

Nano D-Irresolute Maps and Some Separation Axioms in Nano Topological Spaces

R.Uma Mageshwari¹ , K.Dass²,

¹ Research Scholar(Reg No.18221072092005)

PG and Research Department of Mathematics,

The M.D.T.Hindu College,Tirunelveli-627010

Affiliated to Manonmaniam Sundaranar University,

Tirunelveli,Tamilnadu,India.

²Associate professor,Research Supervisor,

PG and Research Department of Mathematics,

The M.D.T. Hindu College, Tirunelveli-627010.

Affiliated to Manonmaniam Sundaranar University,

Tirunelveli,Tamilnadu,India.

Abstract:

In this paper, we introduce the new concept of Nano D-irresolute function and some separation axioms in nano topological spaces and obtained some of their properties using nano D- closed sets. Further, we have given some appropriate examples to understand the abstract concepts clearly.

Key Words: nD -irresolute function, $nD-T_0$, $nD-T_1$, $nD-T_2$.

1.Introduction:

Lellis Thivagar introduced the notion of Nano topology by using theory approximations and boundary region of a subset of an universe in terms of equivalence relation on it and also defined Nano closed sets, Nano-interior (briefly $nint$) and nano closure (briefly ncl) in nano topological spaces. The notion nano D-open sets in nano topological spaces were introduced by K Dass et al and studies some of their properties. Bhuvaneshwari and Ezhilarasi introduced irresolute maps and semi generalized irresolute maps in nano topological spaces

Khalaf and Ahmed Elomoasry discussed slightly nano separation axioms in nano topological spaces. The purpose of this paper is to discuss the concepts of nano D-irresolute maps and nano separation axioms in nano topological spaces via nD -open sets. Moreover $nD-T_0$,

$nD-T_1$, $nD-T_2$, spaces are introduced and some of their properties are discussed

2.Preliminaries:

Definition 2:1:[5]

Let U be a non empty finite set of objects called Universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernibility with one another. The pair (U, R) is said to be the Approximation space.

Let $X \subseteq U$

• The Lower approximation of X with respect to R is the set of all objects which can be certain classified as X with respect to R and is denoted by $L_R(X)$.

$$L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}.$$

• The Upper approximation of X with respect to R is the set of objects which can be possibly classified as X with respect to R and is denoted by $U_R(x)$.

$$U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \emptyset\}.$$

• The boundary region of X with respect to R is the set of all objects which can be classified neither as x nor not X with respect to R and is denoted by $B_R(x)$.

$$B_R(X) = U_R(X) - L_R(X).$$

Definition 2.2 :[6]

Let U be the universe R be an equivalence relation on U and $\tau_R(X) = \{U, \emptyset, L_R(x), U_R(x), B_R(x)\}$, where $X \subseteq U$.

Then the property $\tau_R(X)$, satisfies the following axioms

i. U and $\emptyset$ belongs to $\tau_R(X)$.

ii. The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

iii. The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

Thus, $\tau_R(X)$ is a topology on U is said to be Nano Topology on U with respect to X . $(U, \tau_R(X))$ as the Nano Topological Spaces.

The element of $\tau_R(X)$ are called as Nano-open sets and complement of Nano-open sets are called Nano-closed sets.

Example 2.3:

Let $U = \{a, b, c, d\}$ $U/R = \{\{a\}, \{b\}, \{c\}, \{d\}\}$.

Let $X = \{a, c\} \subseteq U$. Then the nano topology is $\tau_R(X) = \{\emptyset, \{a\}, \{c, d\}, \{a, c, d\}, U\}$.

Definition 2.4 :[5]

Let $(U, \tau_R(X))$ be a nano topological space and $H \subseteq U$. Then H is said to be

i) Nano Semi-open if $H \subseteq \text{Ncl}(\text{Nint}(H))$

ii) Nano regular open if $H = \text{Nint}(\text{Ncl}(H))$

iii) Nano π -open if it is finite union of all nano-regular open sets.

Definition 2.5 :[5]

If $(U, \tau_R(X))$ is a nano topological space and if $H \subseteq U$, then

i) The **nano -interior** of H is defined as the union of all nano open sets contained in H and is denoted by $\text{Nint}(H)$ $\text{Nint}(H)$ is the largest nano open set .

ii) The **nano closure** of the set H is defined as the intersection of all nano closed sets containing H and is denoted by $Ncl(H)$. $Ncl(H)$ is the smallest nano closed set.

Definition 2.6 :[4]

A subset H of a nano topological space $(U, \tau_R(X))$ is called a Nano $\hat{g}$ -closed ($N\hat{g}$ -closed) if $Ncl(H) \subseteq V$ whenever $H \subseteq V$ and V is semi open in $(U, \tau_R(X))$.

Definition 2.7:[13]

Let $(U, \tau_R(X))$ be a nano topological space and $H \subseteq U$. Then H is said to be nano D-closed (resp nano D-open) set if $Npcl(H) \subseteq nint(V)$ whenever $H \subseteq V$ and V is nano $\hat{g}$ -open.

Definition 2.8:[13]

Let $(U, \tau_R(X))$ be a nano topological space and let $H \subseteq U$. Then the nano D-interior of H is the union of all nano D-open sets contained in H and denoted by $nDint(H)$

Definition 2.9:[13]

Let $(U, \tau_R(X))$ be a nano topological space and let $H \subseteq U$. Then the nano D-closure of H is the intersection of all nano D-closed sets containing H and is denoted by $nDcl(H)$.

Definition 2.10[7]:

A mapping $f: (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ is said to be nano continuous on U , if the inverse image of every nano closed in V is nano closed in U .

Definition 2.11[14]:

A mapping $f: (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ is said to be nano D-continuous if for each nano-closed sets F of V , the set $f^{-1}(F)$ is nano D-closed set of U .

Definition 2.12:[11]

Let $(U, \tau_R(X))$ and $(V, \tau_R(Y))$ be two nano topological spaces. A mapping

$f: (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ is said to be nano irresolute on U if the inverse image of every nano closed in $(V, \tau_R(Y))$ is nano closed in $(U, \tau_R(X))$.

Definition 2.13:[10]

A nano topological space is called nano- T_0 space if for any two points $l \neq m$, there exists a nano open set G such that $l \in G$ and $m \notin G$.

Definition 2.14:[10]

A nano topological space is called nano- T_1 space if for any two points $l \neq m$, there exist disjoint nano open sets G and H with $l \in G$, $m \notin G$ and $l \notin H$, $m \in H$.

Definition 2.15:[13]

A nano topological space is called nano- T_2 space if for any two points $l \neq m$, there exist disjoint nano open sets G and H with $l \in G$ and $m \in H$

3. Nano D-irresolute function:

In this section ,we introduce and study a new concept of nD-irresolute functions in Nano topological spaces and discuss their properties.

Definition 3.1:

Let $(U, \tau_R(X))$ and $(V, \tau_R(Y))$ be two nano topological spaces. A mapping

$f: (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ is nano D-irresolute (nD-irresolute) on U if the inverse image of every nano D-Closed in $(V, \tau_R(Y))$ is nano D-closed in $(U, \tau_R(X))$.

Example 3.2:

Let $U=\{a,b,c,d\}$ with $X/R=\{\{c\}, \{d\}, \{a,b\}\}$

Let $X=\{a,b\}$.

Then $\tau_R(X) = \{\varnothing, \{c\}, \{a,b\}, \{a,b,c\}, U\}$

Let $V=\{a,b,c,d\}$ with $Y/R=\{\{a\}, \{c\}, \{b,d\}\}$

Let $Y=\{a,b\}$.

Then $\tau_R(Y)=\{\varnothing, \{a\}, \{b,d\}, \{a,b,d\}, V\}$

Define $f: (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ by $f(a)=a; f(b)=b; f(c)=d; f(d)=c$

It is observed that the inverse image of every nD-closed set in $(V, \tau_R(Y))$ is nD-closed set in $(U, \tau_R(X))$.

Therefore, $f: (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ is nD-irresolute.

Theorem 3.4:

Let $(U, \tau_R(X))$ and $(V, \tau_R(Y))$ and $(W, \tau_R(Z))$ be a nano topological spaces.

If $f: (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ be nano D-irresolute and

$g: (V, \tau_R(Y)) \rightarrow (W, \tau_R(Z))$ be nano D-continuous.

Then $g \circ f: (U, \tau_R(X)) \rightarrow (W, \tau_R(Z))$ is nano D-continuous.

Proof:

Let G be any nano closed set of $(W, \tau_R(Z))$.

Since g is nano D-continuous, $g^{-1}(G)$ is nD-closed set in $(V, \tau_R(Y))$.

Since f is nano D-irresolute, $f^{-1}(g^{-1}(G))=(g \circ f)^{-1}(G)$ is nD-closed set in $(U, \tau_R(X))$.

Therefore, $g \circ f: (U, \tau_R(X)) \rightarrow (W, \tau_R(Z))$ is nD-continuous.

Theorem 3.5:

Let $f: (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ and $g: (V, \tau_R(Y)) \rightarrow (W, \tau_R(Z))$ be two maps.

If f and g are both nD-irresolute, then $g \circ f$ is nD-continuous

Proof:

Let G be nano closed in $(W, \tau_R(Z))$.

Since every nano closed is nano D-closed and also g is nD-irresolute,

$g^{-1}(G)$ is nD-closed in $(V, \tau_R(Y))$.

Since f is nD-irresolute, $f^{-1}(g^{-1}(G))$ is nD-closed in $(U, \tau_R(X))$.

Thus $(g \circ f)^{-1}(G)=f^{-1}(g^{-1}(G))$ is nD-closed set in $(U, \tau_R(X))$ for every nano closed G in

$(W, \tau_{R^*}(Z))$.

Therefore, gof is nD-continuous.

Theorem 3.6:

If $f: (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ is nD-irresolute and $g: (V, \tau_R(Y)) \rightarrow (W, \tau_{R^*}(Z))$ is nD-irresolute, then $\text{gof}: (U, \tau_R(X)) \rightarrow (W, \tau_{R^*}(Z))$ is nD-irresolute.

Proof:

Let G be a nD-closed in $(W, \tau_{R^*}(Z))$.

Since g is nD-irresolute, $f^{-1}(g^{-1}(G))$ is nD-closed in $(U, \tau_R(X))$.

Thus $(\text{gof})^{-1}(G) = f^{-1}(g^{-1}(G))$ is nD-closed in $(U, \tau_R(X))$, for every nD-closed in $(W, \tau_{R^*}(Z))$.

Hence gof is nD-irresolute.

Theorem 3.7:

A mapping $f: (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ is nD-irresolute if and only if the inverse image $f^{-1}(G)$ is nano D-open in $(U, \tau_R(X))$ for every nano D-open in $(V, \tau_R(Y))$.

Proof:

Let G be nD-open in $(V, \tau_R(Y))$.

Then $V-G$ is nD-closed in $(V, \tau_R(Y))$.

Since f is nD-irresolute, $f^{-1}(V-G)$ is nano D-closed in $(U, \tau_R(X))$.

But $f^{-1}(V-G) = U - f^{-1}(G)$.

Hence $f^{-1}(G)$ is nD-open in $(U, \tau_R(X))$.

Conversely, assume that inverse image $f^{-1}(G)$ is nD-open in $(U, \tau_R(X))$,

for every nD-open set G in $(V, \tau_R(Y))$.

Let F be nD-closed in $(V, \tau_R(Y))$.

Then $V-F$ is nD-open in $(V, \tau_R(Y))$.

By assumption, $f^{-1}(V-F)$ is nD-open in $(U, \tau_R(X))$.

But $f^{-1}(V-F) = U - f^{-1}(F)$.

Then $f^{-1}(F)$ is nD-closed in $(U, \tau_R(X))$.

Hence f is nD-irresolute.

Theorem 3.8:

A mapping $f: (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ is nD-irresolute if and only if

$f(\text{nDcl}(F)) \subseteq \text{nDcl}(f(F))$ for every subset F of $(U, \tau_R(X))$.

Proof:

Suppose $f: (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ is nD-irresolute.

Let $F \subseteq U$.

Then $f(F) \subseteq V$.

Hence $nDcl(f(F))$ is nD -closed in V .

Since f is nD -irresolute, $f^{-1}(nDcl(f(F)))$ is nD -closed in $(U, \tau_R(X))$.

Since $f(F) \subseteq nDcl(f(F))$, which implies $F \subseteq f^{-1}(nDcl(f(F)))$.

Since $nDcl(F)$ is the smallest nD -closed set containing F , $nDcl(F) \subseteq f^{-1}(nDcl(f(F)))$.

Conversely, assume that $f(nDcl(F)) \subseteq nDcl(f(F))$ for every subset F of U .

Let G be nD -closed in V .

Now, $f^{-1}(G) \subseteq U$.

Therefore, $f(nDcl(f^{-1}(G))) \subseteq nDcl(f(f^{-1}(G))) = nDcl(G)$,

Therefore, $nDcl(f^{-1}(G)) \subseteq f^{-1}(nDcl(G)) = f^{-1}(G)$.

Therefore, $f^{-1}(G)$ is nD -closed in U for every nD -closed G in V .

Hence f is nD -irresolute.

Theorem 3.9:

A mapping $f: (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ is nD -irresolute if and only if

$f^{-1}(nDint(G)) \subseteq nDint(f^{-1}(G))$ for every subset G of $(V, \tau_R(Y))$.

Proof:

Let $f: (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ be nD -irresolute.

Let $G \subseteq V$.

Then $nDint(G)$ is nD -open in V .

Since f is nD -irresolute, $f^{-1}(nDint(G))$ is nD -open in $(U, \tau_R(X))$.

Hence $nDint(f^{-1}(nDint(G))) = f^{-1}(nDint(G))$

Since $nDint(G) \subseteq G, f^{-1}(nDint(G)) = nDint(f^{-1}(nDint(G)))$
 $\subseteq nDint(f^{-1}(G)).$

Thus $f^{-1}(nDint(G)) \subseteq nDint(f^{-1}(G)).$

Conversely, assume that $f^{-1}(nDint(G)) \subseteq nDint(f^{-1}(G))$ for every subset G of V .

Let G be nD -open in V .

Hence $nDint(G) = G$.

By our assumption, $f^{-1}(G) \subseteq nDint(f^{-1}(G)).$

But $nDint(f^{-1}(G)) \subseteq f^{-1}(G).$

Hence $f^{-1}(G) = nDint(f^{-1}(G)).$

Thus $f^{-1}(G)$ is nD -open in U for every subset G of Y .

Hence f is nD -irresolute.

Theorem 3.10:

A mapping $f: (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ is nD-irresolute if and only if

$nDcl(f^{-1}(F)) \subseteq f^{-1}(nDcl(F))$ for every subset F of V .

Proof:

Suppose $f: (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ is nD-irresolute.

Let $F \subseteq V$.

Then $nDcl(F)$ is nD-closed in $(V, \tau_R(Y))$

Since f is nD-irresolute, $f^{-1}(nDcl(F))$ is nD-closed in U .

Thus $nDcl(f^{-1}(nDcl(F))) = f^{-1}(nDcl(F))$.

Since $F \subseteq nDcl(F)$, then $f^{-1}(F) \subseteq f^{-1}(nDcl(F))$.

Now $nDcl(f^{-1}(F)) \subseteq nDcl(f^{-1}(nDcl(F))) = f^{-1}(nDcl(F))$

Therefore, $nDcl(f^{-1}(F)) \subseteq f^{-1}(nDcl(F))$, for every subset F of V .

Conversely, assume that $nDcl(f^{-1}(F)) \subseteq f^{-1}(nDcl(F))$ for every subset F of V .

Let F be nD-closed in V .

Hence $nDcl(F) = F$.

By our assumption, $nDcl(f^{-1}(F)) \subseteq f^{-1}(F)$

But $f^{-1}(F) \subseteq nDcl(f^{-1}(F))$.

Hence $f^{-1}(F) = nDcl(f^{-1}(F))$.

Then $f^{-1}(F)$ is nD-closed in U for every subset F of V .

Hence f is nD-irresolute.

4. Nano D- Separation Axioms:

In this section, the basic properties of nD- T_0 , nD- T_1 , nD- T_2 spaces are established.

Definition 4.1:

A nano topological space $(U, \tau_R(X))$ is called nano D- T_0 (brief nD- T_0) space if for any two points $l \neq m$, there exists a nD-open set G such that $l \in G$ and $m \notin G$.

Example 4.2:

Let $V = \{a, b, c\}$, $Y = \{b, c\}$

Then $V/R = \{\{a, b\}, \{c\}\}$

Now $\tau_R(Y) = \{\emptyset, \{c\}, \{a, b\}, V\}$

$ND_o(\tau_R(Y)) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, V\}$

Therefore $(V, \tau_R(Y))$ is nD- T_0 space.

Theorem 4.3:

Every nD- T_0 space is nD- T_0 space.

Proof:

Assume that l and m are distinct points in nT_0 -space $(U, \tau_R(X))$.

Since $(U, \tau_R(X))$ is a nT_0 space, there exists a nano open set G in $(U, \tau_R(X))$ such that

$l \in G$ and $m \notin G$.

Since every nano open is nano D-open, there exists a nano D-open G such that $l \in G$ and $m \notin G$.

Therefore, $(U, \tau_R(X))$ is $nD-T_0$ space.

Every nT_0 -space is $nD-T_0$ space.

Theorem 4.3:

A nano topological space $(U, \tau_R(X))$ is a $nD-T_0$ space iff nD -closures of distinct points are disjoint.

Proof:

Let l, m be two distinct points of U .

Since U is a $nD-T_0$ space, there exists a nD -open set G such that $l \in G$ and $m \notin G$.

Consequently, $U-G$ is a nD -closed set containing m but not l .

But $nDcl(\{m\})$ is the intersection of all nD -closed sets containing $\{m\}$.

Hence $m \in nDcl(\{m\})$ but $l \notin nDcl(\{m\})$ as $l \in U-G$.

Therefore, $nDcl(\{l\}) \neq nD-cl(\{m\})$

Conversely, suppose $nD-cl(\{l\}) \neq nD-cl(\{m\})$ for $l \neq m$.

Then there exists at least one point $n \in U$ such that $n \in nD-cl(\{l\})$ but $n \notin nD-cl(\{m\})$.

Suppose $l \notin nD-cl(\{m\})$.

If $l \in nD-cl(\{m\})$, then $\{l\} \subset nDcl(\{m\})$

Therefore, $nD-cl(\{l\}) \subset nD-cl(\{m\})$

So, $n \in nD-cl(\{m\})$, which is a contradiction

Hence $l \notin nD-cl(\{m\})$.

Therefore, $l \in U-nD-cl(\{m\})$, which is a nD -open set containing l but not m .

Hence U is a $nD-T_0$ space.

Theorem 4.4:

If $f: (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ is an injective, nD -continuous function and V is $nD-T_0$ space, then U is a $nD-T_0$ space.

Proof:

Let l and m be any two distinct points in U .

Since f is injective, $f(l)$ and $f(m)$ are distinct points in V .

Since V is $nD-T_0$ space, there exists a nD -open set G in V containing $f(l)$ but not $f(m)$.

Again, since f is nD -continuous, $f^{-1}(G)$ is a nD -open set in U containing l but not m .

Therefore, U is a $nD-T_0$ space.

Definition 4.5:

Let $(U, \tau_R(X))$ be a nano topological space. A space $(U, \tau_R(X))$ is said to be nano $D-T_1$ (briefly $nD-T_1$) space, if any two points $l \neq m$, there exist disjoint nD -open sets G and H

with $l \in G$, $m \notin G$ and $l \notin H$, $m \in H$.

Theorem 4.6:

Every nT_1 space is $nD-T_1$ space.

Proof:

Assume that l and m are distinct points in nT_1 -space $(U, \tau_R(X))$.

Since $(U, \tau_R(X))$ is a $nD-T_1$ space, there exist disjoint nano open sets G and H in

$(U, \tau_R(X))$ such that $l \in G$, $m \notin G$ and $l \notin H$, $m \in H$.

Therefore, $l \in \text{nint}(\{l\})$, $m \in \text{ncl}(\{m\}) \subset G^c$ and $m \in \text{nint}(\{m\})$, $l \in \text{ncl}(\{l\}) \subset G^c$.

Since every nano open is nano D -open, there exist nano D -open G and H such that $l \in G$,

$m \notin G$ and $l \notin H$, $m \in H$ and $G \cap H = \emptyset$.

Therefore, $(U, \tau_R(X))$ is $nD-T_1$ space.

Every nT_1 -space is $nD-T_1$ space.

Examples 4.7:

Let $U = \{a, b, c\}$, $U/R = \{\{a\}, \{b, c\}\}$

Let $X = \{a, b\}$, then $\tau_R(X) = \{\emptyset, \{a\}, \{b, c\}, U\}$

$nDo(\tau_R(X)) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{b, c\}, U\}$.

Therefore, $(U, \tau_R(X))$ is $nD-T_1$ space.

Theorem 4.8:

A nano topological space $(U, \tau_R(X))$ is a $nD-T_1$ space iff every single on subsets $\{l\}$ of U is a nD -open set.

Proof:

Let U be a $nD-T_1$ space and $l \in U$.

Let $m \in U - \{l\}$.

Then for $l \neq m$, there exists nD -open set G_m such that $m \in G_m$ and $l \notin G_m$.

Consequently, $m \in G_m \subset U - \{l\}$.

That is, $U - \{l\} = \bigcup \{G_m : m \in U - \{l\}\}$, which is the union of nD -open sets and hence nD -open set.

Therefore, $\{l\}$ is a nD -closed set

Conversely, suppose $\{l\}$ is a nD -closed set for every $l \in U$.

Let $l, m \in U$ with $l \neq m$.

Now $l \neq m$ implies $m \in U - \{l\}$.

Hence $U - \{l\}$ is a nD -open set G_m containing m but not l .

Similarly $U - \{m\}$ is a nD -open set containing l but not m .

Hence U is a $nD-T_1$ space.

Theorem 4.9:

If $f: (U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ is a injective, nD -irresolute function and V is a $nD-T_1$, then U is also a $nD-T_1$ space.

Proof:

Let l_1 and l_2 be pair of distinct points in U .

Since f is injective, there exists two distinct points m_1 and m_2 of V such that $f(l_1)=m_1$ and $f(l_2)=m_2$.

Since V is $nD-T_1$ space there exists a nD -open set G and H in V such that $m_1 \in G$, $m_2 \notin G$ and $m_1 \in H$ and $m_2 \notin H$.

That is, $l_1 \in f^{-1}(G)$, $l_1 \notin f^{-1}(H)$ and $l_2 \in f^{-1}(H)$, $l_2 \notin f^{-1}(G)$.

Since f is nD -irresolute, $f^{-1}(G)$ and $f^{-1}(H)$ are nD -open sets in U .

Thus for two distinct points l_1 and l_2 of U , there exist nD -open sets $f^{-1}(G)$ and $f^{-1}(H)$ such that $l_1 \in f^{-1}(G)$, $l_1 \notin f^{-1}(H)$ and $l_2 \in f^{-1}(H)$, $l_2 \notin f^{-1}(G)$.

Therefore, U is $nD-T_1$ space.

Definition 4.10:

A nano topological space $(U, \tau_R(X))$ is called nano $D-T_2$ (briefly $nD-T_2$) space if for any two points $l \neq m$, there exist disjoint nD -open sets G and H with $l \in G$ and $m \in H$.

Theorem 4.11:

Every nT_2 -space is $nD-T_2$ space.

Proof:

Assume that l and m are distinct points in nT_2 -space $(U, \tau_R(X))$.

Since $(U, \tau_R(X))$ is a $nD-T_2$ space, there exist nano open sets G and H in $(U, \tau_R(X))$ such that $l \in G$ and $m \in H$ and $G \cap H = \emptyset$.

Therefore, $l \in \text{nint}(\{l\})$ and $m \in \text{nint}(\{m\})$.

Since every nano open is nano D -open, there exist nano D -open sets G and H such that

$l \in G$, $m \in H$ and $G \cap H \neq \emptyset$.

Therefore, $(U, \tau_R(X))$ is nT_2 space.

Every nT_2 -space is $nD-T_2$ space.

Example 4.12:

Let $U = \{a, b, c\}$; $U/R = \{\{a\}, \{b, c\}\}$

Let $X = \{a, b\}$ then $\tau_R(X) = \{\emptyset, \{a\}, \{b, c\}, U\}$

$nD_0(\tau_R(X)) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{b, c\}, U\}$

Therefore, $(U, \tau_R(X))$ is $nD-T_2$ space.

Theorem 4.13:

If $f:(U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ is a nD-continuous, injective and V is nT_2 space, then U is $nD-T_2$ space.

Proof:

For any two distinct points l_1 and l_2 of U , there exists two distinct points m_1 and m_2 of V such that $f(l_1)=m_1$ and $f(l_2)=m_2$.

Since V is a nT_2 space, there exists a disjoint nano open sets G and H in V such that $m_1 \in G$ and $m_2 \in H$.

That is, $l_1 \in f^{-1}(G)$ and $l_2 \in f^{-1}(H)$.

Since f is a nD-continuous function, $f^{-1}(G)$ and $f^{-1}(H)$ are nD-open sets in U .

Further f is injective, $f^{-1}(G) \cap f^{-1}(H) = f^{-1}(G \cap H) = f^{-1}(\varnothing) = \varnothing$.

Thus for two distinct points l_1 and l_2 of U there exist nD-open sets $f^{-1}(G)$

and $f^{-1}(H)$ such that $l_1 \in f^{-1}(G)$ and $l_2 \in f^{-1}(H)$.

Therefore, U is $nD-T_2$ space.

Theorem 4.12:

If $f:(U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$ is a injective, nD-irresolute function and V is a $nD-T_2$ space then U is also a nT_2 -space.

Proof:

Let l_1 and l_2 be a pair of distinct points in U .

Since f is injective, there exist two distinct points m_1 and m_2 of V such that $f(l_1)=m_1$ and $f(l_2)=m_2$.

Since V is $nD-T_2$ space, there exist disjoint nD-open sets G and H in V such that $m_1 \in G$ and $m_2 \in H$.

That is, $l_1 \in f^{-1}(G)$ and $l_2 \in f^{-1}(H)$

Since f is a nD-irresolute function, $f^{-1}(G)$ and $f^{-1}(H)$ are disjoint nD-open sets in U .

Thus for two distinct points l_1 and l_2 of U , there exist disjoint nD-open sets

$f^{-1}(G)$ and $f^{-1}(H)$ such that $l_1 \in f^{-1}(G)$ and $l_2 \in f^{-1}(H)$

Therefore U is $nD-T_2$ space.

Theorem 4.13:

A nano topological space is a $nD-T_2$ space if and only if for each $l \neq m$, then there exists a

nD-open set G such that $l \in G$ and $m \notin nDcl(G)$.

Proof:

Assume that U is a $nD-T_2$ space.

Let $l, m \in U$ and $l \neq m$, then there exists disjoint nD-open sets G and H such that $l \in G$ and $m \in H$ Clearly, $U-H$ is a nD-closed sets.

Since $G \cap H = \varnothing$, $G \subset U-H$

Therefore, $nD-cl(G) \subset nDcl(U-H) = U-H$

Now, $m \notin U-H$ implies $m \notin nDcl(G)$.

Conversely, suppose $l, m \in U$ and $l \neq m$.

By hypothesis, there exists a nD-open set G such that $l \in G$ and $m \notin \text{nDcl}(G)$

This implies, there exists a nD-closed set H such that $m \notin H$.

Therefore, $m \in U-H$ is a nD-open set.

Then there exists two disjoint nD-open sets G and $U-H$ such that $l \in G$ and $m \in U-H$

Therefore, U is a nD- T_2 space

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