate's vibration with various forms and boundary conditions. Saxena and Singh [4] examined the variable-thickness triangular plates' transverse vibration. Chen et al. [5] investigated the free vibration of the thick trapezoidal plates that were cantilevered and symmetrically laminated. Bambill et al. [6] investigated the transverse vibrations of cantilever plates that were rectangular, trapezoidal, and triangular orthotropic. Saliba [7] studied simply supported symmetrical trapezoidal plate-free vibration analysis. Deshpande and Krishnan [8] investigated the trapezoidal plates' free vibration. Lam and Liew [9] have investigated the vibrational action of trapezoidal composite plates that were symmetrically laminated under point restrictions. Lim and Liew [10] investigated the transverse vibration of variable-thickness symmetric trapezoidal plates.

2. Geometry of the Plate

A thin, symmetric, as well as non-homogeneous trapezoidal plate having different densities and thicknesses were considered for the research of transverse vibration. Figure 1 depicts the plate's geometry.

Figure 1: Geometry of the trapezoidal plate



3. Thickness and Density

The plate's thickness, which changes in both directions in a parabolic manner, may be written as

$$h(\xi) = h_0 \left[1 - (1 - \beta_1) \left(\xi + \frac{1}{2} \right)^2 \right] \left[1 - (1 - \beta_2) \left(\eta + \frac{1}{2} \right)^2 \right] \quad (1)$$

here h_0 indicates the thickness along $\eta = y/b$ and $\xi = x/a$.

The non-homogeneity is considered to result from the "linear change in density along with plate length and occurs in the bodies as a result of material imperfection.

$$\rho = \rho_0 \left[1 - (1 - \alpha_1) \left(\xi + \frac{1}{2} \right) \right] \quad (2)$$

The temperature difference on the plate is deemed to be bilinear such as linear in ξ direction and η direction as:

$$\tau = \tau_0 \left(\frac{1}{2} - \xi \right) \left(\frac{1}{2} - \eta \right) \quad (3)$$

Where τ_0 presents the temperature excess over the reference temperature on end $\xi = -\frac{1}{2}$ and τ denotes a excess over

the reference temperature at a distance $\xi = \frac{x}{a}$

The elasticity modulus for elastic materials is defined as:

$$F = F_O (1 - \gamma \tau), \quad (4)$$

Where F_O indicates the Young's modulus value over reference temperature at $\tau = 0$ and γ represents the variation slope of F and τ .

Using Eq. (3) in Eq. (4), we get

$$F = F_O \left(1 - \alpha \left(\frac{1}{2} - \xi \right) \left(\frac{1}{2} - \eta \right) \right), \quad (5)$$

Where $\alpha = \gamma \tau_O$ ($0 \leq \alpha \leq 1$).

4. Governing differential Equation

The differential equation that determines the kinetic energy K as well as strain energy S of a non-homogeneous trapezoidal plate whose thickness changes bi-parabolically may be written as

$$K = \frac{ab}{2} w^2 \int h(\xi) \rho w^2 dA \quad (6)$$

$$S = \frac{ab}{2} \int D(\xi) \left[\left(\frac{1}{a^2} \frac{\partial^2 w}{\partial \xi^2} + \frac{1}{b^2} \frac{\partial^2 w}{\partial \eta^2} \right)^2 - 2(1-\nu) \left[\frac{1}{a^2 b^2} \frac{\partial^2 w}{\partial \xi^2} \frac{\partial^2 w}{\partial \eta^2} - \left(\frac{1}{ab} \frac{\partial^2 w}{\partial \xi \partial \eta} \right)^2 \right] \right] dA \quad (7)$$

Plate's Flexural rigidity $D(\xi)$ is expressed as

$$D(\xi) = D_O \left\{ \left[1 - (1 - \beta_1) \left(\xi + \frac{1}{2} \right)^2 \right] \left[1 - (1 - \beta_2) \left(\eta + \frac{1}{2} \right)^2 \right] \right\}^3 \quad (8)$$

Where $D_O = \frac{F h_O^3}{12(1-\nu^2)}$ indicates the plate's flexural rigidity on side $\xi = -\frac{1}{2}$

The deflection function of 2-term w is considered as

$$\begin{aligned}
 w = & \left[\xi + \frac{1}{2} \right] \left[\eta - \left(\frac{b-c}{2} \right) \xi + \left(\frac{b+c}{4} \right) \right] \\
 & \times \left[\xi - \frac{1}{2} \right] \left[\eta + \left(\frac{b-c}{2} \right) \xi - \left(\frac{b+c}{4} \right) \right] \\
 & \times \left[A_1 + A_2 \left(\xi + \frac{1}{2} \right) \left(\eta - \left(\frac{b-c}{2} \right) \xi + \left(\frac{b+c}{4} \right) \right) \times \left(\xi - \frac{1}{2} \right) \left(\eta + \left(\frac{b-c}{2} \right) \xi - \left(\frac{b+c}{4} \right) \right) \right] \quad (9)
 \end{aligned}$$

Where A_1 and A_2 indicates the constants.

$$\begin{aligned}
 \eta = \frac{c}{4b} - \frac{\xi}{2} + \frac{1}{4} + \frac{c\xi}{2b}, \quad \eta = -\frac{c}{4b} + \frac{\xi}{2} - \frac{1}{4} - \frac{c\xi}{2b}, \\
 \xi = -\frac{1}{2}, \quad \xi = \frac{1}{2} \quad (10)
 \end{aligned}$$

5. Methodology

The frequency equation is resolved with Rayleigh-Ritz method. It is necessary for the greatest strain energy in this procedure to match the highest kinetic energy. Thus, to solve the issue under consideration that

$$\delta(K - S) = 0. \quad (11)$$

By using (1), (2), (6), (7), (8) and (9), (11) becomes

$$\delta(S_1 - \lambda^2 K_1) = 0 \quad (12)$$

Where

$$\begin{aligned}
 K_1 = & \frac{ab}{2} \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\frac{c}{4b} - \frac{\xi}{2} + \frac{1}{4} + \frac{c\xi}{2b}}^{\frac{c}{4b} - \frac{\xi}{2} + \frac{1}{4} + \frac{c\xi}{2b}} \left[1 - (1 - \beta_1) \left(\xi + \frac{1}{2} \right)^2 \right] \left[1 - (1 - \beta_2) \left(\eta + \frac{1}{2} \right)^2 \right] \times \left[1 - (1 - \alpha_1) \left(\xi + \frac{1}{2} \right) \right] w^2 d\eta d\xi \\
 S_1 = & \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-\frac{c}{4b} - \frac{\xi}{2} + \frac{1}{4} + \frac{c\xi}{2b}}^{\frac{c}{4b} - \frac{\xi}{2} + \frac{1}{4} + \frac{c\xi}{2b}} \left[\left[1 - (1 - \beta_1) \left(\xi + \frac{1}{2} \right)^2 \right] \left[1 - (1 - \beta_2) \left(\eta + \frac{1}{2} \right)^2 \right] \right]^3 \times \left(1 - \alpha \left(\frac{1}{2} - \xi \right) \left(\frac{1}{2} - \eta \right) \right) \\
 & \left\{ \left(\frac{1}{a^2} \frac{\partial^2 w}{\partial \xi^2} + \frac{1}{b^2} \frac{\partial^2 w}{\partial \eta^2} \right)^2 - 2(1 - \nu) \left(\frac{1}{a^2 b^2} \frac{\partial^2 w}{\partial \xi^2} \frac{\partial^2 w}{\partial \eta^2} - \left(\frac{1}{ab} \frac{\partial^2 w}{\partial \xi \partial \eta} \right)^2 \right) \right\} d\eta d\xi
 \end{aligned}$$

$$\lambda^2 = \frac{12\rho_o a^4 \omega^2 (1-\nu^2)}{F_o h_o^2}$$

Denote the frequency parameter.

Eq. (12) contains 2 unknowns A_1 A_2 that arise as a consequence of applying the deflection function.

These 2 unknowns may be assessed from Eq. (12) as follows:

$$\begin{aligned} \frac{\partial}{\partial A_1} (S_1 - \lambda^2 K_1) &= 0, m=1,2. \\ \frac{\partial}{\partial A_2} (S_1 - \lambda^2 K_1) &= 0, m=1,2. \end{aligned} \quad (13)$$

On simplifying Eq. (22), we get:

$$b_{m1} A_1 + b_{m2} A_2 = 0, m=1,2 \quad (14)$$

here $b_{m1}, b_{m2} (m=1,2)$ includes the frequency parameter and the parametric constant.

The coefficient determinant in Eq. (14) should be zero for the solution to be non-zero. Thus, the frequency eq. obtained as

$$\begin{vmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{vmatrix} = 0 \quad (15)$$

The quadratic equation λ^2 is attained via eq. (15) which displays the 2 values λ^2 referred to as 1st and 2nd modes of vibration correspondingly.

6. Result and discussion

The current literature examines the “vibration characteristics of a non-uniform trapezoidal plate with bi-parabolic thickness variation and bi-linear temperature variation. Equation (15) yields the frequency parameter values for numerous values of the above-mentioned parameters. For the first 2 vibration modes, the natural frequencies are approximated. Poisson's ratio is calculated to be 0.33. Every result has been shown with the use of graphs.

Figure 1: It is evident that in both cases, when $\alpha_1 = 0.4$ and $\alpha_1 = 1$, the frequency value drops as the temperature gradient α increases from 0 to 1. Additionally, it is noted that when the taper constant rises from 0 to 0.4, the frequency of both vibration modes increases in both scenarios.

Figure 2: When the tapering parameters β_1, β_2 rise from 0 to 1 for both situations ($\alpha_1 = 0.4$ and $\alpha_1 = 1$), the frequency parameter values for both modes increase. Additionally, the graph showed that, in the two aforementioned examples, the frequency of both vibration modes reduces as the temperature gradient value rises from 0 to 0.4.

Figure 3: It is highlighted that when the aspect ratio c/b rises from 0.25 to 1, frequency” falls; however, when the graph is viewed from left into right for the mentioned cases, frequency increases for both modes. Case1: $\alpha = \beta_1 = \beta_2 = 0$ and $\alpha = 0, \beta_1 = \beta_2 = 0.4$

Case2: $\alpha = 0.4, \beta_1 = \beta_2 = 0$ and $\alpha = 0.4, \beta_1 = \beta_2 = 0.4$

Figure 4: It is clear that a rise in a non-homogeneity α_1 range between 0 to 1 indicates that the vibrational frequency for modes declines in one scenario, whereas in the other, the frequency rises. This is demonstrated by observing the graph for the following scenarios from left to right.

Case1: $\alpha = \beta_1 = \beta_2 = 0$ and $\alpha = 0, \beta_1 = \beta_2 = 0.4$

Case2: $\alpha = 0.4, \beta_1 = \beta_2 = 0$ and $\alpha = 0.4, \beta_1 = \beta_2 = 0.4 (\alpha)$

Figure-1: Thermal Gradient α versus Frequency λ

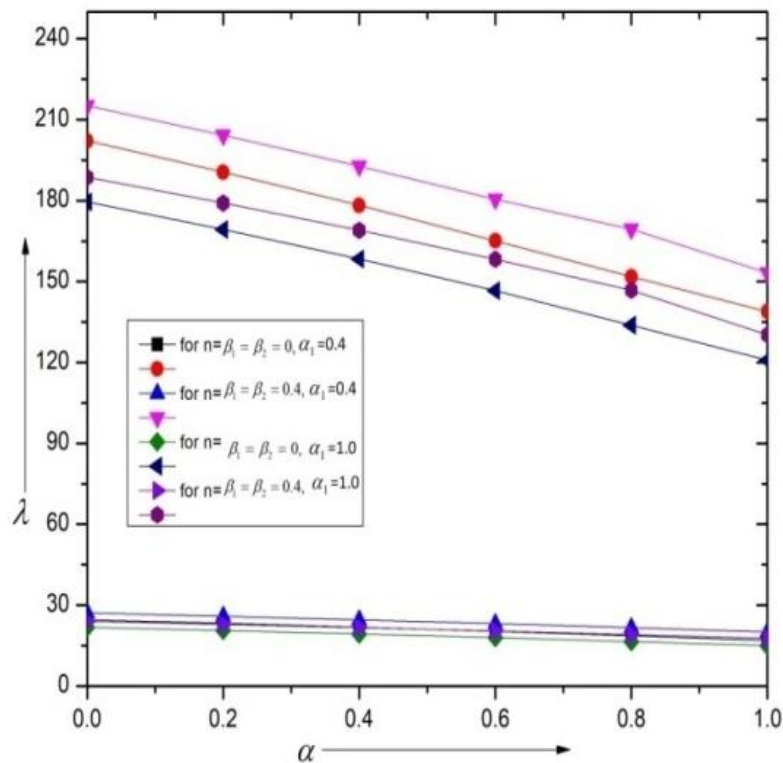


Figure-2: Taper Constant $\beta_1=\beta_2$ versus Frequency λ

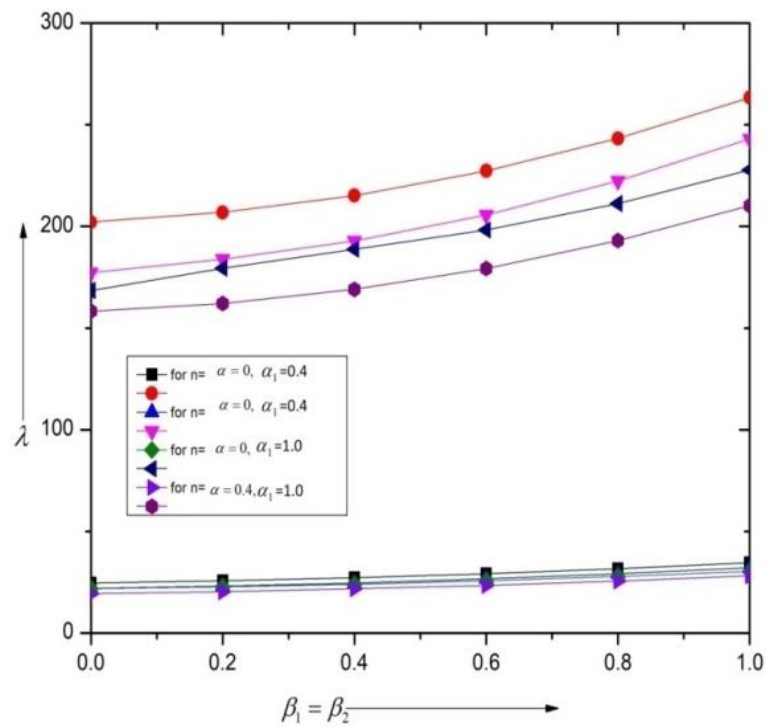


Figure-3: Aspect Ratio (c/b) versus Frequency λ

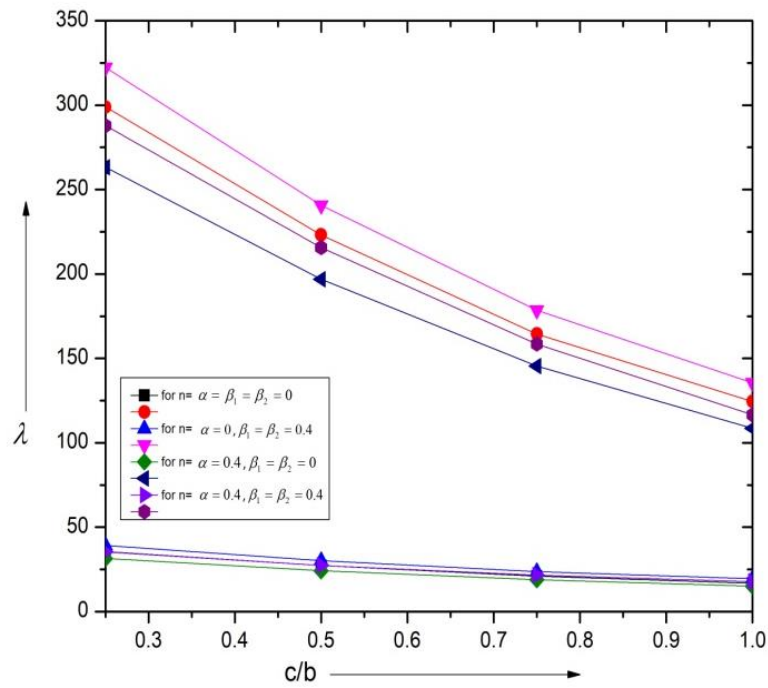
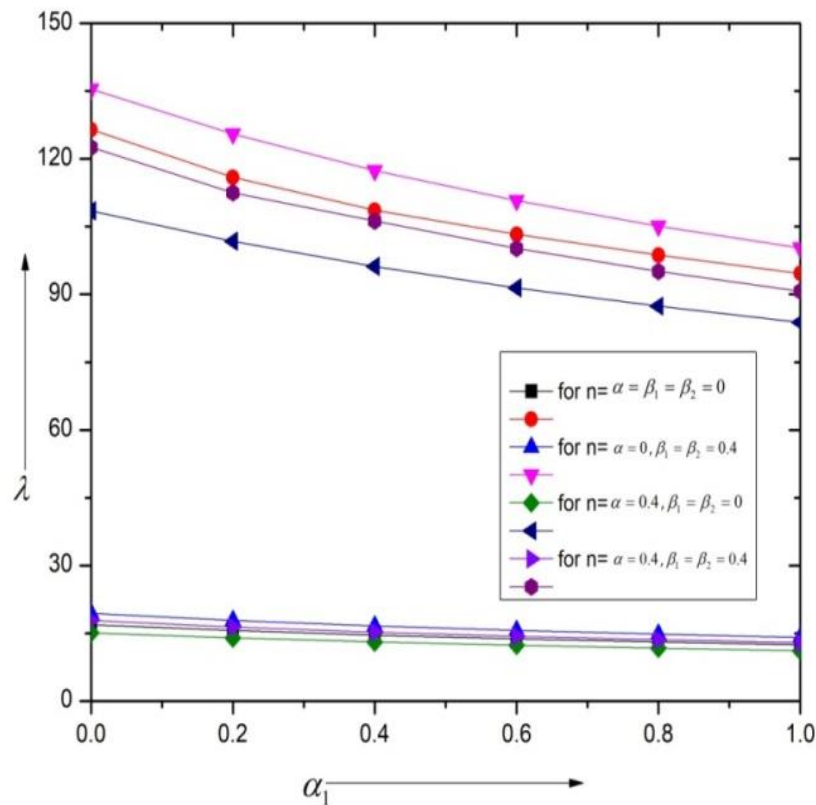


Figure-4: Non Homogeneity Constant α_1 versus Frequency λ



Conclusion

The current study used the Rayleigh-Ritz approach to investigate the temperature impact on the vibrations of a symmetric, non-homogeneous trapezoidal plate made of isotropic material along with all four edges that are simply supported assisted boundary conditions. Additional factors affecting the plate, including aspect ratios, taper constants, thermal gradient, as well as non-homogeneity constants, were also taken into account.

References

- [1] A.W. Leissa, Vibration of plate, NASA SP-160(1969).
- [2] A. Sharma, A.K. Raghav, V. Kumar, Mathematical study of vibration on non-homogeneous parallelogram plate with thermal gradient, International Journal of Mathematical Science, 36(1)(2016), 1801-1809.
- [3] S.K. Sharma, A.K. Sharma Mathematically study on vibration of visco-elastic parallelogram plate, Mathematical Models in Engineering, Vol. 1, Issue 1, 2015, p.12-19.
- [4] Khanna, Anupam, and Ashish Kumar Sharma. "Vibration analysis of visco-elastic square plate of variable thickness with thermal gradient." International Journal of Engineering and Applied Sciences 3.4 (2011): 1-6.
- [5] Qatu, M.S. (1994) Vibrations of Laminated Composite Completely Free Triangular and Trapezoidal Plates. International Journal of Mechanical Sciences, 36, 797-809.

- [6] Kumar, Y. and Lal, R. (2012) Vibrations of Non-Homogeneous Orthotropic Rectangular Plates with Bilinear Thickness Variation Resting on Winkler Foundation. *Meccanica*, 47, 893-915.
- [7] Bambill, D.V., Laura, P.A.A. and Rossi, R.E. (1998) Transverse Vibrations of Rectangular, Trapezoidal and Triangular Orthotropic, Cantilever Plates. *Journal of Sound and Vibration*, 210, 286-290.
- [8] Gupta, A.K. and Sharma, S. (2011) Study the Effect of Thermal Gradient on Transverse Vibration of Non-Homogeneous Orthotropic Trapezoidal Plate of Parabolically Varying Thickness. *Applied Mathematics*, 2, 1-10.
- [9] Gupta, A.K. and Sharma, P. (2012) Study of Thermally Induced Vibration of Non-homogeneous Trapezoidal Plate Varying Thickness and Density. *American Journal of Computational and Applied Mathematics*, 2, 265-275.
- [10] Ashish Kumar Sharma and Tamanna Study of Free Vibration of Trapezoidal Visco Elastic Plate with Thickness Variation. *International Journal of Applied Engineering & Technology* Vol. 5 No.2, June, 2023.