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Hybrid Nanofluids Impact on Convective Heat Transfer Optimization: A Numerical Study

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Abstract:- The objective of this work is to conduct a numerical investigation for solving a problem related to natural convection in a square cavity containing a base fluid (water) and different compositions of hybrid nanoparticles (NTC-Cu, NTC-Al_2O_3, NTC-Ag, Cu-Al_2O_3, Cu-Ag and Al_2O_3-Ag) 50:50 vol %. The finite difference method is employed to discretize the governing equations. Numerical simulations are presented and discussed for Prandtl number Pr = 6.2 and Rayleigh numbers in the range of $10^3 \leq Ra \leq 10^5$, nanoparticle volume fractions φ between 0 to 10%. The effects of nanoparticle concentration and Rayleigh number on comprehensive heat transfer, characterized by the average Nusselt number, are illustrated and discussed for different hybrid nanoparticle compositions. The presented results show that heat transfer enhancement depends on the type of nanoparticles.

Keywords: *Hybrid nanofluid, natural convection, square cavity, Nusselt, Rayleigh.*

1. Introduction

Nanofluids are new categories or classes of fluids that are being discovered and developed, and are attracting the attention of many researchers in various fields who aim to introduce them into industrial applications. They represent a revolutionary solution aimed at developing efficient heat exchange to meet precise requirements. Above all, they meet the challenges that the world is trying to find relevant solutions to in the development of new technologies and techniques that require less energy waste and pollute less with very high efficiency. Several factors impact on the improvement of heat exchange, among them nanoparticles, which are materials with very high thermal conductivities, and very small dimensions, i.e. they are measured on a nano-scale, bearing properties that are very similar to those of a conventional material. In this context, numerous researchers and studies have explored and developed numerical, analytical models, or experiments aiming to address or find effective solutions to this issue. S. U. S. Choi and J. A. Eastman [1] were the first authors to begin studying nanofluids, they observed a significant rise in the thermal conductivity of the nanofluid in contrast to the base fluids. Based on their studies or research in this field, various numerical investigations have been performed on natural convection in cavities containing nanofluids. S. Maiga et al [2] studied the improvement in heat transfer by using nanofluids (Water-Al_2O_3, EG-Al_2O_3) in forced convection flows. The results show that the introduction of nanoparticles into a base fluid allows for a significant increase in the heat transfer coefficient. Sathiyamoorthy, M. Sathiyamoorthy et al [3] conducted a numerical study of steady natural convection within a square enclosure filled with a porous matrix. To study a natural convection heat transfer in a cavity filled with CuO–EG–Water. E. Abu-Nada and A. J. Chamkha [4] developed a numerical study based on the finite volume method. Various parameters were considered (Rayleigh number (Ra), Nusselt number (Nu) and the volume fraction of nanoparticles (φ)). The obtained results show that an increase in the Rayleigh number (Ra) and the volume fraction leads to an increase in the Nusselt number (Nu). These conclusions in good agreement with the observations made by other authors, such as [5]–[9]. To study the thermal transfer of nanofluids, E. H. Ooi and V. Popov [10] used two different models: R. L. Hamilton and O. K. Crosser [11], S. Mueller et al [12]. The purpose of using these two models was to determine the effective thermal conductivity and viscosity of nanofluids. Both of these models take into account the shape of the nanoparticles as a study parameter. H. Ghodsinezhad et al [13] conducted an experimental study which deals about a natural convection of water-Al_2O_3 nanofluids inside a rectangular enclosure, using a single-step method. They found that increasing nanoparticle concentration improved the heat transfer coefficient to an optimal value of 15% improvement at a volume fraction of 0.1%. M. Zaydan et al [14] assumed that the temperature variation at the edge of the cavity was sinusoidal to examine its effect on heat transfer for different values of Rayleigh number and volume fraction. They found that increasing the number of nanoparticles leads to an increase in the Rayleigh number, so that the volume fraction of the particles also increases. They observed that copper particles incorporated into the fluid have an impact on heat transfer, resulting in an increase in the nanofluid's thermal conductivity.

Recently, a large number of investigations been carried out into the compositions of hybrid nanoparticles. [15]–[17] reviewed reported research on the preparation, properties, characterization techniques and models of hybrid nanofluids. A. M. Rashad et al [18] were explored a convective heat transfer in a triangular cavity using a hybrid nanofluid. Their work revealed that a hybrid nanofluid containing the same percentage of Cu and Al_2O_3 dispersed in water didn't exhibit a significant impact on the Nusselt number (Nu) compared to a single-component nanofluid. V. V. Wanatasanappan et al [19] examined investigated the influence of the Al_2O_3-CuO/water-EG hybrid nanofluid on thermal conductivity. The nanofluid was prepared with varying ratios of nanoparticles (20:80, 40:60, 50:50, and 60:40) at a volumetric concentration of 1.0%. The greatest increase in thermal conductivity of the Al_2O_3-CuO nanofluid was observed for nanoparticle ratio of 60:40 showing a maximum improvement of 12.33% over the base fluid. A. J. Chamkha et al [20] They presented a numerical analysis to investigate the impact of the hybrid nanofluid on free convection inside a square cavity under a magnetic field. The findings reveal that the Nusselt number for the hybrid nanofluid surpasses that of the alumina-water nanofluid and falls below that of the copper-water nanofluid. M. Ghalambaz et al [21] examined the behavior of a novel hybrid nanofluid (Ag-MgO/water) inside a square cavity through natural convection. The results demonstrated that the heat transfer rate increased proportionally with both the Rayleigh number and thermal conductivity ratio. A. B. Çolak et al [22] investigated the thermal capacity measurements of water/Al_2O_3-Cu using the DTA method. The findings revealed an increase in specific heat capacity with rising temperature and a decrease with increasing volumetric concentration. Concerning the type of thermal load applied to boundary conditions, one can explore [23]–[25] and apply either a constant uniform thermal load, a distributed linear thermal load, or a distributed thermal load following a nonlinear law to observe the influence of these different cases on the thermal exchange between the base fluid and nanoparticles.

Our study uses the finite difference method to investigate how nanoparticles dispersed in water, located in a differentially heated square-shaped cavity, affect the system dynamics. The nanoparticles considered in this study have different thermo-physical properties, including density, thermal conductivity, specific heat and viscosity, which will promote heat exchange between this fluid and the external environment by natural convection. Depending on the results obtained concerning the influence of these parameters (Rayleigh number (Ra), nanoparticle volume fractions (φ) and type of nanoparticles) on the transference of heat, we will be able to select the most appropriate type of nanoparticles that offer an effective heat exchange with the basic fluid.

So, after presenting the resolutions and formulations of the problem in the second section, the third section will present results concerning the effects of factors influencing heat transfer between nanoparticles and the base fluid. Finally, we will give a general conclusion.

2. Mathematical modelling

The configuration studied in this work is illustrated in Figure 1 and is based on a square cavity of length L filled with a mixture of water and hybrid nanofluids. The right and left vertical walls of the cavity are maintained at T_h (hot temperature) and T_c (cold temperature) respectively. The top and bottom horizontal walls are adiabatic. The hybrid nanofluid is assumed to be incompressible and Newtonian, the flow is laminar and two-dimensional and the heat transfer by radiation is negligible. The thermophysical properties of hybrid nanofluid are considered constant, with the exception of mass density variation, which is estimated using the Boussinesq assumption.

Table 1 represents Thermophysical properties values of the base fluid which is considered, Water, and the nanoparticles introduced in this investigation such as (Al_2O_3, Cu, NTC, Ag).

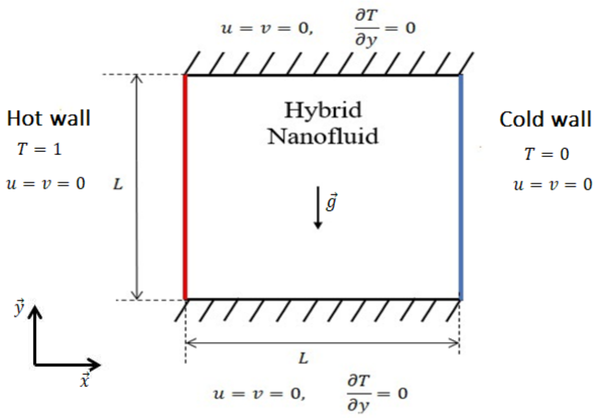


Figure 1: Representation of studied configuration with the considered boundary conditions.

Table 1: Thermophysical properties values of water (H_2O) and nanoparticles (Al_2O_3, Cu, NTC, Ag).

	Pr	ρ (Kg/m³)	CP (J/Kg.K)	K (W/m.K)	β (K^{-1}) ×10-5	α (m²/s) ×10⁻⁷
Water	6,2	997,1	4179	0,613	21	1,47
Al_2O_3	-	$3{,}97\times10^3$	765	40	0,85	131,7
Cu	-	$8{,}933\times10^3$	385	401	1,67	1163,1
NTC	-	$1{,}35\times10^3$	4179,7	3000	0.1	5,23
Ag	-	$1{,}05\times10^4$	235	429	1,89	1738,6

To solve the problem under consideration, as illustrated in Figure 1, we introduce the following of equations, which are expressed in dimensional form as follow:

Poisson's equation:

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}=-w \tag{1}$$

Vorticity equation:

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}=\frac{\mu_{hnf}}{\rho_{hnf}}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)+\frac{(\rho\beta)_{hnf}}{\rho_{hnf}}g\frac{\partial T}{\partial x} \tag{2}$$

Energy equation:

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{hnf}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{3}$$

The previous terms of equations (1-3) will be transformed into dimensionless terms using the as following equations:

$$x=\frac{x'}{L},\quad y=\frac{y'}{L},\quad U=\frac{L}{\alpha_f}u,\quad V=\frac{L}{\alpha_f}v,\quad \Omega=\frac{w\,L^2}{\alpha_f},\quad \Psi=\frac{\psi}{\alpha_f},\quad \theta=\frac{T-T_f}{T_c-T_f} \tag{4}$$

The hybrid nanofluid's thermophysical properties written as follow:

$$\begin{gathered}
\rho_{hnf} = \varphi\rho_{hp} + (1-\varphi)\rho_f \\
\alpha_{hnf} = \frac{k_{hnf}}{(\rho C_p)_{hnf}} \\
(\rho C_p)_{hnf} = \varphi(\rho C_p)_{hp} + (1-\varphi)(\rho C_p)_f \\
(\rho\beta)_{hnf} = \varphi(\rho\beta)_{hp} + (1-\varphi)(\rho\beta)_f \\
\frac{k_{nf}}{k_f} = \frac{k_{hp} + 2k_f - 2\varphi(k_f - k_{hp})}{k_{hp} + 2k_f + \varphi(k_f - k_{hp})} \\
\mu_{hnf} = \frac{\mu_f}{(1-\varphi)^{2.5}}
\end{gathered} \tag{5}$$

Where:

$$\begin{gathered}
\rho_{hp} = \frac{\varphi_{nf1}\rho_{nf1} + \varphi_{nf2}\rho_{nf2}}{\varphi}, \qquad Cp_{hp} = \frac{\varphi_{nf1}Cp_{nf1} + \varphi_{nf2}Cp_{nf2}}{\varphi} \\
\beta_{hp} = \frac{\varphi_{nf1}\beta_{nf1} + \varphi_{nf2}\beta_{nf2}}{\varphi}, \qquad k_{hp} = \frac{\varphi_{nf1}k_{nf1} + \varphi_{nf2}k_{nf2}}{\varphi} \\
\varphi = \varphi_{nf1} + \varphi_{nf2}
\end{gathered} \tag{6}$$

After introducing the dimensionless quantities considered in equation (4) into equations (1-3), we obtain them in dimensionless form as follows:

$$\frac{\partial^2\Psi}{\partial X^2} + \frac{\partial^2\Psi}{\partial Y^2} = -\Omega \tag{7}$$

$$U\frac{\partial\Omega}{\partial X} + V\frac{\partial\Omega}{\partial Y} = \beta_1\left(\frac{\partial^2\Omega}{\partial X^2} + \frac{\partial^2\Omega}{\partial Y^2}\right) + \beta_2\frac{\partial\theta}{\partial X} \tag{8}$$

Where:

$$\beta_1 = \Pr\frac{1}{(1-\varphi)^{2.5}\left((1-\varphi) + \frac{\varphi\rho_{hp}}{\rho_f}\right)} \tag{9}$$

$$\beta_2 = Ra\,\Pr\left[\frac{1}{1 + \frac{\varphi\rho_{hp}}{(1-\varphi)\rho_f}} + \frac{\beta_{hp}}{\beta_f}\frac{1}{1 + \frac{(1-\varphi)\rho_f}{\varphi\rho_{hp}}}\right] \tag{10}$$

$$U\frac{\partial\theta}{\partial X} + V\frac{\partial\theta}{\partial Y} = \beta_3\left(\frac{\partial^2\theta}{\partial X^2} + \frac{\partial^2\theta}{\partial Y^2}\right) \tag{11}$$

$$\beta_3 = \frac{\lambda_{nf}}{\lambda_f\left[(1-\varphi) + \varphi\frac{(\rho C_p)_{hp}}{(\rho C_p)_f}\right]} \tag{12}$$

$$Pr = \frac{\mu_f}{\rho_f \alpha_f} = \frac{\nu_f}{\alpha_f}, \qquad Ra = \frac{g\, \beta_f\, L^3\, (T_c - T_f)}{\nu_f\, \alpha_f}$$

The local Nusselt number (Nu) and the average Nusselt number ($\overline{Nu}$) are given by the following expression in equations (13) and (14):

$$Nu = -\frac{k_{nf}}{k_f}\frac{\partial T}{\partial x} \tag{13}$$

$$\overline{Nu} = \int_0^1 Nu(y)dy \tag{14}$$

3. Results and discussions

In this present section, the obtained results, will be presented and discussed. We will concentrate on the display of isotherms, streamlines and the average Nusselt number ($\overline{Nu}$). These results are obtained for water and the mixture of water/carbon nanotubes-copper (Water/NTC-Cu) with Ra (Rayleigh number) varying from 10^3 to 10^5 and φ (nanoparticles volume fraction) ranging from 0 to 10%. Moreover, we compared various combinations of nanoparticles to understand their influences on heat transfer.

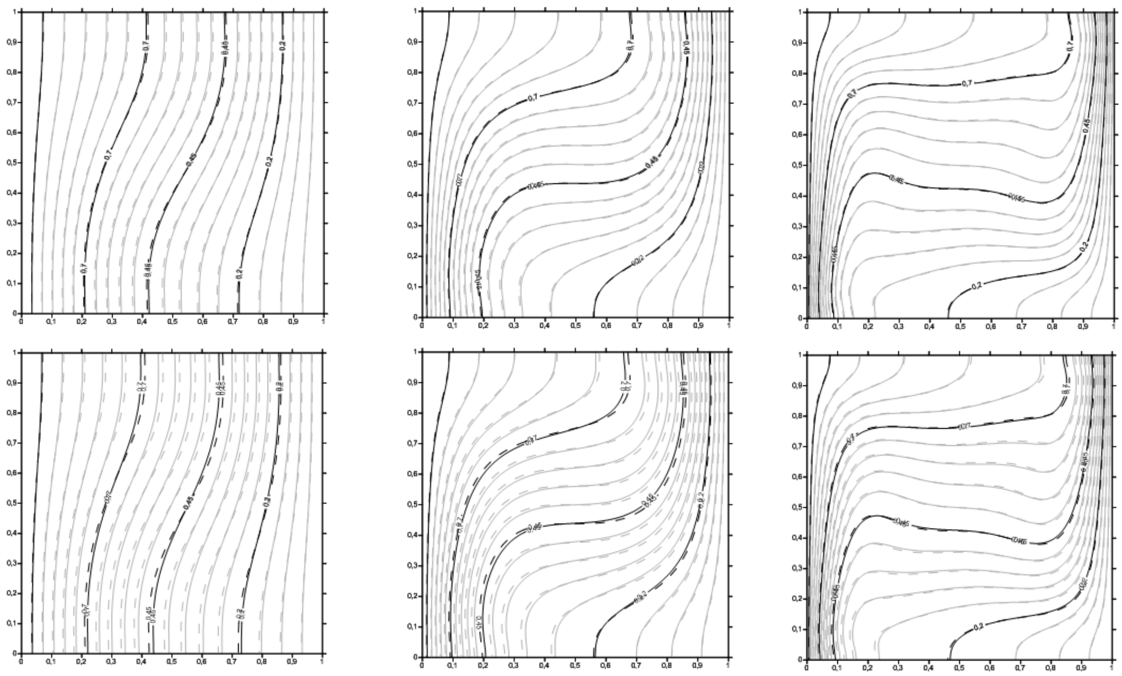
a. Isotherms and streamlines

Figure 2 represents isotherms and streamlines for the hybrid nanofluid (Water/NTC-Cu) at various Rayleigh numbers (Ra = 10^3, 10^4 and 10^5) and different volume fraction values (φ = 2%, 6%, and 10%).

Ra = 10^3 **Ra = 10^4** **Ra = 10^5**

φ = 2%

φ = 6%



$\varphi = 10\%$

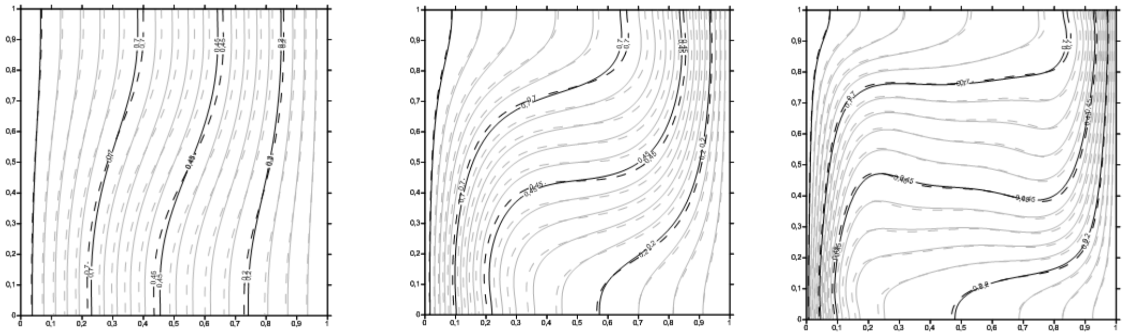


Figure 2: The isotherms of the Water (- - - -) and the hybrid nanofluid (Water/NTC-Cu) (——) for different values of φ and Ra.

Ra = 10^3 | Ra = 10^4 | Ra = 10^5

$\varphi = 2\%$

$\varphi = 6\%$

$\varphi = 10\%$

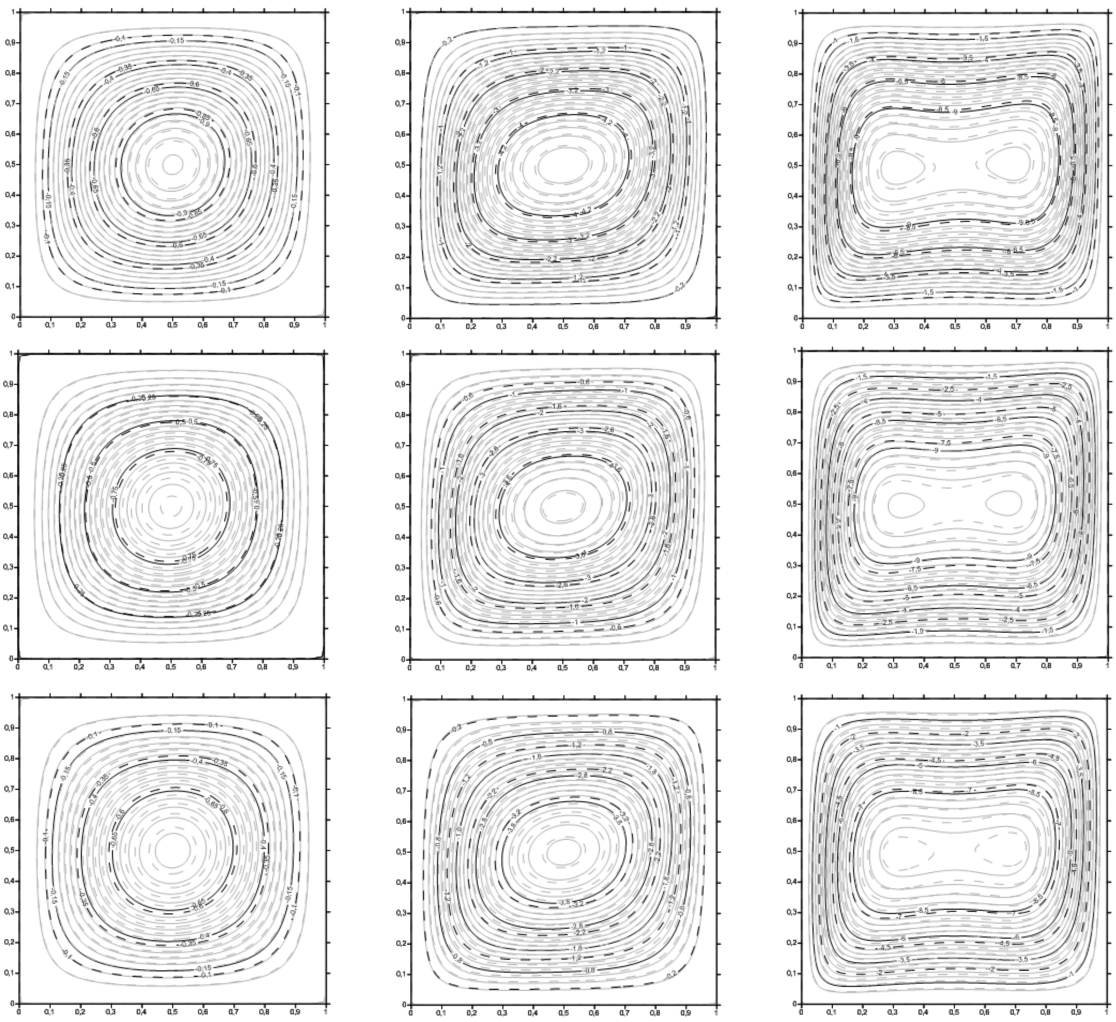


Figure 3: The streamlines of the Water (- - - -) and the hybrid nanofluid (Water/NTC-Cu) (——) for various values of φ and Ra.

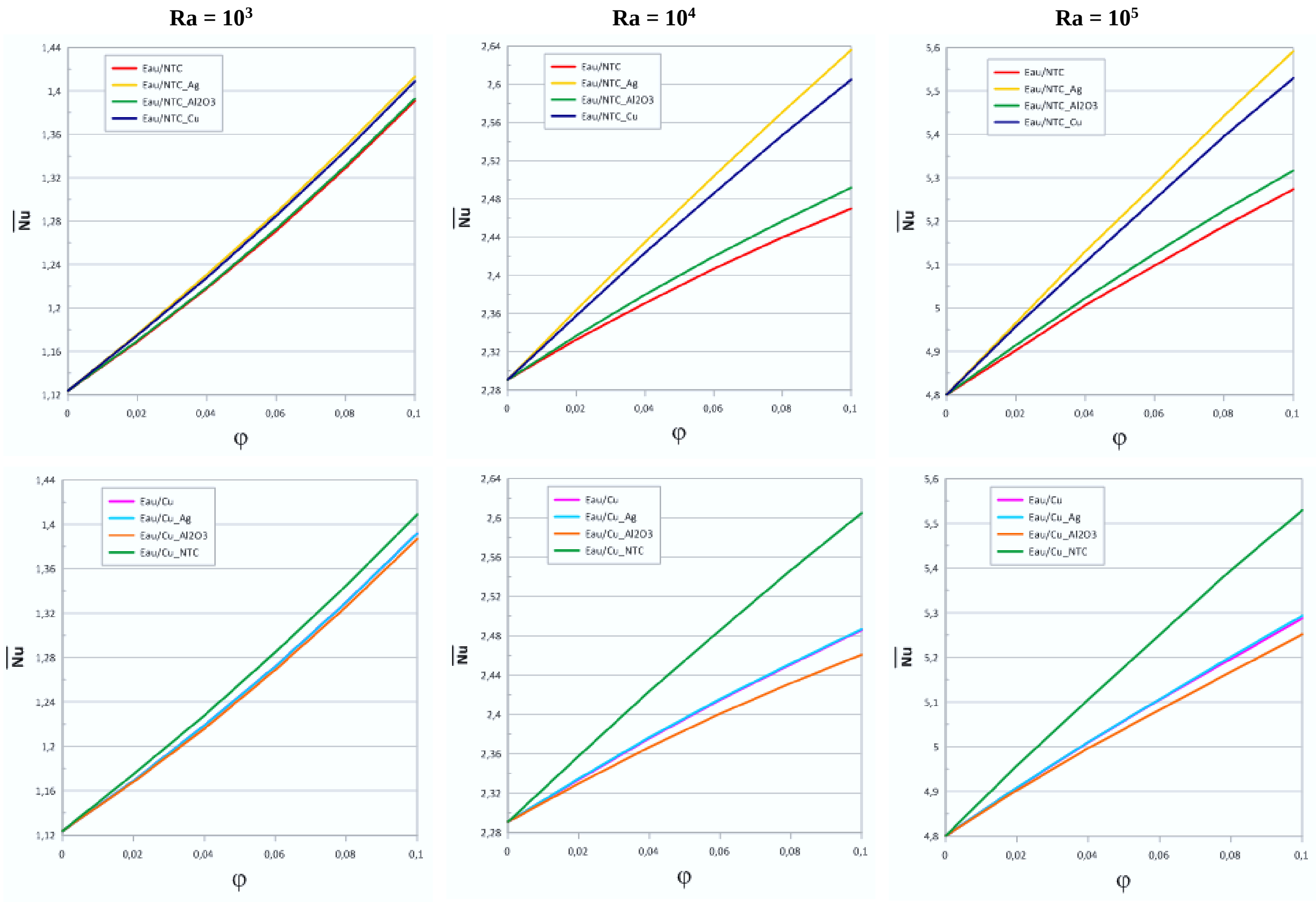
The temperature distribution and the flow pattern of the water and the hybrid nanofluid are shown in Figures 2 and 3, respectively, for different values of the volume fraction φ and the Rayleigh number Ra.

The figures demonstrate that the temperature contours (isotherms) become more bent and move closer to the walls as Ra and φ increase. This implies that the heat transfer rate is enhanced by the combined effects of natural convection and nanofluid properties. A noticeable difference in the isotherms between the hybrid nanofluid and the pure fluid is also observed, indicating that the hybrid nanofluid has a higher thermal conductivity and a lower thermal diffusivity than the water. Moreover, the figures reveal that the flow lines (streamlines) tend to concentrate near the lateral walls as ϕ increases, implying that the flow velocity is higher in those regions, which is typical of the boundary layer flow. The figures also show the existence of dynamic and thermal boundary layers along the isothermal walls, where the velocity and temperature gradients are high.

b. Effect of different nanoparticle combinations on the average Nusselt number

Figure 4 demonstrates the effect of nanoparticle type on the variation of the average Nusselt number for the following combinations:

- Group 1: Water/NTC, Water/NTC-Ag, Water/NTC-Al_2O_3 and Water/NTC-Cu.
- Group 2: Water/Cu, Water/Cu-Ag, Water/Cu-Al2O3, Water/Cu-NTC.
- Group 3: Water/Al_2O_3, Water/Al_2O_3-Ag, Water/Al_2O_3-Cu, Water/Al_2O_3-NTC.



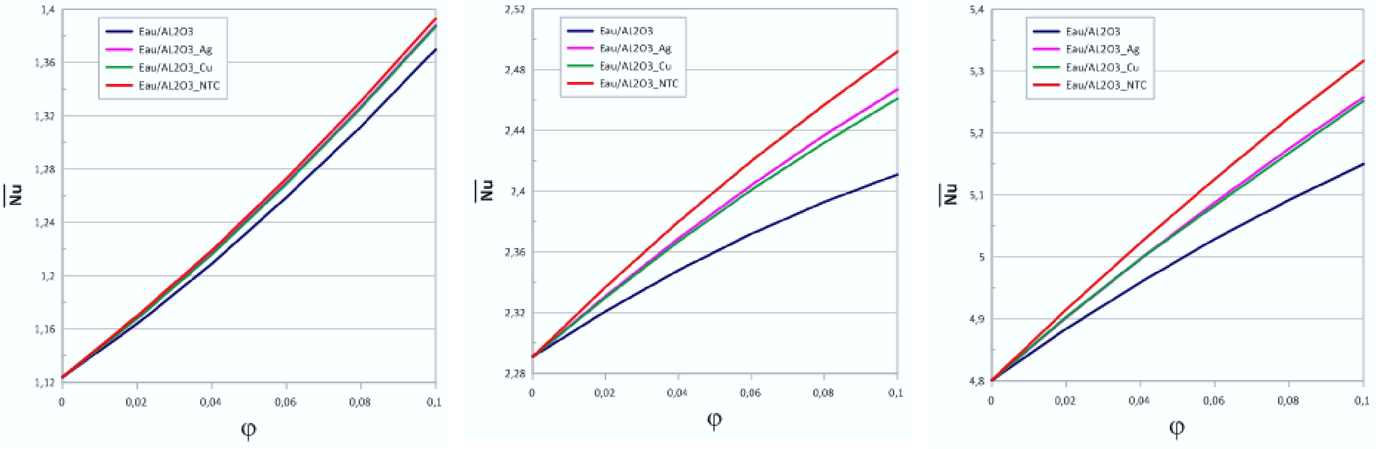


Figure 4: Variation of the average Nusselt number as a function of volume fraction for different nanoparticle combinations.

Figure 4 shows the heat transfer rate of different hybrid nanofluids based on their composition and the Rayleigh number.

According to Figure 4, the heat transfer rate is highest for the hybrid nanofluid Water/NTC-Ag among the first group of combinations, while the hybrid nanofluid Water/Cu-NTC is the most effective among the second group of combinations. The hybrid nanofluid Water/Al_2O_3-NTC is the most balanced among the third group of combinations, with a moderate heat transfer rate. Figure 4 also displays how the average Nusselt number ($\overline{Nu}$) changes with the volume fraction of the nanoparticles for different Rayleigh numbers. It is evident that the average Nusselt number ($\overline{Nu}$) increases with the volume fraction of the nanoparticles, which means that adding more nanoparticles to water improves the heat transfer performance.

4. Conclusion

This work presents a numerical investigation of convective heat transfer in a square cavity filled with a mixture of water and various nanoparticle combinations. The cavity walls were maintained at different temperatures. Our main results indicate that the influence of nanoparticles on convection is particularly pronounced at high Rayleigh numbers. An increase in the nanoparticle volume fraction enhances further heat transfer. The increase in Rayleigh number and volume fraction significantly affects both streamlines and isotherms. Heat transfer in the presence of different hybrid nanofluid combinations, namely Water/NTC-Ag (Group 1), Water/Cu-NTC (Group 2), and Water/Al_2O_3-NTC (Group 3), improves relative to other combinations. Consequently, the type of nanoparticle plays crucial role in improving heat transfer.

References

[1] S. U. S. Choi and J. A. Eastman, 'Enhancing thermal conductivity of fluids with nanoparticles', 1995 International mechanical engineering congress and exhibition, 12-17, 1995.

[2] S. E. B. Maïga, S. J. Palm, C. T. Nguyen, G. Roy, and N. Galanis, 'Heat transfer enhancement by using nanofluids in forced convection flows', Int. J. Heat Fluid Flow, vol. 26, no. 4, pp. 530–546, 2005.

[3] M. Sathiyamoorthy, T. Basak, S. Roy, and I. Pop, 'Steady natural convection flows in a square cavity with linearly heated side wall(s)', Int. J. Heat Mass Transf., vol. 50, no. 3, pp. 766–775, 2007.

[4] E. Abu-Nada and A. J. Chamkha, 'Effect of nanofluid variable properties on natural convection in enclosures filled with a CuO–EG–Water nanofluid', Int. J. Therm. Sci., vol. 49, no. 12, pp. 2339–2352, 2010.

[5] A. H. Bhuiyana, Md. S. Alam, and M. A. Alim, 'Natural Convection of Water-based Nanofluids in a Square Cavity with Partially Heated of the Bottom Wall', Procedia Eng., vol. 194, pp. 435–441, 2017.

[6] A. Saha, K. Ghosh, and N. K. Manna, 'Analysis of nanofluid based natural convection in a square cavity applying corner heating and cooling', IOP Conf. Ser. Mater. Sci. Eng., vol. 1080, no. 1, p. 012049, 2021.

[7] A. A. A. Arani, M. Mahmoodi, and M. Amini, 'Free Convection in a Nanofluid Filled Square Cavity with an Horizontal Heated Plate', Defect Diffus. Forum, vol. 312–315, pp. 433–438, 2011.

[8] W. Bentoto, M. Zaydan, H. A. Alaoui, E. Essaghir, and R. Sehaqui, 'Mixed convection of a MHD oscillatory laminar flow of a nanofluid (Gold-Kerosene oil) in a vertical channel', Stat. Optim. Inf. Comput., vol. 11, no. 1, Art. no. 1, 2023.

[9] E. C. Okonkwo, I. Wole-Osho, I. W. Almanassra, Y. M. Abdullatif, and T. Al-Ansari, 'An updated review of nanofluids in various heat transfer devices', J. Therm. Anal. Calorim., vol. 145, no. 6, pp. 2817–2872, Sep. 2021, doi: 10.1007/s10973-020-09760-2.

[10] E. H. Ooi and V. Popov, 'Numerical study of influence of nanoparticle shape on the natural convection in Cu-water nanofluid', Int. J. Therm. Sci., vol. 65, pp. 178–188, 2013.

[11] R. L. Hamilton and O. K. Crosser, 'Thermal Conductivity of Heterogeneous Two-Component Systems', Ind. Eng. Chem. Fundam., vol. 1, no. 3, pp. 187–191, 1962.

[12] S. Mueller, E. W. Llewellin, and H. M. Mader, 'The rheology of suspensions of solid particles', Proc. R. Soc. Math. Phys. Eng. Sci., vol. 466, no. 2116, pp. 1201–1228, 2009.

[13] H. Ghodsinezhad, M. Sharifpur, and J. P. Meyer, 'Experimental investigation on cavity flow natural convection of Al2O3–water nanofluids', Int. Commun. Heat Mass Transf., vol. 76, pp. 316–324, 2016.

[14] M. Zaydan et al '(PDF) Fourth-Order Compact Formulation for the Resolution of Heat Transfer in Natural Convection of Water-Cu Nanofluid in a Square Cavity with a Sinusoidal Boundary Thermal Condition 2016'.

[15] D. Dhinesh Kumar and A. Valan Arasu, 'A comprehensive review of preparation, characterization, properties and stability of hybrid nanofluids', Renew. Sustain. Energy Rev., vol. 81, pp. 1669–1689, 2018.

[16] M. Hemmat Esfe, M. Kiannejad Amiri, and A. Alirezaie, 'Thermal conductivity of a hybrid nanofluid', J. Therm. Anal. Calorim., vol. 134, no. 2, pp. 1113–1122, 2018.

[17] M. M. Rashidi et al., 'Thermophysical Properties of Hybrid Nanofluids and the Proposed Models: An Updated Comprehensive Study', Nanomaterials, vol. 11, no. 11, Art. no. 11, 2021.

[18] A. M. Rashad, A. J. Chamkha, M. A. Ismael, and T. Salah, 'Magnetohydrodynamics Natural Convection in a Triangular Cavity Filled With a Cu-Al2O3/Water Hybrid Nanofluid With Localized Heating From Below and Internal Heat Generation', J. Heat Transf., vol. 140, no. 072502, 2018.

[19] V. V. Wanatasanappan, M. Z. Abdullah, and P. Gunnasegaran, 'Thermophysical properties of Al2O3-CuO hybrid nanofluid at different nanoparticle mixture ratio: An experimental approach', J. Mol. Liq., vol. 313, p. 113458, 2020.

[20] A. J. Chamkha, R. Yassen, M. A. Ismael, A. M. Rashad, T. Salah, and H. A. Nabwey, 'MHD Free Convection of Localized Heat Source/Sink in Hybrid Nanofluid-Filled Square Cavity', J. Nanofluids, vol. 9, no. 1, pp. 1–12, 2020.

[21] M. Ghalambaz, A. Doostani, E. Izadpanahi, and A. J. Chamkha, 'Conjugate natural convection flow of Ag–MgO/water hybrid nanofluid in a square cavity', J. Therm. Anal. Calorim., vol. 139, no. 3, pp. 2321–2336, 2020.

[22] A. B. Çolak, O. Yıldız, M. Bayrak, A. Celen, A. Dalkilic, and S. Wongwises, 'Experimental Study on the Specific Heat Capacity Measurement of Water-Based Al2O3-Cu Hybrid Nanofluid by using Differential Thermal Analysis Method', Curr. Nanosci., vol. 15, 2019.

[23] Y. El Khouddar et al, 'An analytical approach to geometrically nonlinear free and forced vibration of piezoelectric functional gradient beams resting on elastic foundations in thermal environments', Mech. Adv. Mater. Struct., vol. 30, no. 1, pp. 131–143, 2021.

[24] Y. El Khouddar et al, 'Influence of hygro-thermal effects on the geometrically nonlinear free and forced vibrations of piezoelectric functional gradient beams with arbitrary number of concentrated masses', Arch. Appl. Mech., vol. 92, no. 9, pp. 2767–2784, 2022.

[25] Y. El Khouddar et al, 'Non-linear forced vibration analysis of piezoelectric functionally graded beams in thermal environment.

Nomenclature

Symboles latins:

C_p	Specific heat of fluid at constant pressure	$[J.Kg^{-1}K^{-1}]$
$g, \vec{g}$	Acceleration and acceleration vector of gravity	$[m.s^{-2}]$
L	Cavity length	$[m]$
T	Temperature	$[K]$
θ	Dimensionless temperature	
(u, v)	Cartesian components of the velocity vector	$[m.s^{-1}]$
(U, V)	Dimensionless components of the velocity vector	
V	Volume	$[m^3]$
(x', y')	Cartesian coordinates	$[m]$
(x, y)	Dimensionless Cartesian coordinates	

Greek symbols:

α	Thermal diffusivity	$[m^2.s^{-1}]$
μ	Dynamic viscosity	$[N.s.m^{-2}]$
ν	Kinematic viscosity	$[m^2.s^{-1}]$
ρ	Density	$[Kg.m^{-3}]$
β	Coefficient of thermal expansion	$[K^{-1}]$
K	Thermal conductivity	$[W.m^{-1}.K^{-1}]$
ω	Vorticity function	$[s^{-1}]$
Ω	Dimensionless vorticity function	
ψ	Current function	$[m^2.s^{-1}]$
Ψ	Dimensionless current function	
φ	Volume fraction	

Dimensionless parameters:

$P_r = \frac{\nu_f}{\alpha}$	Prandtl number
$Ra = \frac{g\,\beta_f\,L^3\,(T_c - T_f)}{\nu_f\,\alpha_f}$	Rayleigh number
$Nu = \frac{h\,L}{\lambda}$	Nusselt number

Subscripts:

C	Hot
F	Cold
f	Fluid
hnf	Hybrid nanofluid
hp	Hybrid nanoparticles