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Evaluation and Selection of Suppliers Using Intuitionistic Fuzzy DEA -TOPSIS Methods

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Abstract:- In the dynamic landscape of global supply chains, the effective evaluation and selection of suppliers play a pivotal role in ensuring organizational success and sustainability. This study proposes a comprehensive approach that integrates Intuitionistic Fuzzy Data Envelopment Analysis (IF-DEA) with Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) methods to enhance the accuracy and robustness of supplier assessment. The Intuitionistic Fuzzy DEA framework enables the consideration of uncertainties and vagueness inherent in supplier performance data, providing a more realistic representation of the evaluation process. It considers not only the crisp data but also the hesitancy and ambiguity associated with subjective judgements. Furthermore, the integration of TOPSIS allows for the determination of the optimal supplier from a set of alternatives by considering both ideal and nadir solutions. The proposed methodology is illustrated through a practical case study, wherein the performance of suppliers is assessed based on multiple criteria, including cost efficiency, delivery reliability, and product quality. The results demonstrate the efficacy of the combined IF-DEA-TOPSIS approach in offering a systematic and comprehensive decision-making tool for supplier evaluation, considering both quantitative and qualitative aspects. This research contributes to the evolving field of supplier management by providing a novel and flexible methodology that addresses the challenges posed by uncertainty and subjectively in supplier assessment. The proposed approach not only aids organizations in making informed and strategic decisions but also contributes to the advancement of decision science in the context of supplier selection in supply chain management.

Keywords: Supply chain management; DEA; TOPSIS. Intuitionistic Fuzzy sets

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1. Introduction

At the source level, decision-making regarding the selection of a supplier at the top of the supply chain network is crucial for delivering value. Choosing an efficient supplier that best meets the organization's needs is a challenging and time-consuming task due to the diverse performance/evaluation criteria. These criteria encompass not only quantitative measures such as cost and just-in-time delivery rates but also qualitative aspects like trustworthiness, management ability, environmental regulation compliance, long-term relationship viability, and process and design capabilities. Notably, cost and benefit criteria play a pivotal role in decision-making, dependent on the evaluation of various qualitative and quantitative factors. Numerous Multicriteria Decision-Making (MCDM) systems have been created to enable decision-makers to evaluate alternative suppliers.

De Boer et al. [1] explored various supplier selection approaches, including issue characterization, performance requirements, and supplier credentials, offering new ideas and strategies. Choosing a provider involves a process that requires selecting an option from a range of choices. Dickson [2] provides a comprehensive review of corporate and individual vendor selection processes, along with a list of variables to consider. Chai et al. [3] conducted extensive studies on selection strategies, developing a paradigm of process analysis based on logical aspects of the problem ecosystem, and approach.

Farrell and Michael [4] established the DEA approach, later expanded by Charnes et al. [5]. Friedman and Sinuany [27] developed a special model for linear programming to measure proficiency and analyze operating performance. Soteriou and Stavrinides [6], Ramanathan[7], and Koster et al.[8] attempted to establish benchmarks for less effective values using the DEA approach, predominantly used in educational institutions. Bessent et al.[9] mentioned DEA's use in assessing educational programs, while Tomkins and Green[10] evaluated UK institution departments using DEA.

The DEA technique assesses supplier efficiency, classifying them as inefficient or ineffective. However, selecting the best supplier satisfying manufacturer requirements is challenging, as efficient suppliers are often equally good. Consequently, the TOPSIS strategy is employed in the second phase for shortlisting the best supplier. Hwang and Yoon[14] developed an approach using TOPSIS for speedy supplier selection in MCDM problems.

TOPSIS assumes that the optimal solution is closed to the ideal one on the positive side and farthest on the negative side. Various studies have applied TOPSIS to handle fuzzy/imperfect data. Tsaur et al. [15] used centroid defuzzification. Chu [16] applied TOPSIS in a fuzzy environment for plant positioning, and Chen and Tzeng [17] used fuzzy integrals for fuzzy multi-criteria DM problems. Byun and Lee [18] developed a modified TOPSIS technique for fast prototyping. Chen et al.[19] recommended a TOPSIS-based strategy for supplier selection under fuzzy settings. Boran et al.[20] used TOPSIS with an intuitionistic fuzzy average processor to assess supplier problems.

Wang and Luo[21] proposed the reversal of ranking in MCDM tasks, and Lotfi et al.[22] employed TOPSIS to generate a scope based on efficiency. Chitnis and Vaidya [23] proposed a tie-breaking method for DEA-TOPSIS efficiency counts. Different DEA ranking systems, including cross efficiency, super efficiency, and benchmarking, may or may not share the same viewpoint. To resolve this disagreement, the suggested solution in the second stage employs the TOPSIS method to assist the evaluator in selecting the most suitable provider.

It is noteworthy that, in any situation, the best supplier can be chosen through selection techniques, considering criteria that should be thoroughly evaluated and weighed. The supplier selection process generally involves three stages: identifying criteria, determining evaluation procedures based on these criteria, and choosing suppliers according to assessment findings. While research publications often explain the derivation of decision criteria, they predominantly focus on the supplier selection process. Various criteria exist in the literature for evaluating suppliers, with Dickson[2] being the first to conduct a study and Ellram[28] emphasizing qualitative parameters for long-term collaboration, categorized into financial considerations, corporate culture, technology, and other aspects. The detailed discussion of supplier selection in a close loop of the supply chain networking and highlighted applications further enriches the understanding of the process.

2. Preliminaries

Intuitionistic Fuzzy Set (IFS) [29]

Suppose Universal and subsets be X and $\tilde{F}^I$.

The set $\tilde{F}^I = \{(x, m_{\tilde{F}^I}(x), n_{\tilde{F}^I}(x))/x \in X\}$ is referred to as "IFS" where x is variable. $m_{\tilde{F}^I}(x): X \to [0,1]$, $n_{\tilde{F}^I}(x): X \to [0,1]$ represents the truth values of x (values of Membership of x and Non-membership of x such that $m_{\tilde{F}^I} + n_{\tilde{F}^I} \le 1 \; \forall \; x \in X$. Further, $h_{\tilde{F}^I}(x) = 1 - m_{\tilde{F}^I}(x) - n_{\tilde{F}^I}(x) \; \forall \; x \in X$ with $0 \le h_{\tilde{F}^I}(x) \le 1$ represents Hesitancy Degree regarding x in $\tilde{F}^I$."IFS" $\tilde{F}^I$ becomes a FS $\tilde{F}$ when $h_{\tilde{A}^I}(x) = 0$ and $n_{\tilde{F}^I}(x) = 1 - m_{\tilde{F}^I}(x)$.

Intuitionistic Fuzzy numbers

These numbers F^I are fuzzy-sets (Intuitionistic) $\tilde{F}^I$ on real line with the conditions described below

(i) $\tilde{F}^I$ is normal i.e., there exist $x_i, x_j \in R$ with $m_{\tilde{F}^I}(x_i) = 1$ and

$n_{\tilde{F}^I}(x_j) = 0$.

(ii) $\tilde{F}^I$ is convex in $m_{\tilde{A}^I}$ and concave in $n_{\tilde{A}^I}$.

i.e.,$m_{\tilde{F}^I}(kx_1 + (1-k)x_2) \ge min\left(m_{\tilde{F}^I}(x_1), m_{\tilde{F}^I}(x_2)\right)$

$$n_{\tilde{F}^I}(kx_1 + (1-k)x_2) \le max\left(n_{\tilde{F}^I}(x_1), n_{\tilde{F}^I}(x_2)\right)$$

(iii) $m_{\tilde{F}^I}(x)$ is upper semicontinuous and $n_{\tilde{F}^I}(x)$ lower semicontinuous

(iv) Support of $\tilde{F}^I$ is bounded.

Operations of Generalized Trapezoidal and Triangular Intuitionistic Fuzzy Numbers:

Defining fuzzy operation through trapezoidal intuition as presented by De and Das, (2014). "Trapezoidal-IFN"s are the source of "Triangular-IFN", and the same logistics apply.

Let $P^I = (p_1', p_1, p_2', p_2, p_3, p_3', p_4, p_4' \,; m_p, n_p)$ and

$Q^I = (q_1', q_1, q_2', q_2, q_3, q_3', q_4, q_4' ; m_q, n_q)$ be GTIFNs then

(i) $P^I + Q^I = (p'_1 + q'_1, p_1 + q_1, p'_2 + q'_2, p_2 + q_2, p_3 + q_3, p'_3 + q'_3, p_4 + q_4, p'_4 + q'_4; min(m_p, m_q), max(n_p, n_q))$

(ii) $P^I - Q^I = (p'_1 - q'_4, p_1 - q_4, p'_2 - q'_3, p_2 - q_3, p_3 - q_2, p'_3 - q'_2, p_4 - q_1, p'_4 - q'_1 : min(m_p, m_q), max(n_p, n_q))$

(iii) $kP^I = (kp'_1, kp_1, kp'_2, kp_2, kp_3, kp'_3, kp_4, kp'_4; m_p, n_p)$

Fuzzification:

Process of expressing the crisp data in terms of fuzzy data is called fuzzification.

Defuzzification:

It is the reverse process to fuzzification which involves conversion of a fuzzy value into the closest crisp value.

Ranking Function :

Triangular IFN

$$\tilde{A} = (a_1, a_2, a_4; m_a, n_a)$$
$$R(A^I) = \frac{2}{81}(a_1 + 7a_2 + a_3)(1 + m_a - n_a)$$

Trapezoidal IFN

$$\tilde{A} = (a_1, a_2, a_3, a_4); (a'_1, a'_2, a'_3, a'_4)$$
$$R(\tilde{A}) = \frac{1}{81}[(a_1 + a'_1) + 2(a_2 + a'_3) + 5(a_3 + a'_2) + (a_4 + a'_4)]$$

3.Supplier-Selection Through Mathematical Modelling

Consider a scenario where there are a group of suppliers available from among whom one party which best meets the needs of the organization must be chosen. The procedure is divided into two stages. Using the Data envelope analysis(DEA) method, the performance of suppliers is evaluated in stage 1, and inefficient suppliers are eliminated. The TOPSIS method is used for selection in stage 2. Figure 1 depicts the selection process.

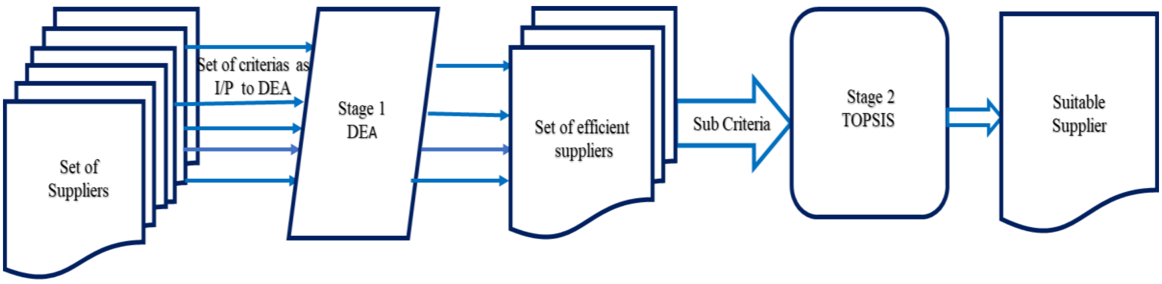

Fig.1 Supplier selection process

Stage-1 Evaluation process

Step 1: Consider the intuitionistic fuzzy DEA model represented by equation (1). The weights assigned for input and output values of all criteria for each specific supplier should be represented in Intuitionistic triangular fuzzy number.

Symbols used

i-The suppliers	x_{ri} – Output variable
r – the Output values	V_{si}^{I} –Weight assigned for input
s – Input values	U_{ri}^{I} – Output weightage
y_{si} – Input -Variable	EFS_i- Efficiency score of the supplier

$$\text{Max } \mathrm{EFS}_i = \sum_{r=1}^{n} U_{ri}^{I} x_{ri}$$

subject to:

$$\sum_{s=1}^{m} V_{si}^{I} y_{si} = 1$$
$$\sum_{r=1}^{n} U_{ri}^{I} x_{ri} - \sum_{s=1}^{m} V_{si}^{I} y_{si} < 0 \quad \forall i$$
$$x_{ri}, y_{si} \geq 0 \quad \forall i, r, s \qquad \textbf{(1)}$$

Step 2 :On input and output weights, use the ranking function described in section 2 to transform the model in step 1 to the equivalent crisp model represented by Eq. (2).

$$\text{Max } \mathrm{EFS}_i = \sum_{r=1}^{n} R(U_{ri}^{I}) x_{ri}$$

subject to:

$$\sum_{s=1}^{m} R(V_{si}^{I}) y_{si} = 1$$
$$\sum_{r=1}^{n} R(U_{ri}^{I}) x_{ri} - \sum_{s=1}^{m} R(V_{si}^{I}) y_{si} < 0 \quad \forall i$$
$$x_{ri}, y_{si} \geq 0 \quad \forall i, r, s \qquad \textbf{(2)}$$

Step 3 : The ratio of any vendor's weighted sum of output values to that of input values is called productivity score. Every supplier's optimum value is enhanced by using the productivity score, which is also the efficiency, with the best score not surpassing 1.

Stage -2 Selection process

The shortlisted suppliers from Stage 1 are checked under a collection of selection' criteria and choose the actual supplier who is best suited for an organization. In this stage, the TOPSIS method is used. It entails the steps below.

Let $S = \{S_i : i = 1,2,3 \ldots m\}$ be the set of shortlisted suppliers from stage 1.

$C = \{C_j : j = 1,2,3..n\}$ be the set of criteria for selection.

$D = \{DM_k ; k = 1,2,3..l\}$ represent the "l" set of people called decision makers who choose the suppliers based on given criteria set.

S_{ij} represent j^{th} criteria for i^{th} supplier expressed in GTIFN as

$$S_{ij} = \left(\underline{s}_{ij}, s_{ij}, \bar{s}_{ij} : m_{s_{ij}}, n_{s_{ij}}\right)$$

Step 1. Calculate the importance of each decision maker.

A group of experts decides which suppliers a manufacturing company should use. Each decision maker's weight is important, and it is computed as follows.

Let $DM_k = \left(\underline{\lambda}_k, \lambda_k, \bar{\lambda}_k : m_k, n_k\right)$ be linguistic score representing significance of each decision maker expressed as GTIFN. The equation for calculation of weightage of any decision maker say k^{th} is as follows.

$d_k = \frac{R(DM_k)}{\sum_{k=1}^{l} R(DM_k)}$ **(3)**

where $R(DM_k) = \frac{2}{81}(\underline{\lambda}_k + 7\lambda_k + \bar{\lambda}_k) \times (1 + m_k - n_k)$is the rank of DM_k obtained by using ranking function defined in section 2 and $d_k \in [0,1]$ such that $\sum_{k=1}^{l} d_k = 1$.

Step 2. In the purview of any decision maker, an accumulated intuitionistic fuzzy decision matrix is constructed. All individual decision opinions are combined in this step to form a group opinion and represented in the matrix form given by $R = (r_{ij})_{mxn}$

Where

$$r_{ij} = \left(\underline{r}_{ij}, r_{ij}, \bar{r}_{ij} : m_{r_{ij}}, n_{r_{ij}}\right) = \begin{pmatrix} \sum_{k=1}^{l} d_k \left(\underline{s}_{ij}\right)_k, \sum_{k=1}^{l} d_k \left(s_{ij}\right)_k, \sum_{k=1}^{l} d_k \left(\bar{s}_{ij}\right)_k : \\ 1 - \prod_{k=i}^{l} \left(1 - m_{S_{ij}}\right)_k^{d_k}, \prod_{k=1}^{l} \left(n_{S_{ij}}\right)_k^{d_k} \end{pmatrix} \quad (4)$$

Step3. Determine the weighting of various criteria $W = \{W_j : j = 1,2,..n\}$for consideration of assessment (of a decision maker) in importance specific to grades.

It is possible that not all criteria are equally important.

Let $wc_j = \left(\underline{wc}_j, wc_j, \overline{wc}_j : m_{wc_j}, n_{wc_j}\right)$ denotes the set of importance grades. The views about the relevance of every specific criteria of decision maker are added together to give each criterion a specific weight.

The weight given to each criterion j *is* $W_j = \left(\underline{w}_j, w_j, \overline{w}_j : m_{W_j}, n_{W_j}\right)$

$$= \left(\sum_{k=1}^{l} d_k \underline{wc}_j, \sum_{k=1}^{l} d_k wc_j, \sum_{k=1}^{l} d_k wc_j : 1 - \prod_{k=1}^{l} \left(1 - m_{wc_j}\right)_k^{d_k}, \prod_{k=1}^{l} \left(n_{wc_j}\right)_k^{d_k}\right) \quad (5)$$

Step 4 Development of a weighted-IF decision matrix that has been accumulated

$$T = (t_{ij})_{mXn}$$

$$t_{ij} = \left(\underline{t}_{ij}, t_{ij}, \bar{t}_{ij} : m_{t_{ij}}, n_{t_{ij}}\right)$$
$$= \left(\underline{r}_{ij}\underline{w}_j, r_{ij}w_j, \bar{r}_{ij}\overline{w}_j : m_{r_{ij}}m_{W_j}, n_{r_{ij}} + n_{W_j} - n_{r_{ij}}n_{W_j}\right) \quad (6)$$

Step 5. Determination of solutions based on intuitionistic fuzzy Optimistic and pessimistic values.

Using the ranking function defined in section 2 identify maximum t_{ij} and min t_{ij} for each criteria j according type of criteria which are classified as Benefit criteria *B* Cost criteria *C*. The optimistic and pessimistic intuitionistic values are derived as responses as follows.

$O^+ = (t_1^+, t_2^+, \ldots t_n^+)$ and $P^- = (t_1^-, t_2^-, \ldots t_n^-)$.

$$t_j^+ = \left[\left(\max_i t_{ij} / j \in B\right)\left(\min_i t_{ij} / j \in C\right)\right] = \left(\underline{t}_j^+, t_j^+, \bar{t}_j^+ : m_{t_j^+}, n_{t_j^+}\right) \text{ and}$$

$$t_j^- = \left[\left(\min_i t_{ij} / j \in B\right)\left(\max_i t_{ij} / j \in C\right)\right] = \left(\underline{t}_j^-, t_j^-, \bar{t}_j^- : m_{t_j^-}, n_{t_j^-}\right)$$

(7)

Step 6. Deduction of measures of Separation

The Separation metrics SM^+ and SM^- for positive and negative intuitionistic optimistic and pessimistic responses can be derived for each supplier *i* by use of what is called Hamming distance method .

$$SM_i^+ = \sum_{j=1}^{n} d\left(t_{ij}, t_j^+\right)$$
$$SM_i^- = \sum_{j=1}^{n} d\left(t_{ij}, t_j^-\right) \quad (8)$$

where $d\left(t_{ij}, t_j^+\right)$ is the measure of distance between t_{ij} and t_j^+ and $d\left(t_{ij}, t_j^-\right)$ is distance between t_{ij} and t_j^- obtained by using the formula.

$$d(t_{ij},t_j^+)=\left(\frac{1}{6}\right)\left[\left|1+m_{t_{ij}}-n_{t_{ij}}\right|\underline{t_{ij}}-\left|1+m_{t_j^+}-n_{t_j^+}\right|\right]\underline{t_j^+}+\left[\left|1+m_{t_{ij}}-n_{t_{ij}}\right|t_{ij}-\left|1+m_{t_j^+}-n_{t_j^+}\right|\right]t_j^+ +\left[\left|1+m_{t_{ij}}-n_{t_{ij}}\right|\overline{t_{ij}}-\left|1+m_{t_j^+}-n_{t_j^+}\right|\right]\overline{t_j^+}$$

$$d(t_{ij},t_j^-)=\left(\frac{1}{6}\right)\left[\left|1+m_{t_{ij}}-n_{t_{ij}}\right|\underline{t_{ij}}-\left|1+m_{t_j^+}-n_{t_j^+}\right|\right]\underline{t_j^-}+\left[\left|1+m_{t_{ij}}-n_{t_{ij}}\right|t_{ij}-\left|1+m_{t_j^+}-n_{t_j^+}\right|\right]t_j^- +\left[\left|1+m_{t_{ij}}-n_{t_{ij}}\right|\overline{t_{ij}}-\left|1+m_{t_j^+}-n_{t_j^+}\right|\right]\overline{t_j^-} \quad \textbf{(9)}$$

The closeness coefficient is measured

$$Cc_i^+=\frac{SM_i^-}{SM_i^+ + SM_i^-} \quad \text{where} \quad 0\le Cc_i^+\le 1 \quad \textbf{(10)}$$

Cc_i^+ is a supplier's relative closeness coefficient, which measures the value's proximity to the optimistic and pessimistic values. The supplier with the highest Cc_i^+ value is the best fit for the organization.

4. Practical Applications in Selection Process

APPLICATION 1

Consider a company's supply chain network which is closed-loop, where manufacturer needs the highest-quality raw materials to make the product. An expert panel known as decision makers assesses the efficiency of five suppliers and chooses the one that best fits the company's needs based on a set of standards.

The model developed which is elaborated in Section 3 for opting a best supplier from amongst those available on the list is highlighted. In stage 1, we assess the supplier's efficacy against numerous parameters and identify the most efficient suppliers using the DEA approach. The group of decision makers selects the supplier that best meets the manufacturer's requirements in stage 2. There are three benefit criteria and one cost criteria among these. Because the given data is linguistic and imprecise, intuitionistic fuzzy numbers are used to represent it.

Stage 1- Evaluation process using Intuitionistic fuzzy DEA method

To evaluate the efficiency of a group of suppliers, use the DEA approach. The following are the input and output variables that were utilized to evaluate the performance of the provided set of suppliers.

Input variables	**Output variables**
y_1 – Quality of the system	x_1– Quality
y_2 – Carbon credits	x_2 –Environmental criteria
y_3 – Material delivery timeline	x_3– just-in-time management
y_4 – Gender equality and the prohibition of child labor.	x_4 – Social criteria
y_5 – Price of the material	x_5 – Financial criteria

Tables 1 and 2 show the input and output criterion weightages for the DEA, TIFN are used to describe these values.

Step 1. De-fuzzify the input and output weights using the ranking method defined in section 2.

$$(a_1,a_2,a_4;m_a,n_a)$$
$$R(A^I)=\frac{2}{81}(a_1+7a_2+a_3)(1+m_a-n_a)$$

Tables 3 and 4 show equivalent crisp weights.

Step 2. The efficiency measure for each supplier is calculated using the linear programming procedure described in equation (2). Table 5 displays each supplier's efficiency score.

From Table 5, we see that efficiency scores for suppliers S_1, S_3 and S_4 are all 1 while the scores of suppliers S_2 and S_5 are less than 1. Hence, we eliminate inefficient suppliers S_2 and S_5. In the second stage, the short-listed suppliers S_1, S_3, and S_4 are tested against the 5 criteria predefined by a group of three experts known as the organization's decision makers, and one of the Suppliers that is most favorable to the organization is chosen using the TOPSIS method.

Stage2 -Selection activity using Intuitionistic fuzzy TOPSIS Technique.

Following steps are applied to find suitable supplier

Step 1: Using equation (3**)** each decision maker opinion weight is calculated and displayed in Table 7 based on the level of importance of each expert opinion given in Table 6.

TABLE -1 Application 1 : Intuitionistic fuzzy input weights of DEA

	V_1^I	V_2^I	V_3^I	V_4^I	V_5^I
S_1	(1,2,3;0.7,0.1)	(1,3,5;0.76,0.2)	(1.5,2.5,3.5;0.65,0.3)	(1.5,2.5,4;0.,0.2)	(0,0,0;1,0)
S_2	(2.5,3,3.5;0.8,0.1)	(2.5,3,4.5;0.7,0.2)	(2.5,3.5,4.5;0.65,0.2)	(1,3,4;0.8,0.12)	(3,5,6.5;0.5,0.35)
S_3	(1.5,2.5,3.5;0.87,0.05)	(2.5,3,4.5;0.7,0.2)	(1,3,5;0.6,0.3)	(1,3,5;0.7,0.15)	(1,2,3;0.75,0.1)
S_4	(1.5,2.5,3.5;0.65,0.2)	(1.5,2.5,3.5;0.7,0.18)	(2.5,3.5,4.5;0.65,0.31)	(1,2.5,5;0.8,0.15)	(2,3,5;0.87,0.05)
S_5	(1,2.5,4,0.7,0.15)	(1.5,2.5,3.5;0.68,0.2)	(0.5,2,3;0.9,0)	(1.5,3,5;0.7,0.3)	(1,2.5,3;0.9,0.1)

TABLE -2 Application 1: Intuitionistic fuzzy output weights of DEA

	U_1^I	U_2^I	U_3^I	U_4^I	U_5^I
S_1	(1,2,3;0.7,0.15)	(1.5,2.5,3.5;0.8,0.2)	(0.8,1,1.2;0.8,0.05)	(0,0,0;1,0)	(1,1.5,3;0.45,0.55)
S_2	(1.5,2.5,3.5;0.86,0.08)	(2,4,5;0.76,0.2)	(2.5,3.5,4.5;0.71,0.22)	(2,4,5;0.7,0.3)	(1.5,3,5;0.6,0.2)
S_3	(3,4,5;0.65,0.25)	(3,4.5,5;0.71,0.1)	(2,4,5;0.8,0.17)	(5.5,6.5,7.5;0.77,0.09)	(2,3.5,4.5;0.81,0.15)
S_4	(2.5,4,5.5;0.67,0.3)	(2,4,5;0.7,0.3)	(1.5,2.5,3.5;0.64,0.25)	(5.5,6.5,7.5;0.81,0.132)	(2,3.5,4.5;0.81,0.15)
S_5	(1,1.2,1.4;0.76,0.2)	(1.5,2.5,3.5;0.8,0.2)	(2,3,4;0.75,0.2)	(0,0,0;1,0)	(1,2.5,4;0.75,0.22)

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TABLE -3 Application 1: Crisp weights of Input variables (V_s)

	V_1		V_2	V_3	V_4	V_5
S_1	0.71		1.04	0.75	0.88	0
S_2	1.13		1.04	1.13	1.08	1.26
S_3	1.01		1.04	0.87	1.03	0.73
S_4	0.81		0.84	1.04	0.96	1.26
S_5	0.86		0.82	0.82	0.95	0.96

TABLE -4 Application 1: Crisp weights of output variables (U_r)

	U_1	U_2	U_3	U_4	U_5
S_1	0.69	0.89	0.39	0	0.32
S_2	0.99	1.35	1.16	1.21	0.95
S_3	1.24	1.57	1.41	2.42	1.27
S_4	1.22	1.21	0.77	2.42	1.27
S_5	0.42	0.89	1.03	0	0.85

TABLE -5 Application 1: Efficiency score of suppliers EFS_i

Suppliers	Efficiency
EFS_1	1
EFS_2	0.86
EFS_3	1
EFS_4	1
EFS_5	0.93

TABLE -6 Application 1: Linguistic Words Score of Decision Makers

Linguist Terms	IFN	Crisp values
Crucial (Cl)	(8.5,9.5,10 :0.85, 0.1)	3.67
Important (Imp)	(7.5,8.5,9.5 :0.7, 0.25)	2.74
Average (Av)	(6.5,7.5,8.5 :0.5, 0.45)	1.75
Depraved (DP)	(5.5,6.5,7.5 :0.45, 0.5)	1.37
Deplorable (D)	(4.5,5.5,6.5 :0.3 ,0.6)	0.86

$$d_k = \frac{R(DM_k)}{\sum_{k=1}^{3} R(DM_k)}$$

$$d_1 = \frac{3.67}{(3.67 + 1.75 + 2.74)} = \frac{3.67}{8.16} = 0.450$$

$$d_2 = \frac{1.75}{(3.67 + 1.75 + 2.74)} = \frac{1.75}{8.16} = 0.214$$

$$d_3 = \frac{2.74}{(3.67 + 1.75 + 2.74)} = \frac{2.74}{8.16} = 0.336$$

TABLE -7 Application 1: Decision-maker opinion weights

	DM_1	DM_2	DM_3
Linguistic Term	Crucial (Cl)	Average (Av)	Important (Imp)
Weights	0.450	0.214	0.336

Step2. In the decision maker's perspective, the Accumulate intuitionistic fuzzy decision matrix $R = (r_{ij})_{mxn}$ is calculated using Table 8 and Table 9 and denoted in tabular form as Table 10.

TABLE -8 Application 1: Criteria based supplier Linguistic rating

Linguistic Terms	IFN
Extraordinary (E)	(8,9,10: 0.85, 0.05)
Reasonable (R)	(6,7,8: 0.7, 0.2)
Average (A)	(4,5,6:0.5, 0.5)
Bad (B)	(2,3,4:0.4, 0.5)
Awful (AW)	(1,2,3:0.2 ,0.65)

TABLE -9 Application 1: Decision makers' perceptions on suppliers for each criterion

SUPPLIERS	CRITERIA	DM_1	DM_2	DM_3
S_1	C_1	E	R	R
	C_2	A	R	R
	C_3	R	A	E
	C_4	A	A	A
	C_5	R	E	E
S_3	C_1	E	A	R
	C_2	A	B	E
	C_3	E	E	E
	C_4	R	R	A
	C_5	B	E	E
S_4	C_1	R	R	R
	C_2	A	E	R
	C_3	E	E	E
	C_4	E	R	A
	C_5	R	B	A

$$R = (r_{ij})_{m\times n}$$
$$= \left(\sum_{k=1}^{l} d_k\,(\underline{s}_{ij})_k, \sum_{k=1}^{l} d_k\,(s_{ij})_k, \sum_{k=1}^{l} d_k\,(\bar{s}_{ij})_k :\ 1-\prod_{k=i}^{l}\left(1-m_{S_{ij}}\right)_k^{d_k}, \prod_{k=1}^{l}\left(n_{S_{ij}}\right)_k^{d_k}\right)$$

$$r_{11} = (6.9,7.9,8.9{:}\,0.780,0.107)$$
$$r_{12} = (4.4,5.4,6.4{:}\,0.552,0.411)$$
$$r_{13} = (5.8,6.8,7.8 : 0.701,0.190)$$
$$r_{14} = (5.8,6.8,7.8 : 0.724,0.153)$$
$$r_{15} = (5.3,6.3,7.3{:}\,0.644,0.272))$$

Similarly calculating other values, we get decision matrix

Step 3. Weights of criteria in each decision maker's view is calculated by equation (3) using data given in Table 11 and Table 12. Obtained weights are represented in Table 13.

$$W_j = \left(\underline{w}_j, w_j, \overline{w}_j : m_{W_j}, n_{W_j}\right)$$
$$= \left(\sum_{k=1}^{l} d_k \underline{wc}_j, \sum_{k=1}^{l} d_k wc_j, \sum_{k=1}^{l} d_k wc_j : 1 - \prod_{k=1}^{l}\left(1 - m_{wc_j}\right)_k^{d_k}, \prod_{k=1}^{l}\left(n_{wc_j}\right)_k^{d_k}\right)$$

$$W_1 = ([6 \times 0.450 + 5 \times 0.214 + 5 \times 0.336], [7.5 \times 0.450 + 6.5 \times 0.214 + 6.5 \times 0.336], [9 \times 0.450 + 8 \times 0.214 + 8 \times 0.336] : [1 - (1 - 0.8)^{0.450} \times (1 - 0.7)^{0.214} \times (1 - 0.7)^{0.336}], [(0.1)^{0.450} \times (0.2)^{0.214} \times (0.2)^{0.336}])$$

$$W_2 = ([7 \times 0.450 + 7 \times 0.214 + 7 \times 0.336], [8.5 \times 0.450 + 8.5 \times 0.214 + 8.5 \times 0.336], [10 \times 0.450 + 10 \times 0.214 + 10 \times 0.336] : [1 - (1 - 0.9)^{0.450} \times (1 - 0.9)^{0.214} \times (1 - 0.9)^{0.336}], [(0)^{0.450} \times (0.0)^{0.214} \times (0.0)^{0.336}])$$

$$W_3 = ([4 \times 0.450 + 6 \times 0.214 + 5 \times 0.336], [5.5 \times 0.450 + 7.5 \times 0.214 + 6.5 \times 0.336], [7 \times 0.450 + 9 \times 0.214 + 8 \times 0.336] : [1 - (1 - 0.5)^{0.450} \times (1 - 0.8)^{0.214} \times (1 - 0.7)^{0.336}], [(0.5)^{0.450} \times (0.1)^{0.214} \times (0.2)^{0.336}])$$

$$W_4 = ([6 \times 0.450 + 5 \times 0.214 + 4 \times 0.336], [7.5 \times 0.450 + 6.5 \times 0.214 + 5.5 \times 0.336], [9 \times 0.450 + 8 \times 0.214 + 7 \times 0.336] : [1 - (1 - 0.8)^{0.450} \times (1 - 0.7)^{0.214} \times (1 - 0.5)^{0.336}], [(0.1)^{0.450} \times (0.2)^{0.214} \times (0.5)^{0.336}])$$

$$W_5 = ([7 \times 0.450 + 6 \times 0.214 + 6 \times 0.336], [8.5 \times 0.450 + 7.5 \times 0.214 + 7.5 \times 0.336], [10 \times 0.450 + 9 \times 0.214 + 9 \times 0.336] : [1 - (1 - 0.9)^{0.450} \times (1 - 0.8)^{0.214} \times (1 - 0.8)^{0.336}], [(0)^{0.450} \times (0.1)^{0.214} \times (0.1)^{0.336}])$$

Step 4. Construction of matrix of weighted-IF decisions that has been accumulated - $T = \left(t_{ij}\right)_{m \times n}$ using equation **(**4) and is shown in Table 14.

$$t_{ij} = \left(\underline{t}_{ij}, t_{ij}, \overline{t}_{ij} : m_{t_{ij}}, n_{t_{ij}}\right)$$
$$= \left(\underline{r}_{ij}\underline{w}_j, r_{ij}w_j, \overline{r}_{ij}\overline{w}_j : m_{r_{ij}}m_{W_j}, n_{r_{ij}} + n_{W_j} - n_{r_{ij}}n_{W_j}\right)$$

Step 5. "IF-optimistic" solution O^+ and "IF-pessimistic" solution P^- based on given criteria set are calculated using the equation in step 4. There are two groups of criteria in the given set. Criteria for determining benefits, B = {C_1, C_2, C_3, C_4}, and for determining cost, C={C_5}

$$\mathrm{R}(S_1C_1) = \left(\frac{2}{81}\right)\left(\underline{t}_{11} + 7 \times t_{11} + \bar{t}_{11}\right)\left(1 + m_{t_{11}} - n_{t_{11}}\right)$$
$$R(S_1C_1) = \left(\frac{2}{81}\right) \times (37.6 + 7 \times 54.9 + 75.2) \times (1 + 0.585 - 0.238) = 16.54$$
$$R(S_3C_1) = \left(\frac{2}{81}\right) \times (16.9 + 7 \times 28.5 + 43.1) \times (1 + 0.343 - 0.594) = 4.931$$
$$R(S_4C_1) = \left(\frac{2}{81}\right) \times (41.3 + 7 \times 59.6 + 80.9) \times (1 + 0.620 - 0.204) = 18.847$$
$$\max_i S_iC_1 = S_4C_1 = t_{41} = (41.3, 59.6, 80.9 : 0.620, 0.204)$$

Similarly, we get

$$\max_i S_iC_2 = S_3C_2 = (56, 76.5, 100 : 0.765, 0.05)$$
$$\max_i S_iC_3 = S_3C_3 = (30.6, 46.5, 65.4 : 0.485, 0.370)$$
$$\max_i S_iC_4 = S_3C_4 = (34.1, 50.7, 70.4 : 0.536, 0.300)$$

$$\min_i S_iC_5 = S_4C_5 = (32.8,42.3,52.2{:}\ 0.575,0.202)$$

Therefore

$$O^+ = \left\{\begin{matrix} (41.3,59.6,80.9{:}\ 0.620,0.204), (56,76.5,100{:}\ 0.765,0.05), \\ (30.6,46.5,65.4{:}\ 0.485,0.370), \\ (34.1,50.7,70.4{:}\ 0.536,0.300), (32.8,42.3,52.2{:}\ 0.575,0.202) \end{matrix}\right\}$$

$$\min_i S_iC_1 = S_3C_1 = (16.9,28.5,43.1{:}\ 0.343,0.573)$$
$$\min_i S_iC_2 = S_4C_2 = (28,42.5,60{:}\ 0.450,0.50)$$

$$\min_i S_iC_3 = S_1C_3 = (27.5,42.4,60.3{:}\ 0.458,0.401)$$

$$\min_i S_iC_4 = S_4C_4 = (18,29.9,44.9{:}\ 0.361,0.528)$$

$$\max_i S_iC_5 = S_3C_5 = (35.7,45.4,71.1{:}\ 0.576,0.225)$$

$$P^- = \left\{\begin{matrix} (16.9,28.5,43.1{:}\ 0.343,0.573), (28,42.5,60{:}\ 0.450,0.50), \\ (27.5,42.4,60.3{:}\ 0.458,0.401), \\ (18,29.9,44.9{:}\ 0.361,0.528), (35.7,45.4,71.1{:}\ 0.576,0.225) \end{matrix}\right\}$$

Step 6. Equations are used to generate separation measures SM^+ and SM^-, which are constructed from corresponding "IF-optimistic" and "IF-pessimistic" solutions. The relative closeness coefficients are determined and are shown in Table 15.

$d(t_{ij}, t_j^+)$ the measure of distance between t_{ij} and t_j^+ and $d(t_{ij}, t_j^-)$, distance between t_{ij} and t_j^- for each supplier is calculated using the equation and SM^+, SM^- are obtained using the equation.

$$SM_i^+ = \sum_{j=1}^{5} d(t_{ij}, t_j^+) \text{ for each } i$$
$$SM_i^- = \sum_{j=1}^{5} d(t_{ij}, t_j^-) \text{ for each } i$$

Closeness coefficient is obtained using equation (4.2.8)

$$Cc_1 = \frac{48.56}{(57.51+48.5)} = 0.4578\ ,Cc_3 = \frac{68.41}{(36.67+68.41)} = 0.6511,$$

$$Cc_1 = \frac{37.246}{(67.85+37.24)} = 0.3544$$

From Table 15 we can see that supplier S_3 is selected as it has the highest Cc_i value and most suitable for the organization.

APPLICATION 2

Let S= {S_1, S_2, S_3, S_4, S_5} be the available set of suppliers. To choose an efficient and suitable supplier for a SCN with a close-loop, the company takes the expert opinion consisting of 3 experts also called decision makers denoted by set D = {DM_1, DM_2, DM_3}. They evaluate the supplier performance efficiency under the criteria set C = {C_1, C_2, C_3, C_4, C_5} and choose the one that suits the company's requirement. The selection process described in Section 3 is used for this, and the results are shown in the tables below.

Stage 1. Finding the efficiency score of each supplier.

Step1 Formulate Intuitionistic fuzzy- DEA model

The following are the DEA's input and output criterion measures for evaluation.

Input criteria	**Output criteria**
y_1 – Quality of the system	x_1 – Quality

y_2 – Carbon credits x_2 – Environmental criteria

y_3 – Delivery time of the material x_3 – just in time management

y_4 – Material variety x_4 – New product development

y_5 – Price of the material x_5 – Financial criteria

In Tables 16 and 17, their evaluations are presented as Intuitionistic fuzzy numbers.

Step 2. Convert the intuitionistic fuzzy DEA model's input and output weights, which are in Symmetric-"Triangular-IFN" to crisp values using the ranking function defined in section 2. The equivalent crisp weights are in Table18 and Table 19.

Step 3. Substituting the above values and solving by TORA software for each supplier yields the efficiency scores shown in Table 20.

According to Table 20, all of the suppliers are equally efficient, as their efficiency scores are all one. This makes the selection process difficult, and the TOPSIS method used in stage 2 assists in selecting a supplier who best meets the company's requirements.

Stage2- Selection process.

In this stage, the given set of suppliers are tested using the same criteria as the DEA outputs by firm's expert panel of three members who takes decision.

Step 1. Calculate each decision maker's individual opinion weightage using equation in step1 based on the significance of each expert's opinion expressed as linguistic IFN in Table 21. Table 22 displays the weight of opinion.

$$d_1 = \frac{3.60}{(3.60 + 1.56 + 2.67)} = \frac{3.60}{7.82} = 0.460$$
$$d_2 = \frac{1.56}{(3.60 + 1.56 + 2.67)} = \frac{1.56}{7.82} = 0.199$$
$$d_3 = \frac{2.67}{(3.60 + 1.56 + 2.67)} = \frac{2.74}{7.82} = 0.341$$

Step 2. Decision makers individual intuitionistic-fuzzy-opinions are accumulated by using Table 23 & Table 24 and represented in the form of a matrix as shown in Table 25.

$$R = (r_{ij})_{m\times n}$$

$r_{11} = (6.9,7.9,8.9: 0.782,0.106)$
$r_{12} = (4.4,5.4,6.4: 0.548,0.417)$
$r_{13} = (5.8,6.8,7.8: 0.700,0.190)$
$r_{14} = (5.9,6.9,7.9: 0.728,0.150)$
$r_{15} = (5.3,6.3,7.3: 0.643,0.273)$
$r_{21} = (3.1,4.1,5.1: 0.456,0.500)$

Finding other values in similar way we get accumulated decision matrix shown in Table 25.

Step3. The weight of each criterion $W_{j\ in}$ the decision makers' opinion is calculated using the data in Tables 26 and 27. Calculated weights are shown in Table 28.

$$W_1 = ([6 \times 0.460 + 5 \times 0.199 + 5 \times 0.341], [7.5 \times 0.460 + 6.5 \times 0.199 + 6.5 \times 0.341], [9 \times 0.460 + 8 \times 0.199 + 8 \times 0.341]: [1 - (1-0.8)^{0.460} \times (1-0.7)^{0.199} \times (1-0.7)^{0.341}], [(0.1)^{0.460} \times (0.2)^{0.199} \times (0.2)^{0.341}])$$

$$W_2 = ([7 \times 0.460 + 7 \times 0.199 + 7 \times 0.341], [8.5 \times 0.460 + 8.5 \times 0.199 + 8.5 \times 0.341], [10 \times 0.460 + 10 \times 0.199 + 10 \times 0.341]: [1 - (1-0.9)^{0.460} \times (1-0.9)^{0.199} \times (1-0.9)^{0.341}], [(0)^{0.460} \times (0.0)^{0.199} \times (0.0)^{0.341}])$$

$$W_3 = ([4 \times 0.460 + 6 \times 0.199 + 5 \times 0.341], [5.5 \times 0.460 + 7.5 \times 0.199 + 6.5 \times 0.341], [7 \times 0.460 + 9 \times 0.199 + 8 \times 0.341]: [1 - (1-0.5)^{0.460} \times (1-0.8)^{0.199} \times (1-0.7)^{0.341}], [(0.5)^{0.460} \times (0.1)^{0.199} \times (0.2)^{0.341}])$$

$$W_4 = ([6 \times 0.460 + 5 \times 0.199 + 4 \times 0.341], [7.5 \times 0.460 + 6.5 \times 0.199 + 5.5 \times 0.341], [9 \times 0.460 + 8 \times 0.199 + 7 \times 0.341]: [1 - (1-0.8)^{0.460} \times (1-0.7)^{0.199} \times (1-0.5)^{0.341}], [(0.1)^{0.460} \times (0.2)^{0.199} \times (0.5)^{0.341}])$$

$$W_5 = ([7 \times 0.460 + 6 \times 0.199 + 6 \times 0.341], [8.5 \times 0.460 + 7.5 \times 0.199 + 7.5 \times 0.341], [10 \times 0.460 + 9 \times 0.199 + 9 \times 0.341]: [1 - (1-0.9)^{0.460} \times (1-0.8)^{0.199} \times (1-0.8)^{0.341}], [(0)^{0.460} \times (0.1)^{0.199} \times (0.1)^{0.341}])$$

Step 4. Matrix formation for of an accumulative -weighted-“IF-decisions”

$$T = (t_{ij})_{m \times n}$$

where

$$t_{ij} = \left(\underline{t}_{ij}, t_{ij}, \overline{t}_{ij}: m_{t_{ij}}, n_{t_{ij}}\right) = \left(\underline{r}_{ij}\underline{w}_j, r_{ij}w_j, \overline{r}_{ij}\overline{w}_j: m_{r_{ij}}m_{W_j}, n_{r_{ij}} + n_{W_j} - n_{r_{ij}}n_{W_j}\right)$$

Table 29 shows the results of using equation in step 4

TABLE -10 Application 1: Matrix of accumulative intuitionistic fuzzy decisions

	C_1	C_2	C_3	C_4	C_5
S_1	(6.9,7.9,8.9 :0.780,0.107)	(4.4,5.4,6.4 :0.552,0.411)	(5.8,6.8,7.8 :0.701,0.190)	(5.8,6.8,7.8 :0.724,0.153)	(5.3,6.3,7.3:0.644,0.272)
S_3	(3.1,4.1,5.1:0.457,0.500)	(8,9,10:0.85,0.050)	(0.741,0.149,0.110)	(0.762,0.126,0.112)	(0.675,0.224,0.101)
S_4	(7.6,8.6,9.6:0.826,0.067)	(4,5,6:0.50,0.50)	(6,7,8:0.700,0.200)	(3.5,4.5,5.5:0.513,0.411)	(5.1,6.1,7.1:0.674,0.202)

TABLE -11 Application 1: Criteria significance linguistic scores

Linguistic Terms	IFN
Very Important (VI)	(7,8.5,10:0.9, 0)
Important (I)	(6,7.5,9:0.8, 0.1)
Medium Important (MI)	(5,6.5,8:0.7, 0.2)
Essential (ES)	(4,5.5,7:0.5, 0.5)
Inessential (IES)	(3,4.5,6:0.3 ,0.5)

TABLE -12 Application 1: Decision makers opinion on Criteria

	DM_1	DM_2	DM_3
C_1	I	MI	MI
C_2	VI	VI	VI
C_3	ES	I	MI
C_4	I	MI	ES
C_5	VI	I	I

TABLE -13 Application 1: Weights of Criteria

Criteria	Weights
C_1	(5.5,7,8.5:0.750, 0.146)
C_2	(7,8.5,10:0.90,0)
C_3	(4.8,6.3,7.8:0.654, 0.260)
C_4	(5.1,6.6,8.1:0.703, 0.199)
C_5	(6.5,7,9.5:0.854, 0)

Step 5. "IF-optimistic" and "IF-pessimistic" values O+ and P- are calculated using the equation in step 5 based on the type of criteria sets Benefit criteria or cost criteria. B= {C_1, C_2, C_3, C_4}, cost criteria C={C_5}

$$R(t_{11}) = \left(\frac{2}{81}\right)(37.8 + 7 \times 55.1 + 75.5)(1 + 0.59 - 0.24) = 16.66$$

$$R(t_{21}) = \left(\frac{2}{81}\right)(16.8 + 7 \times 28.4 + 43)(1 + 0.34 - 0.57) = 4.92$$

$$R(t_{31}) = \left(\frac{2}{81}\right)(45.1 + 7 \times 59.9 + 81.2)(1 + 0.62 - 0.20) = 19$$

$$R(t_{41}) = \left(\frac{2}{81}\right)(32.8 + 7 \times 48.7 + 67.7)(1 + 0.53 - 0.32) = 13.18$$

$$R(t_{51}) = \left(\frac{2}{81}\right)(30.6 + 7 \times 45.9 + 64.3)(1 + 0.50 - 0.35) = 11.84$$

$$\max(16.66,4.92,19,13.1811.84) = 19$$

$$t_j^+ = \left[\left(\max_i \frac{t_{ij}}{j} \in B\right)\left(\min_i \frac{t_{ij}}{j} \in C\right)\right] = \left(\underline{t}_j^+, t_j^+, \bar{t}_j^+ : m_{t_j^+}, n_{t_j^+}\right)$$

$$t_1^+ = t_{31} = (41.5,59.9,81.2: 0.62,0.20)$$

3.

Similarly, we can find $t_2^+ t_3^+, t_4^+ \; and \; t_5^+$ which are given by

$$t_2^+ = t_{22} = (56,76.5,100: 0.77,0.05)$$
$$t_3^+ = t_{53} = (37.9,56.2,77.4: 0.55,0.30)$$
$$t_4^+ = t_{44} = (41,59.6,81.2: 0.60,0.24)$$
$$t_5^+ = t_{45} = (12.9,20.9,37.8: 0.34,0.50)$$

Similarly, we can find

$$t_j^- = \left[\left(\min_i \frac{t_{ij}}{j} \in B\right)\left(\max_i \frac{t_{ij}}{j} \in C\right)\right] = \left(\underline{t}_j^-, t_j^-, \bar{t}_j^- : m_{t_j^-}, n_{t_j^-}\right)$$

$$t_1^- = t_{11} = (16.8,28.4,43:0.34,0.57)$$

$$t_2^- = t_{32} = (28,42.5,60: 0.45,0.50)$$
$$t_3^- = t_{13} = (27.3,42.2,60.1:0.46,0.41)$$
$$t_4^- = t_{54} = (12.3,22.5,35.7:0.30,0.60)$$
$$t_5^- = t_{55} = (40.3,50.3,77.9:0.63,0.14)$$

O^+= {(41.5,59.9,81.2:0.62, 0.20), (56,76.5,100:0.77, 0.05), (37.9,56.2,77.4:0.55,0.30),(41,59.6,81.2:0.60,0.24), (12.9,20.9,37.8:0.34, 0.50)}

P^- = {(16.8,28.4,43:0.34, 0.57), (28,42.5,60:0.45,0.50),(27.3,42.2,60.1:0.46,0.41), (12.3,22.5,35.7:0.30,0.60), (40.3,50.3,77.9:0.63, 0.14)}

TABLE -14 Application 1: Matrix of accumulative-weighted-IF Decisions

	C_1	C_2	C_3	C_4	C_5
S_1	(37.6,54.9,75.2:0.585,0.238)	(31,46.1,64.3:0.497,0.411)	(27.5,42.4,60.3:0.458,0.564)	(29.7,45.1,63.4:0.509,0.321)	(34.4,44,69.2:0.550,0.272)
S_3	(16.9,28.5,43.1:0.343,0.573)	(56,76.5,100:0.765, 0.050)	(30.6,46.5,65.4:0.485, 0.370)	(34.1,50.7,70.4:0.536, 0.30)	(35.7,45.4,71.1:0.576,0.225)
S_4	(41.3,59.6,80.9:0.620,0.204)	(28,42.5,60:0.450, 0.500)	(28.6,43.8,62.1:0.458, 0.408)	(18,29.9,44.9:0.361, 0.528)	(32.8,42.3,52.2:0.575,0.202)

Step 6. Separation measures SM+ and SM- from respective sets O+ & P-, as well as corresponding closeness coefficients for each supplier, are calculated and shown in Table 30 .

$$SM_i^+ = \sum_{j=1}^{5} d\left(t_{ij}, t_j^+\right) \text{ for each } i$$
$$SM_i^- = \sum_{j=1}^{5} d\left(t_{ij}, t_j^-\right) \text{ for each } i$$

$$d(t_{11}, t_1^+) = \frac{1}{6}[|(1+0.59-0.24)37.8-(1+0.62-0.20)41.5)| + |(1+0.59-0.24)55.1-(1+0.62-0.20)59.9)| + |(1+0.59-0.24)75.5-(1+0.62-0.20)81.2)|] = 5.29$$

$d(t_{12}, t_2^+) = 41.21$, $d(t_{13}, t_3^+) = 13.05$, $d(t_{14}, t_4^+) = 13.37$, $d(t_{15}, t_5^+) = 21.32$

Then $SM_1^+ = \sum_{j=1}^{5} d\left(t_{1j}, t_j^+\right) = d(t_{11}, t_1^+) + d(t_{12}, t_2^+) + d(t_{13}, t_3^+) + d(t_{14}, t_4^+) + d(t_{15}, t_5^+) = 5.29 + 41.21 + 13.05 + 13.37 + 21.32 = 94.24$.

$$d(t_{11}, t_1^-) = \frac{1}{6}[|(1+0.59-0.24)37.8-(1+0.34-0.57)16.8)| + |(1+0.59-0.24)55.1-(1+0.34-0.57)28.4)| + |(1+0.59-0.24)75.5-(1+0.34-0.57)43)|] = 26.61$$

$d(t_{12}, t_2^-) = 4.58$, $d(t_{13}, t_3^-) = 0$, $d(t_{14}, t_4^-) = 19.61$, $d(t_{15}, t_5^-) = 10.44$

$SM_1^- = \sum_{j=1}^{5} d\left(t_{1j}, t_j^-\right) = d(t_{11}, t_1^-) + d(t_{12}, t_2^-) + d(t_{13}, t_3^-) + d(t_{14}, t_4^-) + d(t_{15}, t_5^-) = 26.61 + 4.58 + 0 + 19.61 + 10.44 = 61.24$.

$$Cc_1 = \frac{SM_1^-}{(SM_1^+ + SM_+^-)} = \frac{26.61}{(26.61 + 61.24)} = 0.394$$

SM^+, SM^- and their corresponding Cc_i values for other suppliers are calculated in the similar manner.

TABLE 15. Application 1: Separation measures and closeness coefficients

	SM^+	**SM^-**	**Cc_i**
S_1	57.51	48.56	0.4578
S_3	36.67	68.41	0.6511
S_4	67.85	37.24	0.3544

TABLE -16 Application 2: Intuitionistic fuzzy weights of input variables

	V_1^I	V_2^I	V_3^I	V_4^I	V_5^I
S_1	(3.5,4,4.5 ;0.7,0.3)	(1.9,2.1,2.3 ;0.4,0.5)	(7,10,12;0.75,0.2)	(0.65,0.8,0.95;0.8,0.2)	(490,540,575;0.9,0.1)
S_2	(2.8,3.3,3.8 ;0.6,0.2)	(1.4,1.5,1.6 ;0.8,01)	(6,10,15;0.6,0.3)	(1.7,2.1,2.4;0.6,0.35)	(420,450,470;0.65,0.2)
S_3	(4.4,4.9,5.4 ;0.6,0.1)	(2.2,2.6,3 ;0.7,0.2)	(5,9,13;0.65,0.25)	(0.75,0.9,1.27;0.75,0.2)	(520,600,645;0.8,0.2)
S_4	(3.4,4.1,4.8 ;0.4,0.2)	(2.2,2.3,2.4 ;0.85,0.1)	(5.5,7.5,9.5;0.8,0.15)	(1.1,1.5,1.75;0.9,0.05)	(515,550,605;0.7,0.25)
S_5	(5.9,6.5,7.1 ;0.7,0.3)	(3.6,4.1,4.6 ;0.9,0.1)	(5,7,8;0.7,0.25)	(0.6,0.9,1.35;0.65,0.25)	(495,520,565;0.8,0.2)

TABLE -17 Application 2 : Intuitionistic fuzzy weights of output variables

	U_1^I	U_2^I	U_3^I	U_4^I	U_5^I
S_1	(2.4,2.6,2.8;0.9,0.1)	(3.8,4.1,4.4;0.8,0.1)	(6.5,7.4,8.3;0.5,0.2)	(0.7,0.9,1.25;0.7,0.25)	(465,515,550;0.9,0.1)
S_2	(2,2.2,2.4;0.85,0.1)	(3.3,3.5,3.7;0.9,0.1)	(4.4,5.1,5.8;0.8,0.2)	(0.6,0.65,0.71;0.6,0.25)	(500,550,575;0.75,0.2)
S_3	(2.5,3.2,3.7;0.6,0.25)	(4.2,6,6.8;0.5,0.4)	(3.6,4.1,4.6;0.9,0.1)	(0.7,0.85,0.9;0.5,0.15)	(510,600,645;0.9,0.1)
S_4	(2.5,2.8,3.3;0.7,0.1)	(5.5,5.7,5.9;0.4,0.3)	(3.4,4.1,4.8;0.4,0.2)	(0.55,0.7,0.92;0.5,0.3)	(500,550,600;0.6,0.35)
S_5	(4.4,5.2,5.8;0.8,0.2)	(6.5,7.4,8.3;0.5,0.2)	(2.7,3.2,3.7;0.7,0.2)	(0.71,0.9,1.05;0.8,0.2)	(480,540,620;0.7,0.2)

TABLE -18 Application 2: Crisp weights of Input variables (Vs)

Eg-2					
	V_1	**V_2**	**V_3**	**V_4**	**V_5**
S_1	1.24	0.42	3.41	0.28	215.33
S_2	1.03	0.57	2.92	0.58	144.64
S_3	1.63	0.87	2.80	0.32	211.95
S_4	1.09	0.89	2.75	0.61	177.94
S_5	2.02	1.64	2.22	0.29	185.68

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TABLE -19 Application 2: Crisp weights of output variables (U_r)

	U_1	**U_2**	**U_3**	**U_4**	**U_5**
S_1	1.04	1.55	2.14	0.30	205.33
S_2	0.86	1.40	1.81	0.20	188.49
S_3	0.95	1.44	1.64	0.31	238.00
S_4	1.00	1.39	1.09	0.19	152.78
S_5	1.84	2.14	1.07	0.32	180.74

TABLE -20 Application 2: Efficiency scores

Supplier	**Efficiency**
EFS_1	1
EFS_2	1
EFS_3	1
EFS_4	1
EFS_5	1

TABLE -21 Application 2: Linguistic terms rating of Decision makers

Linguistic Terms	IFN	Crisp values
Crucial (CRL)	(8,9,10 ;0.85, 0.05)	3.60
Significant (SFT)	(7,8,9; 0.7, 0.2)	2.67
Standard (SRD)	(6,7,8; 0.5, 0.5)	1.56
Depraved (DPD)	(5,6,7; 0.4, 0.5)	1.20
Deplorable (DE)	(4,5,6; 0.25 ,0.6)	0.72

TABLE -22 Application 2: Decision-maker opinion weights

	DM_1	DM_2	DM_3
Linguistic Term	Crucial (CRL)	Standard (SRD)	Significant (SFT)
Weights	0.46	0.199	0.341

TABLE 23 -Application 2: Criteria based -Supplier Linguistic rating

Linguistic Terms	IFN
Very Good (G)	(8,9,10 :0.85, 0.05
Good (G)	(6,7,8 :0.7,0.2)
Medium Good (MG)	(4,5,6 :0.5, 0.5)
Bad (B)	(2,3,4 :0.4, 0.5)
Very Bad (VB)	(1,2,3 :0.2 ,0.65)

TABLE -24 Application 2: Decision makers opinion on suppliers for each criteria

SUPPLIERS	CRITERIA	DM_1	DM_2	DM_3
S_1	C_1	VG	G	G
	C_2	MG	G	MG
	C_3	MG	G	VG
	C_4	G	B	VG
	C_5	G	G	MG
S_2	C_1	B	MG	MG
	C_2	VG	VG	VG
	C_3	G	VG	G
	C_4	G	G	VG
	C_5	MG	VG	G
S_3	C_1	VG	G	VG
	C_2	MG	MG	MG
	C_3	G	G	G
	C_4	B	G	MG
	C_5	G	VG	B
S_4	C_1	G	G	G
	C_2	MG	G	MG
	C_3	VG	G	VG
	C_4	VG	VG	VG
	C_5	B	B	B
S_5	C_1	G	MG	G
	C_2	MG	MG	MG
	C_3	VG	VG	VG
	C_4	B	MG	B
	C_5	VG	G	MG

TABLE -25 Application 2: Matrix of accumulative "IF- decisions"

	C_1	C_2	C_3	C_4	C_5
S_1	(6.9,7.9,8.9:0.782,0.106)	(4.4,5.4,6.4:0.548,0.417)	(5.8,6.8,7.8:0.700,0.190)	(5.9,6.9,7.9:0.728,0.150)	(5.3,6.3,7.3:0.643,0.273)
S_2	(3.1,4.1,5.1:0.456,0.500)	(8,9,10:0.85,0.050)	(6.4,7.4,8.4:0.739,0.152)	(6.7,7.7,8.7:0.763,0.125)	(5.5,6.5,7.5:0.669,0.231)
S_3	(7.6,8.6,9.6:0.828,0.066)	(4,5,6:0.50,0.50)	(6,7,8:0.700,0.200)	(3.5,4.5,5.5:0.509,0.417)	(5,6,7:0.669,0.207)
S_4	(6,7,8:0.700,0.200)	(4.5,5.4,6.4:0.548,0.417)	(7.6,8.6,9.6:0.828,0.066,	(8,9,10:0.850,0.050)	(2,3,4:0.400,0.500)
S_5	(5.6,6.6,7.6:0.668,0.240)	(4,5,6:0.500,0.500)	(8,9,10:0.850,0.050)	(2.4,3.4,4.4:0.421,0.500)	(6.2,7.2,8.2:0.740,0.144)

TABLE -26 Application 2: IF- Linguistic score for criteria by decision makers

Linguistic Terms	IFN
Very Important (VI)	(7,8.5,10:0.9, 0)
Important (I)	(6,7.5,9 :0.8, 0.1)
Medium Important (MI)	(5,6.5,8 :0.7, 0.2)
Essential (ES)	(4,5.5,7:0.5, 0.5)
Inessential (IES)	(3,4.5,6 :0.3 ,0.5)

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TABLE -27 Application 2: Decision makers opinion on Criteria

	DM_1	DM_2	DM_3
C_1	I	MI	MI
C_2	VI	VI	VI
C_3	ES	I	MI
C_4	I	MI	ES
C_5	VI	I	I

TABLE -28 Application 2: Weights of criteria

Criteria	Weights
C_1	(5.5,7,8.5:0.751,0.145)
C_2	(7,8.5,10:0.9,0)
C_3	(4.7,6.2,7.7:0.650,0.266)
C_4	(5.1,6.6,8.1:0.704,0.199)
C_5	(6.5,7.0,9.5:0.855,0.000)

TABLE -29 Application 2: Matrix of accumulative -weighted-“IF- decisions”

	C_1	C_2	C_3	C_4	C_5
S_1	(37.8,55.1,75.5:0.59,0.24)	(30.3,45.9,64:0.49,0.42)	(27.3,42.2,60.1:0.46,0.41)	(30.1,45.6,64:0.51,0.32)	(34.4,43.9,69.2:0.55,0.27)
S_2	(16.8,28.4,43:0.34,0.57)	(56,76.5,100:0.77,0.05)	(30.3,46.2,65:0.48,0.38)	(34.2,50.8,70.5:0.54,0.30)	(35.4,45,70.7:0.57,0.23)
S_3	(41.5,59.9,81.2:0.62,0.20)	(28,42.5,60:0.45,0.50)	(28.4,43.7,61.9:0.45,0.41)	(17.8,29.6,44.5:0.36,0.53)	(32.5,41.9,51.8:0.57,0.21)
S_4	(32.8,48.7,67.7:0.53,0.32)	(30.8,45.9,64:0.49,0.42)	(41,59.6,81.2:0.54,0.31)	(41,59.6,81.2:0.60,0.24)	(12.9,20.9,37.8:0.34,0.50)
S_5	(30.6,45.9,64.3:0.50,0.35)	(28,42.5,60:0.45,0.50)	(37.9,56.2,77.4:0.55,0.30)	(12.3,22.5,35.7:0.30,0.60)	(40.3,50.3,77.9:0.63,0.14)

TABLE -30 Application 2: Separation measures and closeness coefficients

	SM^+	SM^-	C_i
S_1	94.24	61.24	0.394
S_2	74.41	81.07	0.521
S_3	105.42	50.06	0.322
S_4	56.63	98.85	0.636
S_5	126.73	28.75	0.185

The closeness coefficient value Cc_i of supplier S_4 is the highest of all suppliers, as shown in table 30. As a result, we conclude that Supplier S4 best meets the company's needs and is chosen.

5.Conclusion

In conclusion, this section introduced a supplier selection procedure employing an integrated DEA-TOPSIS technique for CLSCN. The integration enhances product quality, reduces production tie and cost, and proves effective when choosing from a large pool of vendors within a short timeframe. The method aids in identifying the supplier that best aligns with the organization's needs. The mathematical model from section 3 is practically validated in real-time applications 1 and 2 for supplier evaluation and selection. The results demonstrate the effectiveness of the model in identifying the best-suited supplier based on given criteria. In Application Model 1, three efficient suppliers are shortlisted and evaluated further in the second stage. In Application Model 2, where all suppliers exhibit equal efficiency, the method facilitates the selection of the best supplier in the second stage.

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