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4-Total Geometric Mean Cordial Labelling of Star Related Graphs

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Abstract

In this paper, we investigate 4-total geometric mean cordial labeling behavior of star related graphs.

Keywords: Star graph, Corona graph, 4-total geometric mean cordial labeling.

Introduction

Graphs considered here are finite, simple and undirected. Graph labeling we refer to Gallian [2016]. We pursue Harary [1969] for symbols and phrases. The motivation of the works done by Cahit [1987], Vaidya and Shah [2014], Ponraj et al. [2021, 2022]. The notion of k-total geometric mean cordial graphs was introduced by Vennila and Vidhyarani [a]. We investigated 4-total geometric mean cordial graphs of some families of graph in Vennila and Vidhyarani [b]. In this paper, we investigate 4-total geometric mean cordial labeling (4-TGMC) of star related graphs.

Definition 1.1. *Let Z be a* (p,q) *graph. Let* $f : V(Z) \to \{1,2,3,\cdots,k\}$ *be a function where* $k \in N$ *and* $k > 1$ *for each edge st assign the label* $f(st) =$

lp$\overline{f(s)f(t)}$m. *f is called k-total geometric mean cordial labeling of Z if* $|t_{mf}(i) - t_{mf}(j)| \leq 1$, $i,j \in \{1,2,3,\cdots,k\}$ *where* $t_{mf}(y)$ *denotes the total number of vertices and edges labeled with y,* $y \in \{1,2,3,\cdots,k\}$. *A graph that admits the k-total geometric mean cordial (k-TGMC) labeling is called k-total geometric mean cordial graph.*

Definition 1.2. *Ponraj et al. [2022] Let* Z_1 *be a* (p_1,q_1) *graph and* Z_2 *be a* (p_2,q_2) *graph. The corona of* Z_1 *with* Z_2 *is the graph* $Z_1 \odot Z_2$ *obtained by taking one copy of* Z_1, p_1 *copies of* Z_2 *and joining the* j^{th} *dot of* Z_1 *by an edge to every dot in the* j^{th} *copy of* Z_2 *where* $1 \leq j \leq p_1$.

Definition 1.3. *Ponraj et al. [2021] The complement* $\overline{Z}$ *of a graph Z also has* $V(Z)$ *as its dot set, but two dots are adjacent in* $\overline{Z}$ *if and only if they are not adjacent in Z.*

Definition 1.4. *Ponraj et al. [2022] The graph* $(K_{1,3} * K_{1,r})$ *is attained from* $K_{1,3}$ *by attaching root of a star* $K_{1,r}$ *at each pendant dot of* $K_{1,3}$.

Definition 1.5. *Ponraj et al. [2022] Consider two copies of graph Z namely* Z_1 *and* Z_2. *Then the graph* $Z' = (Z_1 \Delta Z_2)$ *is the graph obtained by joining the apex dots of* Z_1 *and* Z_2 *by an edge as well as to new dot x.*

Main Results′

Theorem 2.1. *Let Z be a comb and Z be the graph obtained by attaching $K_{1,2}$*
′ at each pendant dot of a comb graph. Then Z admits 4-TGMC labeling.
′

Proof. Let Z be a comb and Z be the graph obtained by attaching $K_{1,2}$ at each pendant dot of Z. Let whose dots be s_j,t_j,a_j and b_j $(1 \le j \le r)$.

4-Total geometric mean cordial labeling

Case 1: when r is even

The r dots $s_1,s_2,s_3,\cdots,s_r$ are labeled with 1. The r dots $t_1,t_2,t_3,\cdots,t_r$ are labeled with 3. The r dots $a_1,a_2,\cdots,a_{\frac{r}{2}}$ and $b_1,b_2,\cdots,b_{\frac{r}{2}}$ are labeled with 2.

The r dots $a_{\frac{r+2}{2}},a_{\frac{r+4}{2}},\cdots,a_r$ and $b_{\frac{r+2}{2}},b_{\frac{r+4}{2}},\cdots,b_r$ are labeled with 4.

Case 2 : when r is odd

The r dots $s_1,s_2,s_3,\cdots,s_r$ are labeled with 1. The r dots $t_1,t_2,t_3,\cdots,t_r$ are labeled with 3. The r dots $a_1,a_2,\cdots,a_{\frac{r+1}{2}}$ and $b_1,b_2,\cdots,b_{\frac{r-1}{2}}$ are labeled with

2. The r dots $a_{\frac{r+3}{2}},a_{\frac{r+5}{2}},\cdots,a_r$ and $b_{\frac{r+1}{2}},b_{\frac{r+3}{2}},\cdots,b_r$ are labeled with 4.

Clearly $t_{mf}(1) = 2r-1$ and $t_{mf}(2) = t_{mf}(3) = t_{mf}(4) = 2r$.

Hence Z is 4-TGMC graph. □

Theorem 2.2. *The graph $K_{1,2} * K_{1,r}$ admits 4-TGMC labeling.*

Proof. Let $V(K_{1,2} * K_{1,r}) = \{s,s_1,s_2,x_j,y_j : 1 \le j \le r\}$ $E(K_{1,2} * K_{1,r}) = \{ss_1,ss_2\} \bigcup \{s_1x_j,s_2y_j : 1 \le j \le r\}$.

Here $|V(K_{1,2} * K_{1,r})| + |E(K_{1,2} * K_{1,r})| = 4r + 5$.

Allocate the label 1, 3, 4 to the dots s,s_1,s_2. The r dots $x_1,x_2,x_3,x_4,\cdots,x_r$ are labeled with 1. The r dots $y_1,y_2,y_3,y_4,\cdots,y_r$ are labeled with 3.

Clearly $t_{mf}(1) = t_{mf}(3) = t_{mf}(4) = r+1$ and $t_{mf}(2) = r+2$. The function f is 4-TGMC labeling. □

Theorem 2.3. *The graph $K_{1,3} * K_{1,r}$ admits 4-TGMC labeling.*

Proof. Let $V(K_{1,3} * K_{1,r}) = \{s,s_1,s_2,s_3,x_j,y_j,z_j : 1 \le j \le r\}$ $E(K_{1,3} * K_{1,r}) = \{ss_1,ss_2,ss_3\} \bigcup \{s_1x_j,s_2y_j,s_3z_j : 1 \le j \le r\}$.

Here $|V(K_{1,3} * K_{1,r})| + |E(K_{1,3} * K_{1,r})| = 6r + 7$.

Allocate the label 2 to the dot s and 4 to the dots s_1,s_2,s_3.

Case 1: r is even

The $\frac{3r+2}{2}$ dots $x_1,x_2,\cdots,x_r$ and $y_1,y_2,\cdots,y_{\frac{r+2}{2}}$ are labeled with 1. The $\frac{3r-2}{2}$ dots $z_1,z_2,\cdots,z_r$ and $y_{\frac{r+4}{2}},y_{\frac{r+6}{2}},\cdots,y_r$ are labeled with 3.

Case 2: r is odd

The $\frac{3r+3}{2}$ dots $x_1,x_2,\cdots,x_{\frac{r}{2}}$ and $y_1,y_2,\cdots,y_{\frac{r+3}{2}}$ are labeled with 1. The $\frac{3r-3}{2}$ dots $z_1,z_2,\cdots,z_r$ and $y_{\frac{r+5}{2}}, y_{\frac{r+7}{2}}, \cdots, y_r$ are labeled with 3.

The function f is 4-TGMC labeling

Nature of r	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$	$t_{mf}(4)$
r is odd	$\frac{3r+3}{2}$	$\frac{3r+5}{2}$	$\frac{3r+3}{2}$	$\frac{3r+3}{2}$
r is even	$\frac{3r+2}{2}$	$\frac{3r+4}{2}$	$\frac{3r+4}{2}$	$\frac{3r+4}{2}$

□

Theorem 2.4. *The graph* $(K_{1,r}^{(1)} \Delta K_{1,r}^{(2)})$ *admits 4-TGMC labeling.*

Proof. Let $s_1,s_2,\cdots,s_r$ be the pendant dots of $K_{1,r}^{(1)}$ and $t_1,t_2,\cdots,t_r$ be the pendant dots of $K_{1,r}^{(2)}$. Let s and t be the dots of $K_{1,r}^{(1)}$ and $K_{1,r}^{(2)}$ which is adjacent to s_j,t_j $(1 \le j \le r)$ respectively. Assume that a new common dot x is adjacent to s and t.

Here $|V(K_{1,r}^{(1)} \Delta K_{1,r}^{(2)})| + |E(K_{1,r}^{(1)} \Delta K_{1,r}^{(2)})| = 4r + 6$.

Allocate the labels 1,3,4 to the dots x,s,t. Now consider the dots $s_1,s_2,\cdots,s_r$. The r dots $s_1,s_2,\cdots,s_r$ are labeled with 1, and the r dots $t_1,t_2,\cdots,t_r$ are labeled with 3.

Clearly $t_{mf}(1) = t_{mf}(3) = r + 1$ and $t_{mf}(2) = t_{mf}(4) = r + 2$. The function f is 4-TGMC labeling. □

Theorem 2.5. *The graph* $H_{(r)} \odot \overline{K_1}$ *admits 4-TGMC labeling for all* $r \ge 2$.

Proof. Consider $V(H_{(r)}) = \{s_j,t_j : 1 \le j \le r\}$ and let x_j,y_j $(1 \le j \le r)$ be the pendant dots adjacent to s_j,t_j $(1 \le j \le r)$. Here $|V(H_{(r)} \odot \overline{K_1})| + |E(H_{(r)} \odot \overline{K_1})| = 8r - 1$. The $2r$ dots $x_1,x_2,\cdots,x_r$ and $y_1,y_2,\cdots,y_r$ are labeled with 1. The r dots $s_1,s_2,\cdots,s_r$ are labeled with 3, and the r dots $t_1,t_2,\cdots,t_r$ are labeled with 4. Clearly $t_{mf}(1) = t_{mf}(2) = t_{mf}(4) = 2r$ and $t_{mf}(3) = 2r - 1$. The function f is 4-TGMC labeling. □

Theorem 2.6. *The graph* $H_{(r)} \odot \overline{K_2}$ *admits 4-TGMC labeling for all* $r \ge 2$.

Proof. Consider $V(H_{(r)} \odot \overline{K_2}) = \{s_j,t_j,a_j,b_j,x_j,y_j : 1 \le j \le r\}$ and $E(H_{(r)} \odot \overline{K_2}) = \{s_js_{j+1},t_jt_{j+1} : 1 \le j \le r-1\} \bigcup \{s_ja_j,s_jb_j,t_jx_j,t_jy_j : 1 \le j \le r\}$ and let a_j,b_j $(1 \le j \le r)$ be the pendant dots adjacent to s_j $(1 \le j \le r)$ and x_j,y_j $(1 \le j \le r)$ be the pendant dots adjacent to t_j $(1 \le j \le r)$.

Here $|V(H_{(r)} \odot \overline{K_2})| + |E(H_{(r)} \odot \overline{K_2})| = 12r - 1$.

The $2r$ dots $s_1,s_2,\cdots,s_r$ and $y_1,y_2,\cdots,y_r$ are labeled with 3. The $3r$ dots $a_1,a_2,\cdots,a_r$, $b_1,b_2,\cdots,b_r$ and $x_1,x_2,\cdots,x_r$ are labeled with 1, and the r dots $t_1,t_2,\cdots,t_r$ are labeled with 4.

Clearly $t_{mf}(1) = t_{mf}(2) = t_{mf}(4) = 3r$ and $t_{mf}(3) = 3r - 1$. The function f is 4-TGMC labeling. □

Theorem 2.7. *For* $m \ge 2$, *the graph* mK_3, *m copies of* K_3 *is 4-TGMC graph.*

Proof. The graph mK_3 which is m copies of triangles. Let the dot set of mK_3 be

$V = V1 \cup V2 \cup V3 \cup \cdots \cup Vm$, $V = \{V1j, V2j, V3j\}$ Define a function $f : V(G) \to \{1,2,3,4\}$

Case 1: When m is even

For $1 \le j \le \frac{m}{2}$, for the labeling pattern of the dots $V_1^j = V_2^j = 1$, $V_3^j = 2$.

For $\frac{m+2}{2} \le j \le m$, for the labeling pattern of the dots $V_1^j = V_2^j = 3$, $V_3^j = 4$. **Case 2:** When m is odd

For $1 \le j \le \frac{m-1}{2}$, for the labeling pattern of the dots $V_1^j = V_2^j = 1, V_3^j = 2$. *4-Total geometric mean cordial labeling*

For $j = \frac{m+1}{2}$, for the labeling pattern of the dots $V_1^j = 1, V_2^j = 3$.
For $\frac{m+1}{2} \le j \le m, V_3^j = 3$.
For $\frac{m+3}{2} \le j \le m$, for the labeling pattern of the dots $V_1^j = V_2^j = 3$.
For $j = \frac{m+1}{2} \le j \le m, V_3^j = 4$.
The function *f* is 4-TGMC labeling.

Nature of m	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$	$t_{mf}(4)$
m is odd	$\frac{3m-1}{2}$	$\frac{3m+1}{2}$	$\frac{3m-1}{2}$	$\frac{3m+1}{2}$
m is even	$\frac{3m}{2}$	$\frac{3m}{2}$	$\frac{3m}{2}$	$\frac{3m}{2}$

□

Theorem 2.8. *The graph $P_r \odot \overline{K_3}$ is 4-TGMC for all r.*

Proof. Let $p_1, p_2, \cdots, p_r$ be the dots of the path. Let s_j, t_j, w_j be the pendant dots adjacent to p_j $(1 \le j \le r)$.

Note that $|V(P_r \odot \overline{K_3})| + |(P_r \odot \overline{K_3})| = 8r - 1$.
Now consider the dots $p_1, p_2, \cdots, p_r$. The n dots $p_1, p_2, \cdots, p_r$ are labeled with 3. The $2r$ dots $s_1, s_2, \cdots, s_r$ and $t_1, t_2, \cdots, t_r$ are labeled with 1. The r dots $w_1, w_2, \cdots, w_r$ are labeled with 4.
Clearly $t_{mf}(1) = t_{mf}(2) = t_{mf}(4) = 2r$ and $t_{mf}(3) = 2r - 1$. Hence *f* is 4-TGMC labeling. □

Theorem 2.9. *The graph $C_r \odot \overline{K_3}$ is 4-TGMC for all r.*

Proof. Let $c_1, c_2, \cdots, c_r$ be the dots of the cycle. Let s_j, t_j, w_j be the pendant dots adjacent to c_j $(1 \le j \le r)$.

Note that $V|\overline{C_r \odot K_3}| + E|C_r \odot \overline{K_3}| = 8r$.
Now consider the dots $c_1, c_2, \cdots, c_r$. The n dots $c_1, c_2, \cdots, c_r$ are labeled with 3. The $2r$ dots $s_1, s_2, \cdots, s_r$ and $t_1, t_2, \cdots, t_r$ are labeled with 1. The r dots $w_1, w_2, \cdots, w_r$ are labeled with 4.
Clearly $t_{mf}(1) = t_{mf}(2) = t_{mf}(3) = t_{mf}(4) = 2r$. Hence *f* is 4-TGMC labeling. □

Conclusion

Here, we have proved that 4-TGMC labeling behavior of star related graphs and like $K_{1,2} * K_{1,r}$, $(K_{1,3} * K_{1,r})$, $(K_{1,r}^{(1)} \Delta K_{1,r}^{(2)})$, $H_{(r)} \odot K_1$, $H_{(r)} \odot \overline{K_2}$, $P_r \odot \overline{K_3}$ $C_r \odot \overline{K_3}$.

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