

Solid Restricted Burst Error Locating and Correcting Linear Optimal Codes

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Abstract:- In this communication we are presenting the codes that deal with location of the solid restricted bursts errors occurring in byte oriented communication channels/ memory systems during the transmission of information. The code-length for such codes is assumed to be subdivided into bytes of unequal lengths. The necessary and sufficient conditions for the codes those are capable to locate the solid burst errors lying in a single byte. We also provide a construction technique of the solid restricted b_i -byte correcting non-binary linear optimal codes.

Keywords: b_i -byte correcting codes, optimal codes, Syndromes, Byte oriented channels, Parity check matrix, Solid burst errors, restricted burst errors.

1. Introduction

The main purpose of coding theory is construction of codes for error-free transmission of information by detecting and correcting errors that occur during data transmission. The nature of error and the communication channel are known to be related. Study of solid burst errors is an important area in coding theory. Schillinger [3] developed codes that correct solid burst errors in binary system. Solid burst errors can be found in various storage systems (viz. super computer storage system, semiconductor memory data). The primary goal of the early development of solid burst error correcting codes was to detect and correct solid bursts in binary memory systems. We present in this paper the concept of solid restricted burst (SRB Error), which is a modification of Schillinger's [3] definition of a solid burst. The coding theorists are supposed to provide the codes that are helpful to perform the error free transmission of the information through a communication channel. Initially, communication channels used the codes having the capability to detect and correct the errors occurring when the information is transmitted to one place to another place. The nature of

occurrence of the errors may vary from a channel to another channel. Errors can be classified into a number of different categories, including random error, burst error, solid burst error, repeated burst error and key error. Abramson [1] obtained the codes that deal with the errors that are not in a random occurrence in other words these errors occur in clusters. At first, Fire [2] gave the concept of a particular type of clustered errors known as “burst errors”. According to him “A burst of length b is considered as an n -tuple whose only non-zero components are confined to some b consecutive positions, the first and the last of which is non-zero”. The vectors 0012100, 1010000, 0111000, 000201 are examples of bursts of length 3 over GF(3). If all the components of a burst of length b are non-zero then this burst called a “solid burst”. Initially, only those codes were developed that were capable to detect and correct the different types of errors. The error detecting codes use short number of parity check digits, but provide the information about the presence of an error and no other information. The error correcting codes help to remove the errors, but these codes have large number of parity check digits which effects the transmission rate of a channel where these are being used. Wolf and Elspas presented a novel idea called “location of errors” in order to prevent these issues. They were able to locate the error-causing portion of the code length by using the codes they had obtained. These codes have a parity check digit count that falls between the number of parity check digits with codes that can detect and correct these errors, respectively. In other words location is a midway of detection and correction. The study of the codes dealing with the solid burst errors was started by Schillinger [3]. Shiva and Sheng worked on multiple solid bursts. Subsequently, a class of solid burst error-correcting codes derived from a reversible code was obtained by Das. He gave the codes capable of correcting the solid bursts lying in sub-blocks. Etzion initiated to work on the byte oriented channels and obtained the codes that were capable of correct the solid bursts occurring in a byte. Chen [6] developed byte oriented error correcting codes. Recently, Tyagi and Lata [10] gave the perfect binary codes based bytes. Tyagi and Lata [11] introduced the restricted burst errors and in [11] they obtained the byte oriented restricted burst error correcting non-binary optimal codes. According to them a restricted burst is defined as:

Definition-1 A restricted burst of length b is a vector of length n whose all non-zero components are confined to some b consecutive positions, the first and last of which is nonzero with the restriction that all the non-zero consecutive positions contain same field element.

As SRB errors correcting non-binary linear codes for byte-oriented communication channels are of interest to us. The following defines a SRB error of length b :

Definition-2 A solid restricted burst of length b is defined as a n -tuple vector with non-zero entries in some b consecutive positions and zero elsewhere with the restriction that all the non-zero consecutive positions contain same field element.

For example: For $q=3$, $n=4$ and $b=3$, solid restricted burst of length 3 or less are 1110, 2220, 0111, 0222, 1100, 2200, 0110, 0220, 0011, 0022, 1000, 2000, 0100, 0200, 0010, 0020, 0001, 0002.

Reciting that communication channel behaves differently in the presence of different types of errors. Due to having comparatively less parity check digits or in other words have good code rate, the codes capable of detect, correct or locate the solid restricted burst errors will help to enhance the transmission speed of a channel. In this paper, we assume that the whole code length n is subdivided into f number of bytes of different size. Here we obtain the codes that have capability to locate the solid restricted burst errors which are occurring in a single byte. We also provide a construction technique of the solid restricted b_i -byte correcting non-binary linear optimal codes. This paper is divided as follows: Present paper, introduction is given in Section 1. The necessary and sufficient bounds for the existence of SRB error locating codes are provided in Section 2. The solid restricted b_i -byte correcting non-binary linear optimal codes construction method is presented in Section 3.

2. Location of solid restricted burst errors in the bytes

In this part of the paper, we derive two conditions in the form of inequalities for the codes that have capability to locate the solid restricted burst (SRB) errors occurred in one the f bytes of different lengths. We will represent the SRB error locating codes by SRBEL-codes satisfy the following conditions:

- (i) The syndromes due to the SRB errors in a single byte must be different from zero.
- (ii) The syndromes due to the SRB errors in a single byte must be different from the syndromes due to the SRB errors in any other bytes.

To verify these codes, we have also provided an example.

Theorem 1: An $(n = \sum_{s=0}^f \beta_s, k)$ SRBEL-code that locates the SRB errors of length b or less occurring in a single byte satisfies the following condition

$$n - k \geq \log_q \{1 + fb(q - 1)\}$$

Proof: The enumeration of all SRB errors that have to be located in a single byte will give the right hand side of the required result.

Let L be a set that contains all of the vectors that contain all of the restricted non-zero entries in at least b consecutive positions. We assert that no more than two SRB can be found in the same coset. We will use the fact that a code vector is the sum or difference of two bursts in a coset to support this claim. Let x_1 and x_2 be two elements of L lying in the same coset. The sum of x_1 and x_2 i.e. $x_1 + x_2$ or their difference

i.e. $x_1 - x_2$ is a SRB errors. Since x_1 and x_2 are the same coset therefore $x_1 + x_2$ or $x_1 - x_2$ is a code vector. This is a contradiction. Hence our assumption that x_1 and x_2 are in the same coset, was wrong. This proves our claim.

The number of elements in the set L is given by $b(q - 1)$.

Given that the entire code length is divided into f different-length bytes. Therefore the total count of the locatable SRB errors is given as:

$$fb(q - 1) \quad \dots(1)$$

We will get the required result by putting the expression (1) less than q^{n-k} .

$$q^{n-k} \geq f b(q-1).$$

The necessary condition for the existence of SRBEL codes is provided by the following theorem.

Theorem 2: The existence of an $(n = \sum_{s=0}^f \beta_s, k)$ SRBEL-code is ensured if the following inequality holds

$$q^{n-k} > (b-1)(q-1) \left\{ 1 + \sum_{s=1}^{f-1} \sum_{i=1}^b \{(\beta_s - i + 1)(q-1)\} \right\}$$

Proof: Constructing a parity check matrix H with dimension $(n-k) \times n$ is sufficient to derive this theorem. We employ the same methodology as [12] in order to prove the Varshamov Gilbert-Sacks bound. Let us begin by creating the matrix using $n-k$ tuples. Assume that we have selected all of the columns in the first $f-1$ bytes as well as the first $j-1$ columns in the f^{th} . If the conditions (i) - (ii) are met, the last byte β_s 's j^{th} column h_j cannot be expressed as a linear combination of the columns $h_{j-1}, h_{j-2}, \dots, h_{j-(b-1)}$, in accordance with the condition (i). Stated differently, h_j should not equal the linear combination of $b-1$ or fewer columns immediately before it i.e

$$h_j \neq \sigma_1 h_{j-1} + \sigma_2 h_{j-2} + \dots + \sigma_{b-1} h_{j-(b-1)} \quad \dots (2)$$

In the expression (2), the coefficients σ_i are similar as the non-zero entry in a solid restricted burst of length $b-1$ or less. Therefore the total number of linear combinations is given by

$$(b-1)(q-1) \quad \dots (3)$$

Now according to the condition (ii), the column h_j should not be written in the form of linear sum of immediately preceding $b-1$ columns together with the linear sum of any b columns of any other of $j-1$ bytes i.e.

$$h_j \neq \sigma_1 h_{j-1} + \sigma_2 h_{j-2} + \dots + \sigma_{b-1} h_{j-(b-1)} + \delta_1 h_1 + \delta_2 h_2 + \dots + \delta_b h_b. \quad \dots (4)$$

The number of σ_i is same as in the expression (2). That is $(b-1)(q-1)$. The enumeration of the δ coefficients is equivalent to find the solid restricted burst error in a vector of length β_s . Therefore the total number of linear sums due to the expression (4) is given by

$$(b-1)(q-1) \times \sum_{i=1}^b \{(\beta_s - i + 1)(q-1)\} \quad \dots (5)$$

Since there are $j-1$ such bytes, therefore due to the condition (ii) the total number of linear sums is

$$(b-1)(q-1) \times \sum_{s=1}^{j-1} \sum_{i=1}^b \{(\beta_s - i + 1)(q-1)\} \quad \dots (6)$$

given by

Therefore h_j should not be equal to the linear sums given by expression (3) plus linear sums given by expression (6) i.e.

$$(b-1)(q-1)\left\{1 + \sum_{s=1}^{j-1} \sum_{i=1}^b \{(\beta_s - i + 1)(q-1)\}\right\} \quad \dots (7)$$

Since there are at most q^{n-k} cosets, there we have

$$q^{n-k} > (b-1)(q-1)\left\{1 + \sum_{s=1}^{j-1} \sum_{i=1}^b \{(\beta_s - i + 1)(q-1)\}\right\}$$

The required result can be obtained by replacing j by f . We conclude this section by giving an example of a code that is capable of locating SRB errors occurring in a single burst.

Example 2.1: For $\beta_1 = 4, \beta_2 = 5, \beta_3 = 6$ and $\beta_4 = 5, b = 2$, consider a $(20, 15)$ linear code over $GF(3)$ with parity check matrix

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 2 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 2 & 0 & 2 & 2 & 2 & 2 & 0 & 2 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 2 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 2 & 0 & 2 & 0 & 2 & 2 & 0 & 1 & 1 & 0 & 2 & 2 & 2 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 & 1 & 0 & 1 & 1 & 2 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The error patterns and syndromes for the parity check matrix given above can be obtained with the help of MS-EXCEL. It can be seen that all the syndrome corresponding to all solid restricted burst errors are non-zero and distinct in different bytes. This assures that this code locates all the solid restricted burst errors occurring in a single byte.

1. Construction of Solid Restricted b_i -Byte Correcting Non-Binary Linear Optimal Codes

In this section, firstly we give a construction method of an $(n = \sum_{s=0}^f \beta_s, k)$ byte oriented linear optimal code that can correct solid restricted bursts of length b_i or less within the bytes of size $\beta_i, i = 1, 2, 3, \dots, m$, over $GF(q)$, q is prime and $q \geq 3$. The parity check matrix for the requisite code has been constructed by using the construction method given by Etzion [7] (p. 2554).

Let $H = [H_1 \ H_2 \ H_3 \ \dots \ H_m]$ be the parity check matrix for an $(n = \sum_{s=0}^f \beta_s, k)$ code C that corrects all solid restricted bursts of length b_i or less within the bytes of size $\beta_i, i = 1, 2, 3, \dots, m$, over $GF(q)$, q is prime and $q \geq 3$, where $H_i = [h_{i1}, h_{i2}, h_{i3}, \dots, h_{i\beta_i}]$ denotes the sub matrix of parity check matrix H corresponding to i -th byte of size $\beta_i, 1 \leq i \leq m$.

Let $S(H_i)$ denote the set of distinct non-zero syndromes corresponding to all solid restricted bursts of

length b_i or less within the bytes of size β_i for $i = 1, 2, 3, \dots, m$ and $S(H)$ denote the union of sets of distinct non-zero syndromes corresponding to all solid restricted bursts of length b_i or less within the bytes of size β_i for $i = 1, 2, 3, \dots, m$. In other words we can say that, $S(H) = \bigcup_{i=1}^m S(H_i)$

Clearly, each $S(H_i)$ is the union of the sets of the sum of b_i or less adjacent columns of H_i by multiplying the columns of $H_i, i = 1, 2, 3, \dots, m$ with each non-zero field element viz. $1, 2, 3, \dots, (q-1) \in GF(q)$. In other words,

$$S(H_i) = \{h_j + h_{j+1} + h_{j+2} + \dots + h_{j+b_i-1} : 1 \leq j \leq \beta_i - (b_i - 1)\} \cup \{h_j + h_{j+1} + h_{j+2} + \dots + h_{j+b_i-2} : 1 \leq j \leq \beta_i - (b_i - 2)\} \cup \dots \cup \{h_j + h_{j+1} : 1 \leq j \leq \beta_i - 1\} \cup \{h_j : 1 \leq j \leq \beta_i\} \cup \{2h_j + 2h_{j+1} + 2h_{j+2} + \dots +$$

size β_i , therefore each subset of $S(H_i)$ must be distinct for $i = 1, 2, 3, \dots, m$ i.e

$$\{h_j + h_{j+1} + h_{j+2} + \dots + h_{j+b_i-1} : 1 \leq j \leq \beta_i - (b_i - 1)\} \cap \{h_j + h_{j+1} + h_{j+2} + \dots + h_{j+b_i-2} : 1 \leq j \leq \beta_i - (b_i - 2)\} \cap \dots \cap \{h_j + h_{j+1} : 1 \leq j \leq \beta_i - 1\} \cap \{h_j : 1 \leq j \leq \beta_i\} \cap \{2h_j + 2h_{j+1} + 2h_{j+2} + \dots + 2h_{j+b_i-1} : 1 \leq j \leq \beta_i - (b_i - 1)\} \cap \{2h_j + 2h_{j+1} + 2h_{j+2} + \dots + 2h_{j+b_i-2} : 1 \leq j \leq \beta_i - (b_i - 2)\} \cap \dots \cap \{2h_j + 2h_{j+1} : 1 \leq j \leq \beta_i - 1\} \cap \{2h_j : 1 \leq j \leq \beta_i\} \cap \dots \cap \{(q-1)h_j + (q-1)h_{j+1} + \dots + (q-1)h_{j+b_i-1} : 1 \leq j \leq \beta_i - (b_i - 1)\} \cap \{(q-1)h_j + (q-1)h_{j+1} + \dots + (q-1)h_{j+b_i-2} : 1 \leq j \leq \beta_i - (b_i - 2)\} \cap \dots \cap \{(q-1)h_j + (q-1)h_{j+1} : 1 \leq j \leq \beta_i - 1\} \cap \{(q-1)h_j : 1 \leq j \leq \beta_i\} = \Phi \quad \dots (3.2)$$

The non-binary code C is a solid restricted b_i -byte correcting linear optimal code if every column vector of H of length r belongs to $S(H) = \bigcup_{i=1}^m S(H_i)$ and all the column vectors of H are distinct.

We now illustrate the construction of codes using the technique discussed above. In the following, several examples of solid restricted b_i -byte correcting non-binary linear optimal codes have been provided.

Example 3.1: For $\beta_1 = 4, \beta_2 = 9, \beta_3 = 5$ and $b_1 = 3, b_2 = 2, b_3 = 1$, consider a $(18, 15)$ linear code over $GF(5)$ with parity check matrix

$$H = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 1 & 2 & 0 & 3 & 0 & 4 & 0 & 3 & 2 & 3 & 3 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 2 & 2 & 0 & 4 & 0 & 3 & 0 & 1 & 1 & 2 & 3 & 4 & 3 & 4 & 4 \end{bmatrix} \quad \dots (3.3)$$

This matrix has been obtained by the construction method given in the equations (3.1, 3.2) by taking $\beta_1 = 4, \beta_2 = 9, \beta_3 = 5$ and $b_1 = 3, b_2 = 2, b_3 = 1$ over $GF(5)$. It can be easily verified from error-vector and syndrome table (Table 1 and 2) that the syndromes of different solid restricted bursts of length b_i or less within the bytes of size β_i for $i = 1, 2, 3$ are distinct, showing thereby that the linear code that is the null space of the matrix (3.3), corrects all solid restricted bursts of length b_i or less within the bytes of size β_i for $i = 1, 2, 3$ over $GF(5)$.

Table-1: Error-Vector and Syndrome

| Error-Vectors | Syndromes | Error-Vectors | Syndromes |
|----------------------|-----------|----------------------|-----------|
| 1110 000000000 00000 | 231 | 0000 000011000 00000 | 233 |
| 0111 000000000 00000 | 111 | 0000 000001100 00000 | 243 |
| 1100 000000000 00000 | 221 | 0000 000000110 00000 | 241 |
| 0110 000000000 00000 | 110 | 0000 000000011 00000 | 232 |
| 0011 000000000 00000 | 011 | 0000 100000000 00000 | 102 |
| 1000 000000000 00000 | 121 | 0000 010000000 00000 | 012 |
| 0100 000000000 00000 | 100 | 0000 001000000 00000 | 120 |
| 0010 000000000 00000 | 010 | 0000 000100000 00000 | 104 |
| 0001 000000000 00000 | 002 | 0000 000010000 00000 | 130 |
| 2220 000000000 00000 | 412 | 0000 000001000 00000 | 103 |
| 0222 000000000 00000 | 222 | 0000 000000100 00000 | 140 |
| 2200 000000000 00000 | 442 | 0000 000000010 00000 | 101 |
| 0220 000000000 00000 | 440 | 0000 000000001 00000 | 131 |
| 0022 000000000 00000 | 044 | 0000 220000000 00000 | 223 |
| 2000 000000000 00000 | 242 | 0000 022000000 00000 | 214 |
| 0200 000000000 00000 | 200 | 0000 002200000 00000 | 443 |
| 0020 000000000 00000 | 020 | 0000 000220000 00000 | 413 |
| 0002 000000000 00000 | 002 | 0000 000022000 00000 | 411 |
| 3330 000000000 00000 | 143 | 0000 000002200 00000 | 431 |
| 0333 000000000 00000 | 333 | 0000 000000220 00000 | 432 |
| 3300 000000000 00000 | 113 | 0000 000000022 00000 | 414 |
| 0330 000000000 00000 | 330 | 0000 200000000 00000 | 204 |
| 0033 000000000 00000 | 033 | 0000 020000000 00000 | 024 |
| 3000 000000000 00000 | 313 | 0000 002000000 00000 | 240 |
| 0300 000000000 00000 | 300 | 0000 000200000 00000 | 203 |
| 0030 000000000 00000 | 030 | 0000 000020000 00000 | 210 |
| 0003 000000000 00000 | 003 | 0000 000002000 00000 | 201 |
| 4440 000000000 00000 | 324 | 0000 000000200 00000 | 230 |
| 0444 000000000 00000 | 444 | 0000 000000020 00000 | 202 |
| 4400 000000000 00000 | 334 | 0000 000000002 00000 | 212 |
| 0440 000000000 00000 | 440 | 0000 330000000 00000 | 332 |
| 0044 000000000 00000 | 044 | 0000 033000000 00000 | 341 |
| 4000 000000000 00000 | 434 | 0000 003300000 00000 | 112 |
| 0400 000000000 00000 | 400 | 0000 000330000 00000 | 142 |
| 0040 000000000 00000 | 040 | 0000 000033000 00000 | 144 |
| 0004 000000000 00000 | 004 | 0000 000003300 00000 | 124 |
| 0000 110000000 00000 | 114 | 0000 000000330 00000 | 123 |
| 0000 011000000 00000 | 132 | 0000 000000033 00000 | 141 |
| 0000 001100000 00000 | 224 | 0000 300000000 00000 | 301 |

Table-2

| Error-Vectors | Syndromes | Error-Vectors | Syndromes |
|----------------------|-----------|----------------------|-----------|
| 0000 000110000 00000 | 234 | 0000 030000000 00000 | 031 |
| 0000 003000000 00000 | 310 | 0000 000000040 00000 | 404 |
| 0000 000300000 00000 | 302 | 0000 000000004 00000 | 424 |
| 0000 000030000 00000 | 340 | 0000 000000000 10000 | 122 |
| 0000 000003000 00000 | 304 | 0000 000000000 01000 | 133 |
| 0000 000000300 00000 | 320 | 0000 000000000 00100 | 134 |
| 0000 000000030 00000 | 303 | 0000 000000000 00010 | 013 |
| 0000 000000003 00000 | 343 | 0000 000000000 00001 | 014 |
| 0000 440000000 00000 | 441 | 0000 000000000 20000 | 244 |
| 0000 044000000 00000 | 423 | 0000 000000000 02000 | 211 |
| 0000 004400000 00000 | 331 | 0000 000000000 00200 | 213 |
| 0000 000440000 00000 | 321 | 0000 000000000 00020 | 021 |
| 0000 000044000 00000 | 322 | 0000 000000000 00002 | 023 |
| 0000 000004400 00000 | 312 | 0000 000000000 30000 | 311 |
| 0000 000000440 00000 | 314 | 0000 000000000 03000 | 344 |
| 0000 000000044 00000 | 323 | 0000 000000000 00300 | 342 |
| 0000 400000000 00000 | 403 | 0000 000000000 00030 | 034 |
| 0000 040000000 00000 | 043 | 0000 000000000 00003 | 032 |
| 0000 004000000 00000 | 430 | 0000 000000000 40000 | 433 |
| 0000 000400000 00000 | 401 | 0000 000000000 04000 | 422 |
| 0000 000040000 00000 | 420 | 0000 000000000 00400 | 421 |
| 0000 000004000 00000 | 402 | 0000 000000000 00040 | 042 |
| 0000 000000400 00000 | 410 | 0000 000000000 00004 | 041 |

Example 3.2: For $\beta_1 = \beta_2 = 3$, $\beta_3 = 2$ and $b_1 = 3$, $b_2 = 2$, $b_3 = 1$, consider a (8, 5) linear code over $GF(3)$ with parity check matrix

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 2 & 2 & 1 & 2 & 1 & 0 \\ 1 & 2 & 1 & 2 & 2 & 0 & 0 & 1 \end{bmatrix} \quad \dots (3.4)$$

This matrix has been obtained by the construction method given in the equations (3.1, 3.2) by taking $\beta_1 = \beta_2 = 3$, $\beta_3 = 2$ and $b_1 = 3$, $b_2 = 2$, $b_3 = 1$ over $GF(3)$. It can be verified from error-vector and syndrome table (Table 3) that the syndromes of different solid restricted bursts of length b_i or less within the bytes of size β_i for $i = 1, 2, 3$ are distinct, showing thereby that the linear code that is the null space of the matrix (3.4), corrects all solid restricted bursts of length b_i or less within the bytes of size β_i for $i = 1, 2, 3$ over $GF(3)$.

Table-3 Error-Vector and Syndrome

| Error-Vectors | Syndromes | Error-Vectors | Syndromes |
|---------------|-----------|---------------|-----------|
| 111 000 00 | 011 | 000 011 00 | 102 |
| 110 000 00 | 220 | 000 100 00 | 122 |
| 011 000 00 | 200 | 000 010 00 | 012 |
| 100 000 00 | 111 | 000 001 00 | 120 |
| 010 000 00 | 112 | 000 220 00 | 202 |
| 001 000 00 | 121 | 000 022 00 | 201 |
| 222 000 00 | 022 | 000 200 00 | 211 |
| 220 000 00 | 110 | 000 020 00 | 021 |
| 022 000 00 | 100 | 000 002 00 | 210 |
| 200 000 00 | 222 | 000 000 10 | 010 |
| 020 000 00 | 221 | 000 000 01 | 001 |
| 002 000 00 | 212 | 000 000 20 | 020 |
| 000 110 00 | 101 | 000 000 02 | 002 |

Example 3.3: For $\beta_1 = \beta_2 = \beta_3 = 3$ and $b_1 = 1, b_2 = b_3 = 2$, consider a (9, 6) linear code over $GF(3)$ with parity check matrix

$$H = \begin{bmatrix} 2 & 1 & 2 & 2 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 2 & 2 & 1 & 2 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 & 2 & 0 & 1 & 1 \end{bmatrix} \quad \dots (3.5)$$

This matrix has been obtained by the construction method given in the equations (3.1, 3.2) by taking $\beta_1 = \beta_2 = \beta_3 = 3$ and $b_1 = 1, b_2 = b_3 = 2$ over $GF(3)$. It can be easily verified from error-vector and syndrome table that the syndromes of different solid restricted bursts of length b_i or less within the bytes of size β_i for $i = 1, 2, 3$ are distinct, showing thereby that the linear code that is the null space of the matrix (3.5), corrects all solid restricted bursts of length b_i or less within the bytes of size β_i for $i = 1, 2, 3$ over $GF(3)$.

2. Construction

As we know that the byte correcting optimal codes can enhance the rate of transmission of data and also improve the efficiency of the byte-oriented communication systems. The contribution of this paper is to find out the possibility of existence of the non-binary linear codes that can correct solid restricted burst of length b_1 or less in the first byte of size β_1 , solid restricted burst of length b_2 or less in the second byte of size β_2 and so on and solid restricted burst of length b_i or less in the i -th byte of size β_i . We have been able to obtain some codes. This justifies the existence of such solid restricted b_i -byte correcting non-binary linear optimal code.

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