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Laplacian Minimum Pendant Dominating Partition Energy of a Graph

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Abstract:- Let G be any graph. A dominating set S in G is called a pendant dominating set if induced subgraph of S contains at least one pendant vertex. The least cardinality of the pendant dominating set in G is called the pendant domination number of G, denoted by $\gamma_{pe}(G)$. In this paper, we define, compute and study the Laplacian minimum pendant dominating 'k −partition energy of some standard familie's graphs $(1 \leq k \leq n)$. Further we also establish upper and lower bounds for Laplacian minimum pendant dominating k −partition energy of a graph G

Keywords: *Dominating set, Pendant Dominating set, Minimum Pendant Dominating set, Partition Energy, Laplacian Minimum Pendant Dominating k-Partition Energy*

1. Introduction

Let $G = (V, E)$ be a graph with n vertices and m edges. The degree of v_i written by $d(v_i)$ is the number of edges incident with v_i. The maximum vertex of degree is denoted by $\Delta(G)$ and minimum vertex of degree is denoted by $\delta(G)$. The adjacency matrix $A_{pe}(G)$ of G is defined by its entries as $a_{ij} = 1 \; if \; v_iv_j \in E(G)$or $v_i \in D$if $(i = j)$ where D is a pendant dominating set of G and 0 otherwise. The eigen values of graph G are the eigenvalues of its adjacency matrix $A_{pe}(G)$ denoted by $\lambda_1 \geq \lambda_2 \geq \cdots, \geq \lambda_n$. A graph G is said to be singular if at least one of its eigenvalues is equal to zero. For singular graphs, evidently, $detA = 0$. A graph is nonsingular if all its eigenvalues are different from zero.

The energy of a graph G is defined as $E(G) = \sum_{i=1}^{n} | \lambda_i |$. This concept was introduced by I. Gutman in [4]. initially, the graph energy concept did not attract any noteworthy attention of mathematicians, but later they did realize its value and worldwide mathematical research of graph energy started. Nowadays, in connection with graph energy, energy like quantities was considered also for other matrices. I.Gutman and B.Zhou [5] defined the Laplacian energy of a graph G in the year 2006. Let G be a graph with n vertices and m edges. The Laplacian matrix of the graph G, denoted by $L = (L_{i,j})$ is a square matrix of order n. The elements of the Laplacian matrix are defined as

$$L_{ij} = \begin{cases} -1, & \text{if } v_i \text{ and } v_j \text{ are adjacent} \\ 0, & \text{if } v_i \text{ and } v_j \text{ are non adjacent} \\ d_i & if \; i = j \end{cases}$$

Where d_i is the degree of the vertex v_i

Let $\lambda_1, \lambda_2, \ldots, \lambda_n$be the eigen values of Laplacian matrix G. Laplacian energy of G is defined as

$$LE(G) = \sum_{i=1}^{n} \left| \lambda_i - \frac{2m}{n} \right|.$$

The basic properties Laplacian energy including various upper and lower bounds have been established in [5] and it has found that remarkable chemical application, high resolution satellite image classification and segmentation using laplacian graph energy and finding semantic structures in image hierarchies using laplacian graph energy.

In this paper, we are defining a matrix, called the Laplacian minimum pendant dominating k-partition matrix denoted by $LP_kE_{pe}(G)$ and we study its eigenvalues and the energy. Further, we study the mathematical aspects of the Laplacian minimum pendant dominating partition energy of a graph. It may be possible that the Laplacian minimum pendant dominating partition energy which we are considering in this paper have applications in other areas of science such as chemistry and so on. The graphs we are considering are assumed to be finite, simple, undirected having no isolated vertices, and of order at least two. A subset S of $V(G)$ is a dominating set of G if each vertex $u \in V - S$ is adjacent to a vertex in S. The least cardinality of a dominating set is called the domination number of G and is usually denoted by $\gamma(G)$. A dominating set S is called a pendant dominating set if $< S >$ contains at least one pendant vertex. The least cardinality of a pendant dominating set in G is called the pendant domination number of G, denoted by $\gamma_{pe}(G)$. For more details about pendant domination parameter refer[6].

Partition Energy:

Let G be a simple graph of order n with vertex set $V = \{v_1, v_2, v_3, \ldots, v_n\}$ and edge set E. Let $P_k = \{V_1, V_2, V_3, \ldots, V_K\}$ be a partition of a vertex set V. The partition matrix of G is the $n \times n$ matrix is defined by $A(G) = (a_{ij})$, where

$$a_{ij} = \begin{cases} 2, & \text{if } v_i \text{ and } v_j \text{ are adjacent where } v_{i_i}, v_j \in V_r \\ -1, & \text{if } v_i \text{ and } v_j \text{are non adjacent where } v_i, v_j \in V_r \\ 1, & \text{if } v_i \text{ and } v_j \text{ are adjacent between the sets} \\ & V_r \; and \; V_s \; for \; r \neq s \; where \; v_i \in V_r \; and \; v_j \in V_s \\ 0, & otherwise \end{cases}$$

The eigen values of this matrix are called k-partition eigenvalues of G. The k-partition energy $P_kE(G)$ is defined as the sum of the absolute values of k-partition eigenvalues of G. For details about partition energy refer [10]

Minimum Pendant Dominating Partition Energy:

Let G be a simple graph of order n with vertex set $V = \{v_1, v_2, v_3, \ldots, v_n\}$ and edge set E. Let $P_k = \{V_1, V_2, V_3, \ldots, V_K\}$ be a partition of a vertex set V. A subset D of $V(G)$ is a pendant dominating set of G if induced subgraph of D contains at least one pendant vertex. The least cardinality of a pendant dominating set is called the pendant domination number of G and is usually denoted by $\gamma_{pe}(G)$. The minimum pendant dominating k-partition matrix of G is the $n \times n$ matrix defined by $P_kA_{pe}(G) = (a_{ij})$,

Where

$$a_{ij} = \begin{cases} 2, & \text{if } v_i \text{ and } v_j \text{ are adjacent where } v_{i_i}, v_j \in V_r \\ -1, & \text{if } v_i \text{ and } v_j \text{are non adjacent where } v_i, v_j \in V_r \\ 1, & \text{if i = j}, v_i \in \text{D or } v_i \text{ and } v_j \text{ are adjacent between the sets} \\ & V_r \; and \; V_s \; for \; r \neq s \; where \; v_i \in V_r \; and \; v_j \in V_s \\ 0, & otherwise \end{cases}$$

The characteristic polynomial of $A_{pe}(G)$ is denoted by $f_n(G, \lambda) = det\left(\lambda I - A_{pe}(G)\right)$. The minimum pendant dominating k-partition eigenvalues of the graph G are the eigenvalues of $A_{pe}(G)$. Since $A_{pe}(G)$ is real and symmetric, its eigenvalues are real numbers and we label them in non- increasing order $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$. The minimum pendant dominating k-partition energy of G is defined as $P_kE_{pe}(G) = \sum_{i=1}^{n} | \lambda_i |$.

For more details about minimum pendant dominating partition energy of a graph refer [8]

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The Laplacian Minimum Pendant Dominating Partition Energy of a Graph:

Consider a graph $G = (V, E)$ of order n and size m with a partition $P_k = \{V_1, V_2, \ldots, V_k\}$ of V. Let $P_k E_{pe}(G)$ be minimum pendant dominating partition matrix and $D(G)$ represent the diagonal matrix of vertex degrees of the graph G. Then we define the Laplacian minimum pendant dominating partition matrix of G as $LP_k E_{pe}(G) = D(G) - A_{pe}(G)$. The eigen values $\{\mu_1, \mu_2, \ldots, \mu_n\}$ of this matrix$P_k E_{pe}(G)$are called Laplacian minimum pendant dominating k- partition eigen values. We also define auxillary partition eigen values γ_i $i = 1,2,3, \ldots, n$ as $\gamma_i = \mu_i - \frac{2m}{n}$. The Laplacian minimum pendant dominating k- partition energy of G, denoted by $LP_k E_{pe}(G)$ is defined as $\sum_{i=1}^{n} \mid \gamma_i \mid$.

$$i.e., LP_k E_{pe}(G) = \sum_{i=1}^{n} \left| \mu_i - \frac{2m}{n} \right|.$$

Where m is the number of edges of G and $\frac{2m}{n}$ is the average degree of G

2. Methods

Here, we are presenting the methods which are planning to use in our research for proving the result or to explore new ideas or concept in our research area. To establish the upper and lower bounds for the energy of graphs we employ the Standard methods of proofs namely direct methods, by induction or by the contradiction method

3. Results

3.1 The Laplacian Minimum Pendant Dominating Partition Energy of a Graph

Example 3.1: Consider a graph as shown in Figure 1. The possible minimum pendant dominating sets are:

(i) $D_1 = \{v_1, v_2, v_4\}$ (ii) $D_2 = \{v_2, v_3, v_5\}$ (iii) $D_3 = \{v_2, v_3, v_6\}$ (iv) $D_4 = \{v_2, v_3, v_4\}$
(v) $D_5 = \{v_1, v_4, v_3\}$ (vi) $D_6 = \{v_5, v_6, v_2\}$.

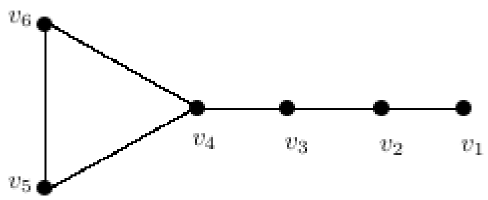

Figure 1

$$A_{pe,D_1}(G) = \begin{pmatrix} & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \\ v_1 & 1 & 2 & -1 & -1 & -1 & -1 \\ v_2 & 1 & 2 & -1 & -1 & -1 & -1 \\ v_3 & -1 & 2 & 0 & 2 & -1 & -1 \\ v_4 & -1 & -1 & 2 & 1 & 2 & 2 \\ v_5 & -1 & -1 & -1 & 2 & 0 & 2 \\ v_6 & -1 & -1 & -1 & 2 & 2 & 0 \end{pmatrix} \text{ and}$$

$$D(G) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}.$$

$$LP_1E_{pe,D_1}(G) = D(G) - A_{pe,D_1}(G) = \begin{pmatrix} 0 & -2 & 1 & 1 & 1 & 1 \\ -2 & 1 & -2 & 1 & 1 & 1 \\ 1 & -2 & 2 & -2 & 1 & 1 \\ 1 & 1 & -2 & 2 & -2 & -2 \\ 1 & 1 & 1 & -2 & 2 & -2 \\ 1 & 1 & 1 & -2 & -2 & 2 \end{pmatrix}.$$

So, $f_n(G,\lambda) = \lambda^6 - 9\lambda^5 - 7\lambda^4 + 168\lambda^3 - 321\lambda^2 - 98\lambda + 152 = 0.$

The Laplacian minimum pendant dominating partition eigen values are $\lambda_1 \approx -4.2573, \lambda_2 \approx -0.7119$, $\lambda_3 \approx 0.6465$, $\lambda_4 \approx 3.1696$, $\lambda_5 \cong 4$, $\lambda_6 \approx 6.1571$. Average degree of the graph $= \frac{2m}{n} = \frac{2\times 6}{6} = \frac{12}{6} = 2$

Hence, Laplacian minimum pendant dominating partition energy, $LP_1E_{pe,D_1}(G) = 17.6534$.

Now, suppose if we choose another minimum pendant dominating set, $D_2 = \{v_2, v_3, v_5\}$. Then

$$A_{pe,D_2}(G) = \begin{pmatrix} & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \\ v_1 & 0 & 2 & -1 & -1 & -1 & -1 \\ v_2 & 2 & 1 & 2 & -1 & -1 & -1 \\ v_3 & -1 & 2 & 1 & 2 & -1 & -1 \\ v_4 & -1 & -1 & 2 & 0 & 2 & 2 \\ v_5 & -1 & -1 & -1 & 2 & 1 & 2 \\ v_6 & -1 & -1 & -1 & 2 & 2 & 0 \end{pmatrix} \text{ and}$$

$$D(G) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{pmatrix}.$$

$$LP_1E_{pe,D_2}(G) = D(G) - A_{pe,D_2}(G) = \begin{pmatrix} 1 & -2 & 1 & 1 & 1 & 1 \\ -2 & 1 & -2 & 1 & 1 & 1 \\ 1 & -2 & 1 & -2 & 1 & 1 \\ 1 & 1 & -2 & 3 & -2 & -2 \\ 1 & 1 & 1 & -2 & 1 & -2 \\ 1 & 1 & 1 & -2 & -2 & 2 \end{pmatrix}$$

Clearly, $f_n(G,\lambda) = \lambda^6 - 9\lambda^5 - \lambda^4 + 166\lambda^3 - 300\lambda^2 - 207\lambda + 324 = 0.$

The Laplacian minimum pendant dominating partition eigen values are $\lambda_1 \approx -4.1456$, $\lambda_2 \approx -1.0821$, $\lambda_3 \approx 0.9278$, $\lambda_4 \approx 3.4223$, $\lambda_5 \cong 6.2208$, $\lambda_6 \approx 3.6568$.

The Laplacian minimum pendant domination partition energy, $LP_1E_{pe,D_2}(G) \approx 17.5998$ Therefore, it is clear from the above example that the Laplacian minimum pendant dominating partition energy of a graph G depends on the minimum pendant dominating set of a graph G.

3.2 Properties of Laplacian Minimum Pendant Dominating Eigen Values of a Graph

Let $G = (V, E)$ be a graph with n vertices and $P_k = \{v_1, v_2, \dots, v_k\}$ be a partitions of G. For $1 \leq i \leq k$, let b_i denote the total number of edges joining the vertices of V_i and c_i be the total number of edges joining the vertices from V_i to V_j for $i \neq j$, $1 \leq i \leq k$ and d_i be the number of non adjacent pairs of vertices within V_i.

Let $m_1 = \sum_{i=1}^{k} | b_i |$, $m_2 = \sum_{i=1}^{k} | c_i |$, $and\ m_3 = \sum_{i=1}^{k} | d_i |$,

Theorem 3.2.1. If D is a minimum pendant dominating set of a graph G and $\lambda_1, \lambda_2, \dots, \lambda_n$ are the minimum pendant dominating k-Partition eigen values of minimum pendant dominating k –partition matrix $A_{pe}(G)$ then

i. $\sum_{i=1}^{n} \gamma_i = | D |$

ii. $\sum_{i=1}^{n} \gamma_i^2 = | D | + 2(4m_1 + m_2 + m_3)$

3.3 Bounds for Laplacian Minimum Pendant Dominating Partition Energy of a Graph

Theorem 3.3.1. Let $G = (V, E)$ be a graph with n vertices and $P_k = \{v_1, v_2, \dots, v_k\}$ be a partition of V. Then $LP_kE_{pe}(G) \leq \sqrt{n[| D | + 2(4m_1 + m_2 + m_3)]}$ where m_1, m_2, m_3 are as defined above for G

Proof. By Cauchy-Schwartz inequality is

$$\left(\sum_{i=1}^{n} a_i b_i\right)^2 \leq \left(\sum_{i=1}^{n} a_i^2\right)\left(\sum_{i=1}^{n} b_i^2\right)$$

If we $a_i = 1$, $b_i = | \gamma_i |$ then

$$\left(\sum_{i=1}^{n} | \gamma_i |\right)^2 \leq \left(\sum_{i=1}^{n} 1\right)\left(\sum_{i=1}^{n} \gamma_i^2\right)$$

$$\Rightarrow \left[LP_kE_{pe}(G)\right]^2 \leq n[| D | + 2(4m_1 + m_2 + m_3)]$$

Theorem 3.3.2. Let G be a simple graph with n vertices and m edges. $P_k = \{v_1, v_2, \dots, v_k\}$ be a partition of V. If D is the minimum pendant dominating set and $P = | detA_{pe}(G) |$ then

$$\sqrt{| D | + 2(4m_1 + m_2 + m_3) + n(n-1)P^{\frac{2}{n}}} \leq LP_kE_{pe}(G) \leq \sqrt{n[| D | + 2(4m_1 + m_2 + m_3)]}$$

Proof. By Cauchy-Schwartz inequality is

$$\left(\sum_{i=1}^{n} a_i b_i\right)^2 \leq \left(\sum_{i=1}^{n} a_i^2\right)\left(\sum_{i=1}^{n} b_i^2\right)$$

If we $a_i = 1$, $b_i = | \gamma_i |$ then

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$$\left(\sum_{i=1}^{n} \mid \gamma_i \mid\right)^2 \leq \left(\sum_{i=1}^{n} 1\right)\left(\sum_{i=1}^{n} \gamma_i^2\right)$$

$$\left[LP_k E_{pe}(G)\right]^2 \leq n[\mid D \mid +2(4m_1 + m_2 + m_3)]$$

$$\Rightarrow \left[LP_k E_{pe}(G)\right]^2 \leq n[\mid D \mid +2(4m_1 + m_2 + m_3)]$$

Now by arithmetic mean and geometric mean inequality

$$\frac{1}{n(n-1)} \sum_{i \neq j} \mid \gamma_i \mid\mid \gamma_j \mid \geq \left[\prod_{i \neq j} \mid \gamma_i \mid\mid \gamma_j \mid\right]^{\frac{1}{n(n-1)}}$$

$$= \left[\prod_{i \neq j} \mid \gamma_i \mid^{2(n-1)}\right]^{\frac{1}{n(n-1)}}$$

$$= \left[\prod_{i \neq j} \mid \gamma_i \mid\right]^{\frac{2}{n}}$$

$$= \mid detA_{pe}(G) \mid^{\frac{2}{n}} = P^{\frac{2}{n}}$$

$$\sum_{i \neq j} \mid \gamma_i \mid\mid \gamma_j \mid \geq n(n-1)P^{\frac{2}{n}} \qquad \longrightarrow \quad 5.1$$

Now consider,

$$\left[LP_k E_{pe}(G)\right]^2 = \left(\sum_{i=1}^{n} \mid \gamma_i \mid\right)^2$$

$$= \left(\sum_{i=1}^{n} \mid \gamma_i \mid\right)^2 + \sum_{i \neq j} \mid \gamma_i \mid\mid \gamma_j \mid$$

$$\therefore \left[LP_k E_{pe}(G)\right]^2 \geq \mid D \mid +2(4m_1 + m_2 + m_3) + n(n-1)P^{\frac{2}{n}} \qquad \text{from (5.1)}$$

$$i.e., LP_k E_{pe}(G) \geq \sqrt{\mid D \mid +2(4m_1 + m_2 + m_3) + n(n-1)P^{\frac{2}{n}}}$$

Theorem 3.3.3. If G be a graph with n vertices, m edges and D is a minimum pendant dominating set of a graph G. Then $L_k E_{pe}(G) \leq \sqrt{2Nn} + 2m$. Where $N = \mid E \mid + \frac{1}{2}\sum_{i=1}^{n}(d_i - h_i)^2$,$h_i = \begin{cases} 1, if\ v_i \in D \\ 0, if\ v_i \notin D \end{cases}$

Proof: Let G be a graph with n vertices and m edges and be the eigen values of G. By using Cauchy's - Schwarz inequality

$$\left(\sum_{i=1}^{n} a_1 b_i\right)^2 \leq \left(\sum_{i=1}^{n} a_i^2\right)\left(\sum_{i=1}^{n} b_i^2\right)$$

Put $a_i = 1, b_i = \gamma_i$ then,

$$\left(\sum_{i=1}^{n} |\gamma_i|\right)^2 \leq \left(\sum_{i=1}^{n} 1\right)\left(\sum_{i=1}^{n} |\gamma_i|^2\right)$$

$$\left(\sum_{i=1}^{n} |\gamma_i|\right)^2 \leq n\, 2N$$

$$\therefore \left(\sum_{i=1}^{n} |\gamma_i|\right) \leq \sqrt{2Nn}$$

By Triangle inequality $\left|\gamma_i - \frac{2m}{n}\right| \leq |\gamma_i| + \left|\frac{2m}{n}\right| \; \forall i = 1,2, \dots, n$

$$i.e., \left|\gamma_i - \frac{2m}{n}\right| \leq |\gamma_i| + \frac{2m}{n}$$

$$\left(\sum_{i=1}^{n} \left|\gamma_i - \frac{2m}{n}\right|\right) \leq \left(\sum_{i-1}^{n} \gamma_i\right) + \left(\sum_{i-1}^{n} \frac{2m}{n}\right)$$

$$\leq \sqrt{2Nn} + 2m$$

$$\therefore \; LP_kE_{pe}(G) \leq \sqrt{2Nn} + 2m$$

3.4 The Laplacian Minimum Pendant Dominating Partition Energy of Some Standard Graphs

Theorem 3.4.1. For $n \geq 4$, the Laplacian minimum pendant dominating 1-partition energy of complete graph K_n is equal to $(2n-5) + \sqrt{4n^2 - 4n + 17}$.

Proof. Let K_n be the complete graph with vertex set $V = \{v_1, v_2, \dots, v_n\}$. The minimum pendant dominating set $D = \{v_1, v_2\}$. Then the Laplacian minimum pendant dominating 1- partition matrix of complete graph is

$$P_1A_{pe}(K_n) = \left(\begin{array}{c|ccccccc} & v_1 & v_2 & v_3 & \dots & v_{n-2} & v_{n-1} & v_n \\ \hline v_1 & 1 & 2 & 2 & \dots & 2 & 2 & 2 \\ v_2 & 2 & 1 & 2 & \dots & 2 & 2 & 2 \\ v_3 & 2 & 2 & 0 & \dots & 2 & 2 & 2 \\ . & . & . & . & \dots & . & . & . \\ . & . & . & . & \dots & . & . & . \\ . & . & . & . & \dots & . & . & . \\ v_{n-2} & 2 & 2 & 2 & \dots & 0 & 2 & 2 \\ v_{n-1} & 2 & 2 & 2 & \dots & 2 & 0 & 2 \\ v_n & 2 & 2 & 2 & \dots & 2 & 2 & 0 \end{array}\right)$$

$$D(K_n) = \begin{pmatrix} n-1 & 0 & 0 & \dots & 0 & 0 \\ 0 & n-1 & 0 & \dots & 0 & 0 \\ 0 & 0 & n-1 & \dots & 0 & 0 \\ . & . & . & \dots & . & . \\ . & . & . & \dots & . & . \\ . & . & . & \dots & . & . \\ 0 & 0 & 0 & \dots & n-1 & 0 \\ 0 & 0 & 0 & \dots & 0 & n-1 \end{pmatrix}.$$

$$LP_1E_{pe}(K_n) = D(K_n) - A_{pe}(K_n) = \begin{pmatrix} n-2 & -2 & -2 & \dots & -2 & -2 \\ -2 & n-2 & -2 & \dots & -2 & -2 \\ -2 & -2 & n-1 & \dots & -2 & -2 \\ . & . & . & \dots & . & . \\ . & . & . & \dots & . & . \\ . & . & . & \dots & . & . \\ -2 & -2 & -2 & \dots & n-1 & -2 \\ -2 & -2 & -2 & \dots & -2 & n-1 \end{pmatrix}$$

The characteristic polynomial of $LP_1E_{pe}(K_n)$ is given by

$$(\lambda - n)\big(\lambda - (n+1)\big)^{n-3}\big(\lambda^2 - \lambda - (n^2 - n + 4)\big) = 0.$$

The Laplacian minimum pendant dominating 1-partition eigen values are:

$$\lambda = n[1 times], \lambda = n+1[(n-3) times], \qquad \lambda = \frac{1 \pm \sqrt{(4n^2 - 4n + 17)}}{2}[one\ time\ each]$$

Average degree of $K_n = \frac{2m}{n} = 2\frac{\frac{n(n-1)}{2}}{n} = n-1$

Hence, the Laplacian minimum pendant dominating 1-partition energy of K_n is

$$LP_1E_{pe}(K_n) = |\, n - (n-1) \,| + |\, (n+1) - (n-1)\,|\, (n-3) + \left|\frac{1 + \sqrt{(4n^2 - 4n + 17)}}{2} - (n-1)\right| + \left|\frac{1 - \sqrt{(4n^2 - 4n + 17)}}{2} - (n-1)\right|$$

$$LP_1E_{pe}(K_{,n}) = 1 + (2n - 6) + \left|\frac{-2n + 3 + \sqrt{(4n^2 - 4n + 17)}}{2}\right| + \left|\frac{-2n + 3 - \sqrt{(4n^2 - 4n + 17)}}{2}\right|$$

$$LP_1E_{pe}(K_{,n}) = (2n - 5) + \sqrt{(4n^2 - 4n + 17)}$$

Theorem 3.4.2. For $n \geq 3$, the minimum pendant dominating n-partition energy of complete graph K_n is equal to $(n-3) + \sqrt{(n^2 - 2n + 9)}$.

Proof. Let K_n be the complete graph with vertex set $V = \{v_1, v_2, v_3, .., v_n\}$. The minimum pendant dominating set is $D = \{v_1, v_2\}$. Then the minimum pendant dominating n-partition matrix of complete graph is

$$P_nA_{pe}(K_n) = \begin{array}{c|ccccccc} & v_1 & v_2 & v_3 & \dots & v_{n-2} & v_{n-1} & v_n \\ \hline v_1 & 1 & 1 & 1 & \dots & 1 & 1 & 1 \\ v_2 & 1 & 1 & 1 & \dots & 1 & 1 & 1 \\ v_3 & 1 & 1 & 0 & \dots & 1 & 1 & 1 \\ . & . & . & . & \dots & . & . & . \\ . & . & . & . & \dots & . & . & . \\ . & . & . & . & \dots & . & . & . \\ v_{n-2} & 1 & 1 & 1 & \dots & 0 & 1 & 1 \\ v_{n-1} & 1 & 1 & 1 & \dots & 1 & 0 & 1 \\ v_n & 1 & 1 & 1 & \dots & 1 & 1 & 0 \end{array}$$

$$D(K_n) = \begin{pmatrix} n-1 & 0 & 0 & \dots & 0 & 0 \\ 0 & n-1 & 0 & \dots & 0 & 0 \\ 0 & 0 & n-1 & \dots & 0 & 0 \\ . & . & . & \dots & . & . \\ . & . & . & \dots & . & . \\ . & . & . & \dots & . & . \\ 0 & 0 & 0 & \dots & n-1 & 0 \\ 0 & 0 & 0 & \dots & 0 & n-1 \end{pmatrix}.$$

$$LP_nE_{pe}(K_n) = D(K_n) - A_{pe}(K_n) = \begin{pmatrix} n-2 & -1 & -1 & \dots & -1 & -1 \\ -1 & n-2 & -1 & \dots & -1 & -1 \\ -1 & -1 & n-1 & \dots & -1 & -1 \\ . & . & . & \dots & . & . \\ . & . & . & \dots & . & . \\ . & . & . & \dots & . & . \\ -1 & -1 & -1 & \dots & n-1 & -1 \\ -1 & -1 & -1 & \dots & -1 & n-1 \end{pmatrix}$$

The characteristic polynomial of $LP_nE_{pe}(K_n)$ is given by

$\big(\lambda - (n-1)\big)(\lambda - n)^{n-3}(\lambda^2 - (n-1)\lambda - 2) = 0.$

The Laplacian minimum pendant dominating n-partition eigen values are:

$\lambda = n - 1[1time], \lambda = n[(n-3)times], \qquad \lambda = \dfrac{(n-1) \pm \sqrt{(n^2 - 2n + 9)}}{2}[one\ time\ each]$

Average degree of $K_n = \frac{2m}{n} = 2\frac{\frac{n(n-1)}{2}}{n} = n - 1$

Hence, the Laplacian minimum pendant dominating 1-partition energy of K_n is

$$LP_nE_{pe}(K_n) = \mid (n-1) - (n-1) \mid + \mid n - (n-1) \mid (n-3) + \left|\frac{(n-1) + \sqrt{(n^2-2n+9)}}{2} - (n-1)\right| + \left|\frac{1 - \sqrt{(n^2-2n+9)}}{2} - (n-1)\right|$$

$$LP_nE_{pe}(K_{,n}) = (n-3) + \left|\frac{-n+1+\sqrt{(n^2-2n+9)}}{2}\right| + \left|\frac{-n+1-\sqrt{(n^2-2n+9)}}{2}\right|$$

$$LP_nE_{pe}(K_{,n}) = (n-3) + \sqrt{(n^2-2n+9)}$$

Discussion

Nowadays, the study of theory of domination and energy of graph is an important area in Graph theory and also remarkable research is going on in this area. In recent years many scholars are working in this area and also they are introducing new domination parameters. In this paper we have initiated the study of pendant domination parameter and extend this parameter toLaplacian partition energy of graph. We have calculated the Laplacian partition energies for some standard family graphs and we have established some bounds for this parameter. Further, we have studied some important properties of Laplacian minimum pendant dominating partition eigenvalues and some bounds related to eigen values.

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