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Anisotropic Dark Energy Bianchi Type III in Brans - Dicke Theory of Gravitation and General Relativity.

V P Kadam
Dept of Math, Gopikabai Sitaram Gawande College, Umarkhed- 445206, India
E-mail: vpkadamgsg@gmail.com.

Abstract

We investigate the spatially homogeneous Bianchi Type III cosmological models with anisotropic dark energy fluid in General theory of relativity and in scalar tensor theory of gravitation proposed by Brans-Dicke [1]. The solutions of the models are obtained by using relation between shear scalar σ and scalar expansion θ. The physical aspects of the dark energy models are discussed.

Keywords: Bianchi Type III Universe, Brans-Dicke Theory of Gravitation, Dark Energy

I. Introduction:

Over the past decade, one of the most remarkable discoveries is that our universe is currently accelerating. The high redshift supernovae Ia was first observed by Astier, P. *et al.* [1], Perlmutter *et al.* [2], Riess *et al.* [3, 4], which later confirmed from the cosmic microwave background radiation by Bennett *et al.* [5, 6], Bemui *et al.* [7] and Spergel *et al.* [8] and from the large scale structure by Abazajian *et al.* [9,10,11], Hawkins *et al.* [12], Tegmark *et al.* [13] and Verde *et al.* [14]. Recent astronomical observations, based on Friedmann cosmological model, indicate that our universe is flat and currently consists of approximately 73% DE, 23% dark matter, and 4% baryon matter and radiation. The nature of DE is unknown, and many radically different models have been proposed, such as, quintessence by Caldwell [15], Frieman *et al.* [16], Liddle and Scherrer [17], Peebles [18], Ratra and Peebles [19], Steinhardt *et al.* [20], Turner and White [21] and Wetterich [22], Chaplygin gas by Kamenshchik [23], phantom energy by Caldwell [24] and DE in brane worlds by Cai [25], Sahni [26], Yang and Wang [27] among many others. (see the review articles Sahni [28, 29], Carroll [30], Padmanabhan [31], Peebles [32])

Many relativists have taken keen interest in studying Bianchi type-IX universe because familiar solutions like Robertson Walker universe, the de-sitter universe, the Taub-NUT solutions etc. Bianchi type-IX cosmological models include closed FRW models. These models allow not only expansion but also rotation and shear and in general are anisotropic. The dynamical effects of Bianchi type-IX models are studied by Waller [33]. Bali and Dave [34] have investigated Bianchi type-IX string cosmological model in general relativity. Bali and Kumawat [35] have investigated Bianchi type-IX stiff fluid tilted cosmological model with bulk viscosity. Tyagi and Sharma [36] investigated Bianchi-IX string cosmological model for perfect fluid distribution in general relativity. Bali and Yadav [37] studied Bianchi type-IX string as well as viscous models. Reddy *et al.* [38] studied Bianchi type-II, VIII and IX models in scale covariant theory of gravitation. Shanthi and Rao [39] studied Bianchi type-VIII and IX models in Lyttleton-Bondi universe. Also Rao and Sanyasi Raju [40, 41] have studied Bianchi type-VIII and IX models in zero mass scalar fields and self creation cosmology. Rahaman *et al.* [42] have investigated Bianchi type-IX string cosmological model in scalar-tensor theory formulated by Sen [43] based on Lyra [44] manifold. Rao *et al.* [45, 46, 47] have studied Bianchi type-II, VIII and IX string cosmological models, perfect fluid cosmological models in Saez-Ballester scalar-tensor theory of gravitation and string cosmological models in general relativity as well as self creation theory of gravitation respectively.

Brans and Dicke [48] theory of gravitation is well known modified version of Einstein's general theory of gravitation. It is a scalar tensor theory in which the gravitational interaction is mediated by a scalar field ϕ as well as the tensor field g_{ij} of Einstein's theory. In this theory the scalar field ϕ has the dimension of the inverse of the gravitational constant. In recent years, there has been a renewed interest of the gravitational constant. Several aspects of Brans-Dicke cosmology have been extensively investigated by many authors. In particular, spatially homogeneous Bianchi models in Brans-Dicke theory in the presence of perfect fluid with or without radiation are quite important to discuss the early stages of evolution of the universe. Nariai [49], Belinskii and Khalatnikov [50], Reddy and Rao [51], Banerjee and Santos [52], Shri Ram [53], Shri Ram and Singh [54], Berman *et al.* [55], Reddy [56], Adhav *et al.* [57] and Rao *et al.* [58, 59] are some of the authors who have investigated several aspects of this Brans-Dicke theory.

$$G_{ij} = -8\pi\phi^{-1}T_{ij} - \omega\phi^{-2}\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) - \phi^{-1}\left(\phi_{i;j} - g_{ij}\phi_{,k}{}^{;k}\right) \tag{1}$$

$$\phi_{;k}^{;k} = \frac{8\pi}{(3+2\omega)}T \tag{2}$$

Where $G_{ij} = R_{ij} - -\frac{1}{2}Rg_{ij}$ is an Einstein tensor, R is the scalar curvature, ω is a dimensional less constant, and T_{ij} is the stress energy tensor of the matter. Also comma and semicolon denote partial and covariant differentiation, respectively.

Also, we have energy conservation equation;

$$T_{;j}^{ij} = 0 \tag{3}$$

The energy momentum tensor of fluid can be written most generally anisotropic diagonal form as follows

$$T_j^i = diag\left[T_1^1, T_2^2, T_3^3, T_4^4\right] \tag{4}$$

$$T_j^i = diag\left[-p_x, -p_y, -p_z, \rho\right]$$

$$T_j^i = diag\left[-\omega_x, -\omega_y, -\omega_z, 1\right]\rho$$

$$T_j^i = diag\left[-(\omega+\delta), -(\omega+\delta), -\omega, 1\right]\rho \tag{5}$$

In this paper we discussed Bianchi type-III space-time filled with DE in Brans-Dicke theory of gravitation and in general relativity. This work is organized as follows: In Section 2, the metric and field equations have been presented. The field equations have been solved in Section 3 by using deceleration parameter. The physical and geometrical properties of the model have been discussed in Section 4 and concluding remarks have been expressed, in the last Section 5,.

2. Metric and Field Equations :

We consider the Bianchi-type III cosmological model given by

$$ds^2 = dt^2 - A^2dx^2 - B^2e^{-2x}dy^2 - C^2dz^2 \tag{6}$$

Where A, B and C are functions of time t alone.

With the help of (2) to (5), the field equations (1) for the metric (6) can be written as

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4}{B}\frac{C_4}{C} + \frac{\omega}{2}\frac{\phi_4{}^2}{\phi^2} + \frac{\phi_{44}}{\phi} + \frac{\phi_4}{\phi}\left(\frac{B_4}{B} + \frac{C_4}{C}\right) = -8\pi\phi^{-1}(\omega+\delta)\rho \tag{7}$$

$$\frac{A_{44}}{A}+\frac{C_{44}}{C}+\frac{A_4}{A}\frac{C_4}{C}+\frac{\omega}{2}\frac{\phi_4{}^2}{\phi^2}+\frac{\phi_{44}}{\phi}+\frac{\phi_4}{\phi}\left(\frac{A_4}{A}+\frac{C_4}{C}\right)=-8\pi\phi^{-1}(\omega+\delta)\rho$$

(8)

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4}{A}\frac{B_4}{B}-\frac{1}{A^2}+\frac{\omega}{2}\frac{\phi_4{}^2}{\phi^2}+\frac{\phi_{44}}{\phi}+\frac{\phi_4}{\phi}\left(\frac{A_4}{A}+\frac{B_4}{B}\right)=-8\pi\phi^{-1}\omega\,\rho$$

(9)

$$\frac{A_4}{A}\frac{B_4}{B}+\frac{A_4}{A}\frac{C_4}{C}+\frac{B_4}{B}\frac{C_4}{C}-\frac{1}{A^2}-\frac{\omega}{2}\frac{\phi_4{}^2}{\phi^2}+\frac{\phi_4}{\phi}\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right)=8\pi\phi^{-1}\,\rho$$

(10)

$$\frac{A_4}{A}-\frac{B_4}{B}=0 \tag{11}$$

$$\rho_4+(1+\omega)\rho\left[\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right]+\delta\rho\left[\frac{A_4}{A}+\frac{B_4}{B}\right]=0$$

(12)

On integrating equation (11), we get

$$A=Bc_1 \tag{13}$$

Without loss of generality, by taking $c_1=1$, we have

$$A=B \tag{14}$$

By using equation (14), equations (7) to (10) and equation (12) becomes

$$\frac{A_{44}}{A}+\frac{C_{44}}{C}+\frac{A_4}{A}\frac{C_4}{C}+\frac{\omega}{2}\frac{\phi_4{}^2}{\phi^2}+\frac{\phi_{44}}{\phi}+\frac{\phi_4}{\phi}\left(\frac{A_4}{A}+\frac{C_4}{C}\right)=-8\pi\phi^{-1}(\omega+\delta)\rho$$

(15)

$$2\frac{A_{44}}{A}+\frac{A_4{}^2}{A^2}-\frac{1}{A^2}+\frac{\omega}{2}\frac{\phi_4{}^2}{\phi^2}+\frac{\phi_{44}}{\phi}+2\frac{\phi_4}{\phi}\frac{A_4}{A}=-8\pi\phi^{-1}(\omega+\delta)\rho$$

(16)

$$\frac{A_4{}^2}{A^2}+2\frac{A_4}{A}\frac{C_4}{C}-\frac{1}{A^2}-\frac{\omega}{2}\frac{\phi_4{}^2}{\phi^2}+\frac{\phi_4}{\phi}\left(2\frac{A_4}{A}+\frac{C_4}{C}\right)=8\pi\phi^{-1}\,\rho$$

(17)

$$\rho_4+(1+\omega)\rho\left[2\frac{A_4}{A}+\frac{C_4}{C}\right]+2\delta\,\rho\frac{A_4}{A}=0$$

(18)

Above equations are highly non-linearly containing $A, C, \omega, \phi, \delta$ and ρ six unknowns. To get deterministic solution, we use following plausible physical condition: the shear scalar σ is proportional to scalar expansion θ which leads to relationship between metric potential $A,$ and C i.e.

$$A = C^n \tag{19}$$

By using equation (19) equations (15) to (18) can be reduced to form

$$(n+1)\frac{C_{44}}{C} + n^2\frac{C_4{}^2}{C^2} + \frac{\omega}{2}\frac{\phi_4{}^2}{\phi^2} + \frac{\phi_{44}}{\phi} + (n+1)\frac{\phi_4}{\phi}\frac{C_4}{C} = -8\pi\phi^{-1}(\omega+\delta)\rho$$

(20)

$$2n\frac{C_{44}}{C} + (3n^2 - 2n)\frac{C_4{}^2}{C^2} - \frac{1}{C^{2n}} + \frac{\omega}{2}\frac{\phi_4{}^2}{\phi^2} + \frac{\phi_{44}}{\phi} + 2n\frac{\phi_4}{\phi}\frac{C_4}{C} = -8\pi\phi^{-1}\omega\rho$$

(21)

$$(n^2 + 2n)\frac{C_4{}^2}{C^2} - \frac{1}{C^{2n}} - \frac{\omega}{2}\frac{\phi_4{}^2}{\phi^2} + (2n+1)\frac{\phi_4}{\phi}\frac{C_4}{C} = 8\pi\phi^{-1}\rho$$

(22)

$$\rho_4 + (1+\omega)(2n+1)\rho\frac{C_4}{C} + 2\delta\rho n\frac{C_4}{C} = 0$$

(23)

We define mean Hubble parameter for the Bianchi type III ,

$$H = \frac{a_4}{a} = \frac{1}{3}\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right)$$

(24)

With a being the average scale factor of Bianchi type model and can be expressed as

$$a = (ABC)^{\frac{1}{3}} \tag{25}$$

The special Volume V is given by

$$V = a^3 = ABC = A^2C = C^{2n+1}$$

(26)

The Expansion Scalar θ , Shear scalar σ , the Mean isotropy parameter A_m are defined by

$$\theta = 3H = \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right)$$

(27)

$$\sigma^2 = \frac{3}{2}A_m H^2$$

(28)

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2$$

(29)

3. Solution of Field equation

To find deterministic solution use special form of deceleration parameter for FRW metric is

$$q = -\frac{aa_{44}}{a_4^{\,2}} \tag{30}$$

On integrating equation (30) we get

$$a = (\alpha t + \beta)^{\frac{1}{1+q}} \tag{31}$$

Where α and β are a constant and a is mean scale factor of the universe.

By using equation (14), (19) and (26)

$$A = (\alpha t + \beta)^{\frac{3n}{(1+q)(2n+1)}}$$
(32)

$$B = (\alpha t + \beta)^{\frac{3n}{(1+q)(2n+1)}}$$
(33)

$$C = (\alpha t + \beta)^{\frac{3}{(1+q)(2n+1)}}$$
(34)

Where $\alpha \neq 0$ and β are arbitrary constants and

$$\phi = (\alpha t + \beta)^r \tag{35}$$

Where $r = 0$ or $r = \frac{2[(2n+1)q - (n+2)]}{(2n+1)(1+q)(\omega+2)}$

Since Brans-Dicke theory has to go to Einstein theory in all respect when $r = 0$,

$\phi =$ constant, then r should be of the form

$$r = \frac{2[(2n+1)q - (n+2)]}{(2n+1)(1+q)(\omega+2)}$$
(36)

The metric (6) can be now be written as

$$ds^2 = dt^2 - (\alpha t + \beta)^{\frac{6n}{(1+q)(2n+1)}} dx^2 - (\alpha t + \beta)^{\frac{6n}{(1+q)(2n+1)}} e^{-2x} dy^2 - (\alpha t + \beta)^{\frac{6}{(1+q)(2n+1)}} dz^2$$
(37)

From equation (22), we get the string energy density

$$\rho=\frac{1}{8\pi\phi^{-1}}\left\{\begin{array}{l}\frac{9n(n+2)\alpha^{2}}{(2n+1)^{2}(1+q)^{2}(\alpha t+\beta)^{2}}-\frac{\omega\alpha^{2}r^{2}}{2(\alpha t+\beta)^{2}}\\+\frac{3r(2n+1)\alpha^{2}}{(2n+1)(1+q)(\alpha t+\beta)^{2}}-\frac{1}{(\alpha t+\beta)^{\frac{6n}{(2n+1)(1+q)}}}\end{array}\right\}$$

(38)

From equation (21),

$$\omega=-\left\{\frac{\begin{array}{l}\frac{3n\alpha^{2}[5n-2-2(2n+1)q]}{(2n+1)^{2}(1+q)^{2}(\alpha t+\beta)^{2}}-\frac{1}{(\alpha t+\beta)^{\frac{6n}{(2n+1)(1+q)}}}+\frac{\omega r^{2}\alpha^{2}}{2(\alpha t+\beta)^{2}}\\+\frac{r(r-1)\alpha^{2}}{(\alpha t+\beta)^{2}}+\frac{6nr\alpha^{2}}{(2n+1)(1+q)(\alpha t+\beta)^{2}}\end{array}}{\begin{array}{l}\frac{9n(n+2)\alpha^{2}}{(2n+1)^{2}(1+q)^{2}(\alpha t+\beta)^{2}}-\frac{\omega r^{2}\alpha^{2}}{2(\alpha t+\beta)^{2}}+\frac{3r(2n+1)\alpha^{2}}{(2n+1)(1+q)(\alpha t+\beta)^{2}}\\-\frac{1}{(\alpha t+\beta)^{\frac{6n}{(2n+1)(1+q)}}}\end{array}}\right\}$$

(39)

By using $\omega+\delta=0$, we have

$$\delta=\left\{\frac{\begin{array}{l}\frac{3n\alpha^{2}[5n-2-2(2n+1)q]}{(2n+1)^{2}(1+q)^{2}(\alpha t+\beta)^{2}}-\frac{1}{(\alpha t+\beta)^{\frac{6n}{(2n+1)(1+q)}}}+\frac{\omega r^{2}\alpha^{2}}{2(\alpha t+\beta)^{2}}\\+\frac{r(r-1)\alpha^{2}}{(\alpha t+\beta)^{2}}+\frac{6nr\alpha^{2}}{(2n+1)(1+q)(\alpha t+\beta)^{2}}\end{array}}{\begin{array}{l}\frac{9n(n+2)\alpha^{2}}{(2n+1)^{2}(1+q)^{2}(\alpha t+\beta)^{2}}-\frac{\omega r^{2}\alpha^{2}}{2(\alpha t+\beta)^{2}}+\frac{3r(2n+1)\alpha^{2}}{(2n+1)(1+q)(\alpha t+\beta)^{2}}\\-\frac{1}{(\alpha t+\beta)^{\frac{6n}{(2n+1)(1+q)}}}\end{array}}\right\}$$

(40)

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It is interesting to note that when $r=0$ and hence from (35), we get $\phi=$constant.

From (22), we get the string energy density

$$\rho=\frac{1}{8\pi\phi^{-1}}\left\{\frac{9n(n+2)\alpha^{2}}{(2n+1)^{2}(1+q)^{2}(\alpha t+\beta)^{2}}-\frac{1}{(\alpha t+\beta)^{\frac{6n}{(2n+1)(1+q)}}}\right\}$$

(41)

From equation (21),

$$\omega = -\left\{\frac{\dfrac{3n\alpha^2\left[5n-2-2(2n+1)q\right]}{(2n+1)^2(1+q)^2(\alpha t+\beta)^2}-\dfrac{1}{(\alpha t+\beta)^{\frac{6n}{(2n+1)(1+q)}}}}{\dfrac{9n(n+2)\alpha^2}{(2n+1)^2(1+q)^2(\alpha t+\beta)^2}-\dfrac{1}{(\alpha t+\beta)^{\frac{6n}{(2n+1)(1+q)}}}}\right\}$$

(42)

By using $\omega+\delta=0$, we have

$$\delta = \left\{\frac{\dfrac{3n\alpha^2\left[5n-2-2(2n+1)q\right]}{(2n+1)^2(1+q)^2(\alpha t+\beta)^2}-\dfrac{1}{(\alpha t+\beta)^{\frac{6n}{(2n+1)(1+q)}}}}{\dfrac{9n(n+2)\alpha^2}{(2n+1)^2(1+q)^2(\alpha t+\beta)^2}-\dfrac{1}{(\alpha t+\beta)^{\frac{6n}{(2n+1)(1+q)}}}}\right\}$$

(43)

4.Some Physical Properties:

The physical and kinematical parameter of the model, which are important for the discussion of cosmological model (3.12) are the following.
Spatial volume

$$V=(\alpha t+\beta)^{\frac{3}{(1+q)}}$$

(4.1)
Scalar of expansion

$$\theta=\frac{3\alpha}{(1+q)(\alpha t+\beta)}$$

(4.2)
The mean Hubble parameter

$$H=\frac{\alpha}{(1+q)(\alpha t+\beta)}$$

(4.3)
The average anisotropy parameter,

$$A_m=\frac{6n^2-8n+6}{3(2n+1)^2}$$

(4.5)
Shear scalar,

$$\sigma^2=\frac{(6n^2-8n+6)\alpha^2}{2(2n+1)^2(1+q)^2(\alpha t+\beta)^2}$$

(4.6)

5.Conclusion:

In this paper, we have presented Bianchi type III cosmological model with anisotropic dark energy in scalar tensor theory of gravitation proposed by Brans-Dicke and in general relativity. We observed that model (37) has no singularity at $t=-\dfrac{\beta}{\alpha}$ and for this point, the special volume will be zero, whereas when $t\to\infty$, the special volume will be infinitely large. At $t=-\dfrac{\beta}{\alpha}$, the expansion scalar, the mean Hubble parameter and shear scalar tends to infinity whereas when $t \to \infty$ the expansion scalar, the mean Hubble parameter and shear scalar tends to zero. Equation (4.6) perform that model is anisotropy.

References

1 Astier, P., *et al.*: *The supernova legacy survey: measurement of data set.* (2005) [arXiv:astro-ph/0510447]
2 Perlmutter, S., *et al.*: *[Supernova Cosmology Project Collaboration] Astrophys.* J. 517, 565 (1999) [arXiv:astro-ph/9812133].
3 Riess, A.G., *et al.*: [Supernova Search Team Collaboration] *Astron.* J. 116, 1009 (1998) [arXiv:astro-ph/9805201].
4 Riess, A.G., *et al.*: *Astrophys.* J. 607, 665 (2005) [arXiv:astro-ph/0402512].
5 Bennett, C.L., *et al.: Astrophys. J. Suppl.* 148, 1 (2003) [arXiv:astro-ph/0302207].
6 Bennett, C.L., *et al.: Astrophys. J. Suppl.* 608, 10 (2004) [arXiv:astro-ph/0305097].
7 Bemui, A., Mota, B., Reboucas, M., Tavakol, R.: *Mapping the large-scale anisotropy in WMAP data* (2006) [arXiv:astro—ph/0309003].
8 Spergel, D.N., *et al.: [WMAP Collaboration] Astrophys. J. Suppl.* 148, 175 (2003) [arXiv:astro-ph/0302209].
9 Abazajian, K., *et al.: [SDSS Collaboration], Astron. J.* 126, 2081 (2003) [arXiv:astro-ph/0305492].
10 Abazajian, K., *et al.: [SDSS Collaboration], Astron. J.* 128, 502 (2004) [arXiv:astro-ph/0403325].
11 Abazajian, K., *et al.*: (2004) [arXiv:astro-ph/0410239].
12 Hawkins, E., *et al.: Mon. Not. R. Astron. Soc.* 346, 78 (2003) [arXiv:astro-ph/0212375].
13 Tegmark, M., *et al.: [SDSS Collaboration] Phys. Rev. D* 69, 103501 (2004) [arXiv:astro-ph/0310723].
14 Verde, L., *et al.: Mon. Not. R. Astron. Soc.* 335, 432 (2002) [arXiv:astro-ph/0112161].
15 Caldwell, R.R., Dave, R., Steinhardt, P.J.: *Phys. Rev. Lett.* 80, 1582 (1998) [arXiv:astro-ph/9708069].
16 Frieman, J.A., Hill, C.T., Stebbins, A., Waga, 1.: *Phys. Rev. Lett.* 75, 2077 (1995) [arXiv:astro-ph/9505060].
17 Liddle, A.R., Scherrer, R.J.: *Phys. Rev. D* 59, 023509 (1999) [arXiv:astro-ph/9809272].
18 Peebles, P.J.E., Ratra, B.: *Astrophys. J.* 325, L17 (1988).
19 Ratra, B., Peebles, P.J.E.: *Phys. Rev.* D 37, 3406 (1988).
20 Steinhardt, P.J., Wang, L.M., Zlatev, 1.: *Phys. Rev.* D 59, 123504 (1999) [arXiv:astro-ph/9812313].
21 Turner, M.S., White, M.J.: *Phys. Rev.* D 56, 4439 (1997) [arXiv:astro-ph/9701138].
22 Wetterich,C.:*Nucl.Phys.*B302,668(1988).
23 Kamenshchik, A., Moschella, U., Pasquier, V.: *Phys. Lett.* B 511, 265 (2001) [arXiv:gr-qc/0103004].
24 Caldwell, R.R.: *Phys. Lett.* B 545, 23 (2002) [arXiv:astro-ph/9908168].
25 Cai, R.-G., Wang, A.: *JCAP* 03, 002 (2005) [arXiv:hep-th/0411025].
26 Sahni, V.: *Cosmological surprises from Braneworld models of dark energy.* (2005) [arXiv:astro-ph/04502032].
27 Yang, G., Wang, A.: *Gen. Relat. Grav.* 37, 2201 (2005) [arXiv:asLro—ph/0510006]
28 Sahni V.: *Dark matter and dark energy.* (2004) [arXiv:astro-ph/0403324].
29 Sahni, V.: *Cosmological surprises from Braneworld models of dark energy.* (2005) [arXiv:astro-ph/04502032].
30 Carroll, S.M.: *Living Rev. Relat.* 4, 1 (2001) [arXiv:astro-ph/0004075].
31 Padmanabhan, T.: *Phys. Rep.* 380, 235 (2003).
32 Peebles, P.] Ratra, B.: *Rev. Mod. Phys.* 75, 559 (2002) [arXiv:astro-ph/0207347].
33 Waller, S.M., (1984) *Phys. Rev.* D. 29, 2, 176-185.
34 Bali, R & Dave S., *Pramana J. of physics*, 56, 4, (2001).
35 Bali, R & Kumawat P, 2010 *EJTP* 7, 24 , 383-394.
36 Tyagi,A & Sharma K (2010) *Chin. Phys. Lett.*, Vol 27, No 7079801.
37 Bali Raj, Yadav, M.K.: *Pramana J. Phys.* 64(2), 187 (2005).
38 Reddy, D.R.K., Patrudu, B.M., Venkateswarlu, R.: *Astrophys. Space Sci.* 204, 155 (1993).
39 Shanthi, K., Rao, V.U.M.: *Astrophys. Space Sci.* 179, 163 (1991).
40 Rao, V.U.M., Sanyasi Raju, Y.V.S.S.: *Astrophys. Space Sci.* 187, 113 (1992).
41 Rao, V.U.M, Sanyasi Raju, Y.V.S.S.: *Astrophys. Space Sci*. 189, 39 (1992).
42 Rahaman, F., Chakraborthy, S., Begum, N., Hossian, M., Kalam, M.: *Pramana J. Phys.* 60, 1153 (2003).
43 Sen, D.K.: *Z. Phys.* 149, 311 (1957).
44 Lyra, G.:*Math. Z.* 54, 52 (1951).
45 Rao, V.U.M., Vijaya Santhi, M., Vinutha, T.: *Astrophys. Space Sci.* 314, 73 (2008c).

46 Rao, V.U.M., Vijaya Santhi, M., Vinutha, T.: *Astrophys. Space Sci.* 317, 27 (2008d).
47 Rao, V.U.M., Vijaya Santhi, M., Vinutha, T.: *Astrophys. Space Sci*. 317, 83 (2008e).
48 Brans, C.H., Dicke, R.H.: *Phys. Rev.* 124, 925 (1961).
49 Nariai, H.: *Prog. Theor. Phys.* 47, 1824 (1972).
50 Belinskii, V.A., Khalatnikov, I.M.: *Sov. Phys. JETP* 36, 591 (1973).
51 Reddy, D.R.K., Rao, V.U.M.: *J. Phys. A, Math. Gen.* 14, 1973 (1981).
52 Banerjee, A., Santos, N.O.: *Nuovo Cimento* 67B, 31 (1982).
53 Shri Ram: *Gen. Relativ. Gravit.* 15, 635 (1983).
54 Shri Ram, Singh, D.K.: *Astrophys. Space Sci.* 103, 21 (1984).
55 Berman, M.S., et al.: *Gen. Relativ. Gravit.* 21, 287 (1989).
56 Reddy, D.R.K.: *Astrophys. Space Sci.* 286, 359 (2003).
57 Adhav, K.S., Nimkar, A.S., Ugale, M.R., Dawande, M.V.: *Astrophys. Space Sci.* 310, 231 (2007).
58 Rao, V.U.M., Vinutha, T., Vijaya Santhi, M., Sireesha, K.V.S.: *Astrophys. Space Sci.* 315, 211 (2008a).
59 Rao, V.U.M., Vinutha, T., Vijaya Santhi, M.: *Astrophys. Space Sci.* 314, 213 (2008b).